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Algebraic Geometry III/IV

Problems, set 1.

Exercise 1. Prove the following fact: Let P_0, P_1, P_2, P_3 be four points in $\mathbb{P}^2_{\mathbb{C}}$ such that no three of them lie on a common projective line. Then there exists a projective transformation $f : \mathbb{P}^2_{\mathbb{C}} \to \mathbb{P}^2_{\mathbb{C}}$ such that $f(P_0) = [1, 0, 0]$, $f(P_1) = [0, 1, 0]$, $f(P_2) = [0, 0, 1]$ and $f(P_3) = [1, 1, 1]$. In fact, this projective transformation is unique, but you do not need to show this.

Exercise 2. Let $P_1, \ldots, P_5$ be five different points in $\mathbb{P}^2_{\mathbb{C}}$. Prove the following facts:

(a) If no three of these points lie on a common projective line, then there is a unique conic C, containing all five points. Moreover, C is irreducible. You may use Exercise 1 for the proof.

(b) If P_1, P_2, P_3 lie on a common projective line L, but P_4, P_5 do not lie on L, then there is also a unique conic C, containing all five points. This time, C is reducible.

(c) If P_1, P_2, P_3, P_4 lie on a common projective line L, then there are infinitely many conics containing all five points. All these conics are reducible.