a) Draw the fundamental domain of the torus, T_2, defined by the identifications

$$\begin{aligned} z \equiv & \quad z+1 \\ z \equiv & \quad z+e^{\pi i/3}. \end{aligned}$$

T_2 is modded out by the Z_3 group generated by rotations through $2\pi/3$. Draw the remaining fundamental domain including fixed points and identified edges. (Hint - begin with the T_2 domain and show how combinations of the above identifications, and Z_3 rotations map some regions into others.)

b) What fraction of the area of the T_2 fundamental domain remains?

c) What is the *deficit* angle around one of the fixed points? (i.e. a vector parrallel transported around the fixed point rotates through ($2\pi-$the deficit angle))

d) A realistic 4-D heterotic orbifold model can be constructed by a Z_3 modding of the 6 additional dimensions compactified on a torus, $T_6 = T_2 \times T_2 \times T_2$. Writing the 6 compactified dimensions as 3 complex ones, the Z_3 twist consists of the identification

$$\{\psi_j, z_j\} \to e^{iv_j}\{\psi_j, z_j\}$$

where $v_j = \frac{1}{3}(1,1,-2)$ and ψ_j are the superpartners of the z_j.

The spectrum contains the massless gravitino states, $\Psi^{\mu=0..3}$ where

$$\Psi^\mu = b_{1,0}^{n_1} b_{2,0}^{n_2} b_{3,0}^{n_3} |0\rangle_L \times \alpha_{-1}^\mu |0\rangle_R$$

$b_{j,0}$ are zeroth modes of ψ_j and $n_j = 0$ or 1. Show that invariance under the Z_3 twist leaves only 1/4 of the original gravitinos (i.e. the model has N=1 supersymmetry).

e) Show that the Z_2 orbifold defined by $\mathrm{z}_{1,2} \to -z_{1,2}$ instead has N=2 supersymmetry.