EPSRC Workshop 2016 “Geometry and Computation on Groups and Complexes”

Newcastle, 6-10 June 2016

1 Schedule

General Information: All long talks are 50 minutes followed by 10 minutes for questions. They take place in LT1, HERB. Tea/Coffee breaks will be in Foyer HERB. The Reception takes place in the Foyer of HERB. Conference Dinner will take place in the Penthouse, HERB.

Monday, 6 June

08.30 - 09.30 Registration (Foyer HERSCHEL building)

09.30 - 10.30 A. Valette, *Expanders and box spaces*

10.30 - 11.00 Tea/Coffee

11.00 - 12.00 A. Khukhro, *Geometry of finite quotients of groups*

12.00 - 12.10 Break

12.10 - 13.10 S. Hermiller, *Trees, flow functions, and algorithms in groups*

13.10 - 14.30 Lunch and Registration (Foyer HERB)

14.30 - 15.30 Sh. Liu, *Higher order Buser inequalities for the graph connection Laplacian*

15.30 - 16.00 Tea/Coffee

16.00 - 17.00 L. Ciobanu, *On conjugacy growth in groups*

17.00 - 17.15 Break

17.15 - 18.15 V. Shpilrain, *Mysteries of* 2×2 *matrices*

18.15 - Reception (Foyer HERB)

Tuesday, 7 June

09.00 - 10.00 J. Brodzki, *Amenable actions of locally compact second countable groups*

10.00 - 10.20 Tea/Coffee

10.20 - 11.20 J. Jost, *On the curvature of graphs*

11.20 - 11.30 Break

11.30 - 12.30 A.Yu. Olshanskii, *On the growth of subgroups in finitely generated groups*

12.30 - 13.30 Lunch (Foyer HERB)

13.30 - 14.30 A. Iozzi, *Irreducible lattices and bounded cohomology*

14.30 - 15.00 Tea/Coffee

15.00 - 16.00 L. Bartholdi, *Decision problems in self-similar groups*

16.00 - 19.00 Poster Session (HERB 3.19); Discussions in separate rooms (HERB TR1,3,4)

19.00 - Conference Dinner (Penthouse)

Wednesday, 8 June

09.00 - 9.40 R. Kangaslampi, *Hyperbolic triangular buildings and periodic apartments*

09.40 - 09.50 Break

09.50 - 10.30 G. Williams, *Fibonacci-type groups and 3-manifolds*

10.30 - 11.00 Tea/Coffee

11.00 - 11.30 N. Radu, *A locally non-Desarguesian $\tilde{A}_2$-building admitting a uniform lattice*

11.30 - 11.40 Break

11.40 - 12.10 N. Runganapirom, *Quaternionic lattice of rank 2 over characteristic 2 with small quotient square complex*

12.30 - Free afternoon / Excursion to Beamish Museum

Thursday, 9 June

09.00 - 10.00 F. Chouraqui, *Knuth-Bendix algorithm and the conjugacy problems in monoids*

10.00 - 10.20 Tea/Coffee

10.20 - 11.20 M. Farber, *Topology of large random spaces*

11.20 - 11.30 Break

11.30 - 12.30 T. Nagnibeda, *Subgroups in branch groups*

12.30 - 13.30 Lunch (Foyer HERB)

13.30 - 14.30 D. Witte Morris, *Horospherical limit points of locally symmetric spaces*

14.30 - 14.40 Break

14.40 - 15.40 N. Boston, *Arboreal Galois Representations*

15.40 - 16.10 Tea/Coffee

16.10 - 17.10 I. Bumagin, *Makanin-Razborov diagrams over relatively hyperbolic groups*

17.10 - 17.20 Break

17.20 - 18.20 M. Sapir, *Subgroups of R. Thompson group F*

Friday, 10 June

09.00 - 10.00 D. Holt, *A new algorithmic approach to proving that groups are hyperbolic*

10.00 - 10.20 Tea/Coffee

10.20 - 11.20 D. Cartwright, *Some lattice subgroups of* $PU(2,1)$

11.20 - 11.30 Break

11.30 - 12.30 C. Lange, *Cayley graphs of finite Coxeter groups, Colin de Verdiere matrices and convex polytopes*

12.30 End of Workshop

2 Abstracts

Laurent Bartholdi (ENS, Paris, France): Decision problems in self-similar groups

Self-similar groups are finitely generated groups "presented" by a recursion, rather than by generators and relators. I will give important examples, survey classical decision problems (word problem, finiteness, conjugacy problem, order problem) and describe a partial solution to the Engel problem, describing in particular all ad-nilpotent elements (i.e. all y with $[x, y, ..., y] = 1$ for all x and all long enough commutators) in the Grigorchuk group.

Nigel Boston (Wisconsin-Madison, USA): Arboreal Galois Representations

The study of p-adic Galois representations has reaped huge rewards in number theory and arithmetic geometry. Galois groups also naturally act on the infinite binary rooted tree and we are developing a parallel theory for this case. I shall describe the general context and then go into detail regarding what is known and expected as regards the images of these representations and of special elements, focusing on a case involving the Basilica group.

Jacek Brodzki (Southampton, UK): Amenable actions of locally compact second countable groups

The study of amenability of groups has been a very active area of research pretty much since von Neumann introduced this notion in 1922. Among the many weaker notions introduced over the years, exactness has gained a prominent place and has generated a lot of interest. In the world of discrete groups, there is a deep connection between exactness and the so called amenability at infinity, but the transition to the locally compact case has been difficult. In this talk I will describe how exactness in the original sense of Kirchberg-Wassermann can be described in terms of amenable actions. In particular, I will show that for a general locally compact second countable group amenability at infinity is equivalent to the exactness of the group, understood as the exactness of the reduced crossed product functor. This is joint work with Chris Cave and Kang Li.

Inna Bumagin (Carleton, Canada): Makanin-Razborov diagrams over relatively hyperbolic groups

Let G be a finitely generated relatively hyperbolic group. Our goal is to give a description of the set of homomorphisms $\mathrm{Hom}(L, G)$ from a finitely generated G-limit group L to G in terms of a finite directed rooted tree. The vertices of the tree correspond to G-limit quotients of L and the edges correspond to epimorphisms. This is joint work with Nicholas Touikan.

Donald Cartwright (Sydney, Australia): Some lattice subgroups of $PU(2,1)$

A fake projective plane (”fpp”) is a compact complex surface with the same Betti numbers as the complex projective plane $\mathbf{P}(\mathbb{C}^2)$, but which is not homeomorphic to $\mathbf{P}(\mathbb{C}^2)$. In an important paper, Prasad and Yeung divided the fpp's into a small number of ”classes” and found an fpp in most of these classes. Subsequently, Tim Steger and I enumerated the fpp's, finding all 50 of them, by finding all fpp's in each class. Part of this work involved eliminating several“matrix algebra cases” left open by Prasad and Yeung. This involved a detailed study of altogether thirteen explicit lattice subgroups of $PU(2,1)$, and torsion-free subgroups thereof. While none of these lattice subgroups gives rise to a fake projective plane, one of them gives a new compact complex surface with interesting properties. Most of the material today is a development of work with Tim Steger, but I shall also be mentioning more recent work with Sai-Kee Yeung and Vincent Koziarz.

Fabienne Chouraqui (Haifa, Israel): Knuth-Bendix algorithm and the conjugacy problems in monoids

The use of string rewriting systems has been proved to be a very efficient tool to solve the word problem. A question that arises naturally is whether the use of rewriting systems may be an efficient tool for solving other decision problems, specifically the conjugacy problem. The complexity of this question is due to some facts, one point is that for monoids the conjugacy problem and the word problem are independent one of another and another point is that in semigroups and monoids, there are several different notions of conjugacy that are not equivalent in general. We present an algorithmic approach to the conjugacy problems in monoids, using rewriting systems. We extend the classical theory of rewriting systems developed by Knuth and Bendix to a rewriting that takes into account the cyclic conjugates.

Laura Ciobanu (Neuchâtel, Switzerland): On conjugacy growth in groups

In this talk I will discuss various aspects of conjugacy growth in several classes of groups, such as hyperbolic, right-angled Artin, Baumslag-Solitar or wreath products.

Michael Farber (Queen Mary, UK): Topology of large random spaces

I will describe several models producing large random topological spaces, I will also present some recent results about topological properties of such spaces (their Betti numbers, fundamental groups etc).

Susan Hermiller (Nebraska, USA): Trees, flow functions, and algorithms in groups

A bounded flow function is a dynamical system on the Cayley complex of a finitely presented group mapping the set of paths into itself, such that path lengths increase in a bounded way and iteration eventually maps every path into a fixed maximal tree. Although a flow function does not imply solvability of the word problem, if the function can be computed by a finite state automaton (FSA), the group is called autostackable and the FSA can be used to solve the word problem for the group. In this talk I'll discuss autostackability for closed 3-manifold groups and relatively hyperbolic groups. This includes joint work with Mark Brittenham, Conchita Martinez-Perez, and Tim Susse.

Derek Holt (Warwick, UK): A new algorithmic approach to proving that groups are hyperbolic

We describe new algorithms developed by Richard Parker, Colva Roney-Dougal, Max Neunhoeffer, Steve Linton and others for verifying that a group defined by a finite presentation is word-hyperbolic. They are based on curvature arguments applied to van Kampen diagrams, and generalise methods that have been applied to groups that satisfy small cancellation conditions. When successful, they calculate an upper bound for the Dehn function of the presentation, which in turn results in an upper bound for the slim or thin triangles constant of the Cayley graph.

In many examples it is possible to perform the calculations by hand. This means that we can sometimes apply them to infinite families of examples, which is an advantage over other algorithms for proving hyperbolicity, such as the one in the speaker's KBMAG package, which can only be applied to individual groups.

We also report on progress on an implementation of these algorithms in GAP.

Alessandra Iozzi (ETH Zürich, Switzerland): Irreducible lattices and bounded cohomology

We show some of the similarities and some of the differences between irreducible lattices in product of semisimple Lie groups and their siblings in product of locally compact groups. In the case of product of trees, we give a concrete example with interesting properties, among which some in terms of bounded cohomology and quasimorphisms.

Jürgen Jost (MPI MIS, Leipzig, Germany): On the curvature of graphs

I shall describe some concepts for the geometric analysis of graphs that were originally motivated by Riemannian geometry and which provides insight into several properties of graphs that are also useful for the analysis of empirical networks.

Riikka Kangaslampi (Aalto, Finland): Hyperbolic triangular buildings and periodic apartments

This talk is motivated by Gromov's famous surface subgroup question: Does every one-ended hyperbolic group contain a subgroup which is isomorphic to the fundamental group of a closed surface of genus at least 2?

We study groups acting simply transitively on the vertices of hyperbolic triangular buildings of the smallest non-trivial thickness. The existing examples of subgroups of groups acting on buildings arise from periodic apartments. With the help of computerized search we show, that most of the buildings we consider do no have any apartments invariant under genus 2 orientable surface group action. The existence of such an action would imply the existence of a surface subgroup, but it is not known, whether the existence of a surface subgroup implies the existence of a periodic apartment. Thus, these groups are the first candidates for groups that do not have surface subgroups arising from periodic apartments.

This is joint work with Alina Vdovina.

Ana Khukhro (Neuchâtel, Switzerland): Geometry of finite quotients of groups

The study of geometric properties of Cayley graphs of groups is known to be extremely fruitful, often providing strong structural results for the group. When the group is rich in finite quotients, it makes sense to look at the geometry of the finite Cayley graphs of these quotients, since they encode both geometric and algebraic information about the group. Studying the connections between the geometric properties that these quotients have uniformly, and the algebraic or analytic properties of the parent group is not only intriguing from a group-theoretic point of view, but can also provide us with examples of metric spaces with interesting properties. We will focus mainly on the relationship between the diameters and the sizes of finite quotients, ending with some open problems.

Carsten Lange (TU Munich, Germany): Cayley gaphs of finite Coxeter groups, Colin de Verdiere matrices and convex polytopes

In 2001, Lovasz obtained an explicit description how certain weighted adjacency matrices of a 3-connected planar graph G relate to realizations of 3-dimensional (convex) polytopes with a vertex-edge graph isomorphic to G. The matrices involved are known as Colin de Verdiere matrices and they link to realizations of polytopes by eigenvectors of the second smallest eigenvalue. In 2013, Ivrissimtzis and Peyerimhoff studied spectral properties of transition matrices on vertex-transitive graphs in general and, in the special case of Cayley graphs associated to Coxeter groups in type A_3, B_3 and H_3, they determined the multiplicity of the second largest eigenvalue of these transition matrices by methods also used by Lovasz and gave a geometric interpretation in terms of Archimedean solids.

In this talk, I review the results mentioned above and extend the results of Lovasz and of Ivrissimtzis and Peyerimhoff to Cayley graphs of finite Coxeter groups and associated polytopes. This presentation is based on joint work with Ivrissimtzis, Liu and Peyerimhoff.

Shiping Liu (Durham, UK): Higher order Buser inequalities for the graph connection Laplacian

In this talk, I will discuss upper bounds for eigenvalues of the connection Laplacian on a graph with an orthogonal group or unitary group signature. Those upper bounds are given in terms of Cheeger type constants in the case of nonnegative Ricci curvature. This can be considered as higher order Buser type inequalities.

This is joint work with Florentin Münch and Norbert Peyerimhoff.

Tatiana Nagnibeda (Geneva, Switzerland): Subgroups in branch groups

The subgroup structure is one of the most basic questions in group theory.

Branch groups can be roughly described as groups whose lattice of subgroups contains a homogeneous rooted tree. They hence have a rich subgroup structure.

We will discuss some of its aspects, in particular maximal and weakly maximal subgroups and subgroup separability.

A.Yu. Olshanskii (Vanderbilt, USA, and Moscow State University, Russia): On the growth of subgroups in finitely generated groups

The growth function of a subgroup H in a group G generated by a finite set A is defined by the formula $f(n) = card\{h \in H; |h|_A \leq n\}$, where $|g|_A$ is the length of $g \in G$ with respect to A. I will discuss the behavior of growth functions in arbitrary finitely generated groups and then focus on the growth (and co-growth) functions of subgroups, in particular subnormal subgroups, with respect to a free basis of a free group.

Nicolas Radu (Louvain, Belgium): A locally non-Desarguesian $\tilde{A}_2$-building admitting a uniform lattice

An $\tilde{A}_2$-building is a simply connected simplicial complex of dimension 2 such that each sphere of radius 1 centered at a vertex is isomorphic to the incidence graph of a projective plane. In 1986, Kantor asked the problem of constructing an $\tilde{A}_2$-building with a cocompact lattice and whose local projective planes are finite and non-Desarguesian. In this talk, I will explain how this problem could be solved thanks to the work of Cartwright-Mantero-Steger-Zappa (1993) and by making use of a computer.

Nithi Runganapirom (Frankfurt, Germany): Quaternionic lattice of rank 2 over characteristic 2 with small quotient square complex

Although there are plenty of quaternionic arithmetic lattices of rank 2, only few of them yields a square complex as the quotient of its action on the product of Bruhat-Tits trees with minimal number of vertices. On the other hand, the fundamental group of a square complex rarely has an arithmetic origin. In this talk Im going to give a construction of such a lattice over characteristic 2. A presentation of such a lattice can be explicitly determined by comparing its action on the product of Bruhat-Tits trees with the action of the corresponding (orbital) fundamental group on the product of trees as the universal covering of the corresponding square complex. This lattice can be used to construct a non-classical fake quadric in characteristic 2.

Mark Sapir (Vanderbilt, USA): Subgroups of R. Thompson group F

Subgroups of F which have been studied for more than 25 years are still very mysterious. For example, a seemingly random collection of elements may generate a maximal subgroup of F or even the whole F. I will talk about our joint work with Gili Golan and her own work which somewhat clarifies the situation.

Vladimir Shpilrain (CUNY, USA): Mysteries of 2×2 matrices

We consider some special 2-generator groups and semigroups of 2×2 matrices over $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{Z}_p$ and address various relevant algorithmic problems and their complexity. The talk is based on joint work with Lisa Bromberg, Anastasiia Chorna, Katherine Geller, and Alina Vdovina.

Alain Valette (Neuchâtel, Switzerland): Expanders and box spaces

Expanders, especially those coming from box spaces of residually finite groups, have been used to test various forms of the coarse Baum-Connes conjecture. The first construction of a pair of expanders, one not coarsely embedding in the other, was provided by Mendel and Naor in 2012. This was extended by Hume in 2014 who constructed a continuum of expanders with unbounded girth, pairwise not coarsely equivalent. In joint work with A. Khukhro, we construct a continuum of expanders with geometric property (T) of Willett-Yu, as box spaces of $SL(3, \mathbb{Z})$. We will discuss the following results: if box spaces of groups G, H are coarsely equivalent, then the groups G, H are quasi-isometric (Khukhro and myself), and moreover G and H are uniformly measure equivalent (K. Das).

Gerald Williams (Essex, UK): Fibonacci-type groups and 3-manifolds

The *Fibonacci groups* $F(2, n)$ are the groups defined by the presentations with generators $x_0, \ldots, x_{n-1}$ and relations $x_i x_{i+1} = x_{i+2}$ (subscripts mod n). Replacing the relations by $x_i x_{i+2} = x_{i+1}$ we obtain the *Sieradski groups* $S(2, n)$. By constructing a suitable face-pairing polyhedron that satisfies the Seifert-Threlfall condition, Sieradski proved that each $S(2, n)$ is a 3-manifold group. Similarly, for each even n, the group $F(2, n)$ is a 3-manifold group (Hilden, Lozano, Montesinos-Amilibia; Helling, Kim, Mennicke; Cavicchioli, Spaggiari; Howie).

The groups of Fibonacci-type $G_n(m,k)$ are defined by presentations with generators $x_0, \ldots, x_{n-1}$ and relations $x_i x_{i+m} = x_{i+k}$, and so generalize $F(2,n)$ and $S(2,n)$. With the exception of two challenging groups, we classify when the group $G_n(m,k)$ is a 3-manifold group. Spoiler alert: only Fibonacci groups $F(2,2m)$, Sieradski groups $S(2,n)$, and cyclic groups can arise.

This is recent joint work with Jim Howie.

Dave Witte Morris (Lethbridge, Canada): Horospherical limit points of locally symmetric spaces

Fix a point x in the symmetric space X associated to $SL(n,\mathbb{R})$. A point z on the visual boundary of X is a "horospherical limit point" if the $SL(n,\mathbb{Z})$-orbit of x intersects every horoball based at z. In the special case of the upper half-plane model of X for $n=2$, it is well known that the horospherical limit points are precisely the irrational numbers on the real line. For larger n, it was proved by T. Hattori that every horospherical limit point satisfies a certain irrationality property. We prove the converse, by applying a special case of Ratner's Theorem on unipotent flows that was established by S.G. Dani. Furthermore, $SL(n,\mathbb{R})$ can be replaced with any semisimple Lie group and $SL(n,\mathbb{Z})$ can be replaced with any S-arithmetic subgroup, if we replace X with the corresponding Bruhat-Tits building. This is joint work with G. Avramidi and K. Wortman.

3 Sponsors

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