a) Draw the fundamental domain of the torus, T_2, defined by

$$\begin{aligned} z &\equiv z+1 \\ z &\equiv z+i. \end{aligned}$$

T_2 is modded out by the Z_4 group generated by rotations through $\pi/2$. Draw the remaining fundamental domain including fixed points and identified edges.

b) What is the deficit angle around one of the fixed points?

c) T_2 can instead be modded out by $Z_2 \times Z_2$, where the first factor is a rotation through π (i.e. $z \to -z$) and the second is a reflection in $x = Re(z)$. Show that the fundamental domain is the same as the Z_4 case. (Hint - begin with the fundamental domain of the first Z_2 and show how it is reduced by the reflections in x.)

d) Why might you have expected this?

e) A realistic 4-D heterotic orbifold model can be constructed by a Z_4 modding of the 6 additional dimensions compactified on a torus, $T_6 = T_2 \times T_2 \times T_2$. Writing the 6 compactified dimensions as 3 complex ones, the Z_4 twist consists of the identification

$$\{\psi_j, z_j\} \to e^{iv_j}\{\psi_j, z_j\}$$

where $v_j = \frac{1}{4}(1, 1, -2)$ and ψ_j are the superpartners of the z_j.

The spectrum contains the massless gravitino states, $\Psi^{\mu=0..3}$ where

$$\Psi^\mu = b_{1,0}^{n_1} b_{2,0}^{n_2} b_{3,0}^{n_3} |0\rangle_L \times \alpha_{-1}^\mu |0\rangle_R$$

$b_{j,0}$ are zeroth modes of ψ_j and $n_j = 0$ or 1. Show that invariance under the Z_4 twist leaves only 1/4 of the original gravitinos (i.e. the model has N=1 supersymmetry).

f) Show that the Z_2 orbifold defined by $z_{1,2} \to -z_{1,2}$ has N=2 supersymmetry.