SUSY and the MSSM for virgins

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Abstract

This is a pared down introduction to supersymmetry intended for an audience of non-experts taken from lectures given in St Andrews in 2001. I begin by giving a pedagogical review of supersymmetry and the MSSM, and discuss some aspects of flavour and CP phenomenology.

1 Introduction

This set of notes is extracted from lectures given in St Andrews in the Summer of 2001, where the brief was to introduce some of the ideas and challenges for supersymmetry coming from flavour and CP, aimed at an audience that includes 'non-experts'.[1] It is usual for courses on this subject to jump in at the superpotential for the Minimal Supersymmetric Standard Model, and leave the reader to sort out for him or herself what the superpotential actually means. In these notes I instead decided to begin at square one, assuming no prior knowledge other than field theory of scalars and fermions. While it might seem a challenge to get from this point to the MSSM in a just page or two of working, this is in fact possible (and only slightly painful) if we do not spend too long worrying about things like 'superspace', which although quite fundamental, do not really have much bearing on the question of phenomenology.[2] Thus the first section will develop the idea of a superpotential by giving a quick and dirty demonstration of how to 'supersymmetrize' a fundamental scalar. By following the (very explicit) steps in this

[1] i.e. people whose immediate interest may not be supersymmetry, and who can therefore be relied upon to examine the grandiose claims sometimes made by the subject with unnerving rigour

[2] This first part of the course was developed with the help of Jeff Forshaw whilst we were both in RAL, to whom I am indebted.

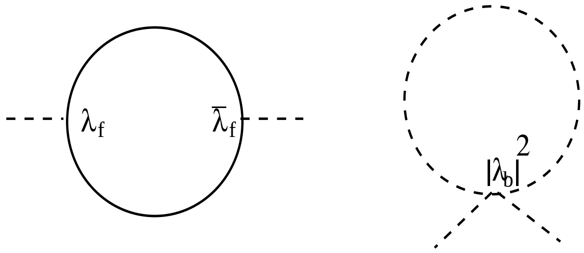
part, those readers who are unfamiliar with supersymmetry can get a grasp of the various concepts involved. In particular we will see first hand the famous cancellation of divergences that makes supersymmetry such an important idea. This will lead naturally to the MSSM superpotential and from there we can go on to analyse the particular problems that flavour and CP throw up for supersymmetry.

The interests of a St Andrews audience are typically rather broad, so the notes are organized as follows. The reader who requires a brief overview of the subject will be able to follow the main body of the text which is self contained. However, since some of the audience may want to develop a deeper knowledge of the subject, I have made copious use of appendices to include more detailed discussions of various aspects as we go along. From these it should be possible for the reader to follow some of the more formal texts in ref.[1].

Why supersymmetry?

Although there are several responses to this question, the most important is undoubtedly cancellation of divergences. This is a remarkable feature of supersymmetric models that gives stability to any theory, specifically stability of mass scales. We shall shortly derive it in detail, but in brief it goes as follows.

Consider a theory involving scalars with self coupling λ_b (which for the sake of argument I shall call Higgses) and also fermions which couple to the Higgses in Yukawa couplings with strength λ_f.



At one loop order, the Higgs potential will get a divergent contribution from the diagrams shown in the figure, which is of the form

$$\delta V_{higgs} \sim (|\lambda_b|^2 - |\lambda_f|^2) H^2 \Lambda^2.$$

The fermionic loop gets an extra minus sign. This type of contribution is a disaster for electroweak symmetry breaking, since we will have to tune our λ_b and λ_f couplings very precisely (to at least one part in 10^{14} if we take $\Lambda > M_{GUT} \sim 10^{16}$GeV) if the physical Higgs mass and hence weak gauge bosons are to end up around the 100GeV mark. Moreover, the λ couplings will themselves have logarithmic radiative corrections from one-loop diagrams, and in fact we need to retune at each order in perturbation theory. Finally the low energy theory will be sensitive to unknown high energy physics - for example an unknown GUT theory or string theory. The problems introduced by the existence of vastly different mass scales are referred to as the *gauge-hierarchy* problem. If you believe that the higgs fields are the result of some non-perturbative process (a top quark condensate perhaps) happening just above the weak scale, or that there are no new scales above M_W, or more generally, that there are no fundamental scalars, then this will not be a problem for you.

Supersymmetry solves these technical problems by ensuring that $\lambda_f = \lambda_b$, and in fact all contributions to the effective potential then vanish. The only renormalisation of the bare couplings which takes place comes from field renormalisations. The price we have to pay for such a useful cancellation is high; we have to introduce a fermionic(bosonic) degree of freedom for every boson(fermion) (known collectively as *superpartners*). The general form of the couplings will be

$$\lambda(\mu_2) \sim \lambda(\mu_1) \log \frac{\mu_1^2}{\mu_2^2}. \tag{1}$$

Now couplings which start off small at the GUT scale remain small. (This holds to all orders in perturbation theory.)

This then is the main reason for taking supersymmetry seriously. Other reasons (in descending order of compulsion) are that the gauge couplings of the supersymmetric standard model (MSSM) appear to exactly unify at some (presumably GUT) scale, or that the only known theory of quantum gravity (*i.e.* string theory) strongly suggests it, or that the cosmological constant is zero. This last reason is probably invalid, since supersymmetry must be broken at some scale, reintroducing a cosmological constant (albeit one which is only out by 52 orders of magnitude instead of 120). One additional interesting hint for extra scalars comes from the fact that for relatively low mass Higgses (around 115 GeV say) the Standard Model vacuum is unstable to decay, as shown in for example ref.[2] and references therein, because the top mass is large. This instability can be removed if there are more scalars than just the Higgs fields.

2 Express Supersymmetry

Our aim in this section is to derive (or at least explain the form of) supersymmetric interactions from their non-supersymmetric counterparts (SM). We begin in this subsection by supersymmetrising a simple lagrangian describing a massless, free boson. We will then discuss how to write down interactions using the superpotential and examine the top quark Yukawa coupling in detail, in particular verifying the cancellation discussed above. Readers already familiar with supersymmetry can skip to section 3. Readers who would like to see the more detailed conventional approach can consult refs.[1].

Let us begin by considering the Lagrangian for a free boson;

$$\mathcal{L} = -(\partial_\mu \phi^*)(\partial^\mu \phi). \tag{2}$$

Since we wish to describe both particles and anti-particles, ϕ is complex with two degrees of freedom. Also ϕ has spin 0 and dimension 1. This lagrangian resembles the higgs which is already present in the SM. Eventually, each chirality (left or right) of matter fields will aquire a scalar superpartner of this type, together forming *chiral* supermultiplets.

Now let us introduce our fermion superpartner which must also have two degrees of freedom for a sensible transformation. Before continuing the reader should familiarize themselves with the Weyl notation for fermions, outlined in appendix A (which has become standard for this subject). The supersymmetry transformation should by definition turn the scalars into fermions so, noting that a two component spinor has the same number of degrees of freedom as the scalar, let us introduce an infinitessimal (linear) supersymmetry transformation δ_ξ on the complex boson above involving such a spinor;

$$\delta_\xi \phi = a\xi^\alpha \chi_\alpha. \tag{3}$$

The fermionic parameter ξ^α describes the magnitude of the transformation, and a is some as yet undetermined constant. We will often drop the fermionic indices α when they are contracted. The spin and dimension on both sides must match, so that ξ^α must have spin 1/2 (it anticommutes with itself and χ) and dimension 1/2. Next we wish to have an equivalent transformation for the spinor itself. Matching spin and dimensions again, this can be of the form,

$$\delta_\xi \chi_\alpha = b\sigma^\mu_{\alpha\dot{\alpha}}\overline{\xi}^{\dot{\alpha}}\partial_\mu \phi + c\xi_\alpha F. \tag{4}$$

where F is some field of spin 0 and dimension 2 whose existence will be explained shortly. Then for F we can have;

$$\delta_\xi F = d\overline{\xi}_{\dot{\alpha}}\overline{\sigma}^{\mu\dot{\alpha}\alpha}\partial_\mu \chi_\alpha. \tag{5}$$

In order to determine the constants a, b, c, d we need to consider the commutation of two transformations. For example,

$$[\delta_\xi, \delta_\eta]\phi = ab(\eta\sigma^\mu\overline{\chi} - \chi\sigma^\mu\overline{\eta})\partial_\mu\phi. \tag{6}$$

This commutator involves only one field on each side, and is therefore related to the underlying symmetry (supersymmetry). It describes a relation between the δ_ξ and ∂_μ operators, and should not depend on which field (ϕ, χ or F) we choose to express it. (The equivalent statement in gauge theory, is that the Lie-algebra $[T^a, T^b] = if_{abc}T^c$ is formed by successive gauge transformations on any of the fields in the theory, irrespective of their representation.) We need to ensure that the algebra is the same for the field χ_α and this is why the field F is required; in fact

$$[\delta_\xi, \delta_\eta]\chi = cd(\eta\sigma^\mu\overline{\chi} - \chi\sigma^\mu\overline{\eta})\partial_\mu\chi, \tag{7}$$

giving $ab = cd$. The extra field F is known as an *auxilliary* field since it is there simply to make the algebra close. It does not propagate, and so it is easily removed from the Lagrangian using its equation of motion. The full chiral supermultiplet consists of the three fields $\Phi = \{\phi, \chi, \mathrm{F}\}$ (and the anti-chiral supermultiplet, $\overline{\Phi} = \{\phi^*, \overline{\chi}, \mathrm{F}^*\}$). Having determined the multiplet structure, we are able to write down the simplest invariant Lagrangian. Using the usual normalisations, our first guess is,

$$\mathcal{L}_{KE} = -i\chi\sigma^\mu\partial_\mu\overline{\chi} + \phi^*\partial^2\phi + F^*F \tag{8}$$

In order for this to be invariant (*i.e.* to transform into itself plus total derivatives under the operations (3)–(5)), we find that we require

$$b = ia \ ; \ d = ic \tag{9}$$

which leaves only one degree of freedom a which normalises the operators δ_ξ. Conventionally we take $a = \sqrt{2}$.

So far we have only the kinetic terms of our Lagrangian. In order to be able to include Yukawa couplings for example, we need to consider additional interaction terms. As a starting point, notice that the auxilliary field F, transforms into a total derivative under a supersymmetry transformation. Thus

$$\mathcal{L}_F = F \tag{10}$$

is a suitable invariant. By itself this is of course not much use, since we also wish to include mass terms and gauge symmetries. In order to do this we must first generalise our infinitessimal transformations to make them finite; a transformation through a finite *spinor* parameter θ_α is given by the operator $\exp \delta_\theta$. Arbitrary transformations through θ_α of the scalar field $\phi(x)$ will change its value in a one-to one manner. We can now construct a scalar quantity, $\Phi(x,\theta,\overline{\theta})$, which gives $\phi(x)$ for arbitrary values of θ_α. Taking $\phi(x)$ as the starting value when $\theta_\alpha = 0$,

$$\Phi(x,0,0) = \phi(x), \tag{11}$$

we can find the general form of Φ by transforming ϕ;

$$\begin{aligned}\Phi(x,\theta,\overline{\theta}) &= \left(1+\delta_\theta+\frac{1}{2!}{\delta_\theta}^2+\frac{1}{3!}{\delta_\theta}^3+\frac{1}{4!}{\delta_\theta}^4+\ldots\right)\phi(x)\\ &= \phi(y)+\sqrt{2}\theta\chi(y)+\theta\theta F(y)\end{aligned} \tag{12}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$. [3] These scalar functions are referred to as chiral-*superfields*. The Taylor expansion in thetas of any function terminates at the $\theta\theta$ term, and consequently any general function of y and θ is a chiral superfield, and its components must transform as the components of a chiral supermultiplet. Beginning with ϕ^* results instead in *anti*-chiral superfields (denoted $\overline{\Phi}$) which are functions of y^* and $\overline{\theta}$ only.

Chiral superfields enable us to build up more general lagrangians, once we notice that (trivially) the product of two or more chiral superfields is also of this form and therefore is also a chiral superfield. So, for example, the F-term of the product of two chiral fields gives mass terms,

$$\mathcal{L}_m = \Phi^2\Big|_F + \text{h.c.} = \phi F - \frac{1}{2}\psi^\alpha\psi_\alpha + \text{h.c.} \tag{13}$$

It is easy to check that this is invariant under the infinitessimal supersymmetry transformations. For the Yukawa interactions we need the product of three superfields,

$$\mathcal{L}_{yuk} = \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k\Big|_F + \text{h.c.} = \lambda_{ijk}F_i\phi_j\phi_k - \lambda_{ijk}\psi_i^\alpha\psi_j\alpha\phi_k + \text{h.c.} \tag{14}$$

The generic indices i,j,k stand for summation over gauge representation and/or field labels (*e.g.* higgs, matter *etc.*). Terms of order higher than three make the theory non-renormalisable and will not be considered further. The most general interaction lagrangian is therefore of the form

$$\mathcal{L}_{int} = W(\Phi_i)|_F + \text{h.c.} \tag{15}$$

where $W(\Phi_i)$ is a third order polynomial of the chiral superfields, known as the *superpotential*. In terms of the superpotential, the the lagrangian is

$$\mathcal{L}_{int} = F_i\frac{\partial W}{\partial\phi_i} - \frac{1}{2}\psi_i\psi_j\frac{\partial^2 W}{\partial\phi_i\partial\phi_j} + \text{h.c.} \tag{16}$$

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[3] We have used the identities $\theta^2\theta_\alpha = 0$ (because θ_α has only two components which anticommute), and $\epsilon^{\alpha\beta}\theta\theta = -2\theta^\alpha\theta^\beta$

where only the scalar part of the potential is taken, and summation over indices is implied. Finally, by using the equations of motion of F_i,

$$F_i = -\frac{\partial W}{\partial \phi_i} \tag{17}$$

we are able to eliminate it, to give,

$$\mathcal{L}_{int} = -\left(\frac{\partial W}{\partial \phi_i}\right)^2 - \frac{1}{2}\psi_i\psi_j\frac{\partial^2 W}{\partial\phi_i\partial\phi_j} + \text{h.c.} \tag{18}$$

In particular we can see that the supersymmetry potential is positive definite, and has a minimum at $\partial W/\partial\phi_i = 0$ where it vanishes.

So we have derived a lagrangian which is able to describe scalars and their fermionic superpartners. For this introduction we have been avoiding the notion of *superspace*; the interested reader can consult appendix B which explains the connection. Now let us progress to the simplest non-trivial example, the top quark Yukawa, in order to see the promised cancellation of one loop divergences.

The top quark Yukawa

To write down a top-quark Yukawa we naturally define the superfields to have the exactly the same charges as they would in the Standard Model. Thus we have

$H_u =$	h_u	$\tilde{h}_u$	F_{h_u}
$Q =$	$\tilde{q}$	q	F_Q
$t_c =$	$\tilde{t}_c$	t_c	F_{t_c}

$$W_{top-Yukawa} = \lambda_t Q H_u t_c \tag{19}$$

leading to a lagrangian of the form

$$\mathcal{L}_{top-Yukawa} = -\lambda_t q h_u t_c - \lambda_t\tilde{q}(\tilde{h}_u t_c) - \lambda_t(q\tilde{h}_u)\tilde{t}_c + \lambda_t^2|h_u\tilde{t}_c|^2 + \lambda_t^2|h_u\tilde{q}|^2 + \lambda_t^2|\tilde{q}\tilde{t}_c|^2. \tag{20}$$

It is customary to write the superpartner of any field as that field with a tilde and we shall follow that convention here. Now consider the one loop contributions to the h_u mass squareds. The internal fermions can only be the pair q and t_c , whereas the scalar 'bubble' diagrams gets two contributions separately from $\tilde{q}$ and $\tilde{t}$. Including the symmetry factor of $1/2$ in the scalar diagram, and -1 for the fermion loop, we find exact cancellation as promised.

As yet we have not included the gauge fields (we have no spin one fields), however we can guess what their Lagrangian might look like from the above. Supersymmetry makes fermions and bosons indistinguishable, so not surprisingly the nett effect on the Yukawa coupling was to introduce duplicate 'super'-Yukawa couplings with the fermions and bosons permuted (the first three terms in eq.20). The same duplication happens in the gauge sector as well. Here the physical components of the vector supermultiplets are $\{\lambda_A, v_A^\mu, D_A\}$ (where A is a group index) which are respectively the gauginos, gauge bosons (of the Standard Model) and auxilliary fields. The matter-gauge-matter couplings become

$$\mathcal{L}_{\Phi_i-\Phi_j-V_A} = g\chi_i\sigma_\mu v_{ij}^\mu\overline{\chi}_j + i\sqrt{2}g\phi_i^*\lambda_{ij}\chi_j + h.c. \quad (21)$$

Where we use the usual notation that $v_{ij}^\mu = v_A^\mu T_{ij}^A$. As with the Yukawa, the introduction of supersymmetry has simply added supersymmetric counterpart couplings with the roles of boson and fermion reversed.

The inclusion of gauge symmetry and the supersymmetrization of the Yang-Mills term involves so-called vector supermultiplets. It is possible (although much more arduous) to build these supermultiplets beginning with the lowest component, as we did above for the chiral supermultiplets. However, for the discussion of CP and flavour phenomenology we now have everything we need; the reader interested in these more technical aspects can turn to appendix C where a discussion of the total gauge invariant Lagrangian is included for completeness.

3 The softly broken MSSM

We now have a general framework for building supersymmetric gauge invariant Lagrangians. Our aim is to build a model which includes this, and yet mimics the Standard Model at low enough energies. A consensus seems to have been established as to the most likely way this might happen in nature. This is referred to as the softly broken Minimal Supersymmetric Standard Model (MSSM).

There are a number of influences guiding the form of the MSSM, one of the most overriding being ‘naturalness’. A case in point is the scale of supersymmetry breaking itself (clearly a requirement). Most would agree that this should not be too high, otherwise fine-tuning is re-introduced. Because of this, a ceiling of a few TeV is usually placed on this parameter. One should bear in mind however that there are no other upper limits on the scale of supersymmetry breaking. This problem is compounded by the fact that there is still a large number of additional parameters left in the model (supersymmetry breaking scalar masses for example). Often these are reduced by considering a particular kind of supersymmetry breaking mediation – for example mediation by supergravity (local supersymmetry) or gauge mediation, but also they are reduced by considering experiment itself. (For example, the absence of large FCNCs is often cited as the reason for taking scalar masses to be degenerate at the GUT scale.) Because of this, getting genuine predictions out of supersymmetry is difficult, and usually the ‘prediction’ is merely that supersymmetry is *consistent* with a large $(g-2)_\mu$ or electric dipole moment of the neutron or $b \to s\gamma$ or whatever. In fact, usually supersymmetry is also consistent with values of $(g-2)_\mu$ *etc.* which totally violate current experimental bounds, thus lending an air of ‘just around the corner’-ness to the whole business.

This difficulty really arises out of our ignorance about the nature of supersymmetry breaking and leads to what has become known as the supersymmetric flavour and CP problems. These general problems will be illustrated in this section, and expanded upon later. First however, let us present the superpotential of the MSSM.

3.1 The MSSM Superpotential

Since we wish to stay as close to the SM as possible, we assume that the same gauge groups and representations will appear. Thus, the group will be $SU(3)_c \times SU(2)_L \times U(1)_Y$. The gauge bosons ($g$, $W_{1,2,3}$ and B) are accompanied by fermionic superpartners, the *gluinos, winos* and *bino.* The matter and higgs fields become chiral supermultiplets. These are usually represented by capital letters, so that we have $Q = \{\tilde{q}_L, q_L\}$, $L = \{\tilde{l}_L, l_L\}$, $U^c = \{\tilde{u}_R^*, u_R^\dagger\}$ and so on for each of the quarks and leptons, and $H = \{h, \tilde{h}\}$ for the higgs doublet. L, Q and H are $SU(2)$ doublets with

$$L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} \qquad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad H = \begin{pmatrix} h^0 \\ h^+ \end{pmatrix} \tag{22}$$

The superpartners are the *squarks, sleptons* and *higgsinos* respectively.

We now encounter a minor problem. In the standard model we were able to make do with only one higgs field, to generate two masses. The Yukawa couplings which did this were of the form

$$\mathcal{L}_{yuk} = \lambda_e \bar{l}_L h e_R + \lambda_d \bar{q}_L h d_R + \lambda_u \bar{q}_L \hat{h} u_R + \text{h.c.}. \tag{23}$$

The h scalar has hypercharge $Y = 1$, and the $\hat{h}$ scalar is given by $\hat{h} = i\tau_2 h^*$ with $Y = -1$. (The τ_i are the Dirac matrix generators of $SU(2)$, acting on isospin indices, not to be confused with the σ_i.) Clearly, if we turn these fields into chiral supermultiplets, the two quark mass terms cannot both be derived from a superpotential since if h corresponds to a chiral superfield then h^* corresponds to an *anti*chiral one. An additional problem comes from triangle anomalies. These are a potential problem in any theory with chiral fermions, and their disappearance requires that $\sum_i Y_i = \sum_i Y_i^3 = 0$. In the standard model, these cancel exactly, but now this is no longer the case since we have introduced an extra fermion, the higgsino, which also couples to isospin. Happily, both these problems are solved by the introduction of a new higgs superfield with hypercharge -1, which replaces $\hat{h}$. Our total particle content has the following assignments and transformations under the standard model gauge group,

$$\begin{array}{lcll}
L & = & \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} & (1,2,-1) \\
Q & = & \begin{pmatrix} u_L \\ d_L \end{pmatrix} & (3,2,1/3) \\
E^c & & & (1,1,2) \\
D^c & & & (\overline{3},1,2/3) \\
U^c & & & (\overline{3},1,-4/3) \\
H_1 & = & \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} & (1,2,-1) \\
H_2 & = & \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} & (1,2,1).
\end{array} \tag{24}$$

where H_1 and H_2 are two entirely separate superfields. (There is, by definition, no right handed neutrino in the MSSM although clearly experimental data now suggests one.

Neutrino physics requires a review in itself and we shall not attempt to cover it here.) The yukawa piece of the superpotential (taking a little more care with our isospin indices for a moment) may now taken to be

$$W_{yuk} = \lambda_E E^c H_1^T \epsilon' L + \lambda_D D^c H_1^T \epsilon' Q + \lambda_U U^c H_2^T \epsilon' Q + \text{h.c.}, \tag{25}$$

where we have suppressed colour and generation indices. The $\epsilon' = i\tau_2$ is the same matrix as ϵ, the prime being there simply to remind us that its indices act on isospin. [4]

The extra higgs superfield has introduced another parameter; $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$. The fact that this ratio can be large is seen as a possible explanation for the large value of the top quark mass.

In addition to the Yukawa terms, we will require some higgs self couplings in order to generate the correct electro-weak breaking;

$$W_{higgs} = \mu H_1^T \epsilon' H_2, \tag{26}$$

where μ has dimensions of mass. In order to get the correct scale for breaking (*i.e.* the correct values of M_W and M_Z) it turns out that we need to have $\mu = \mathcal{O}(M_W)$. The fact that we have an additional dimensional parameter (besides the scale of SUSY breaking) that needs to be set by hand, is known as the 'μ–*problem*'.

3.2 Soft SUSY breaking

If supersymmetry is the correct extension of the Standard Model, then it is expected to be softly broken by dimensionful parameters (such as squark mass terms), in order for the broken supersymmetry to still remove quadratic divergences and hence provide a solution to the problem of maintaining a hierarchy between the electroweak scale and the Planck scale.

The physics of the breaking is not yet well understood and there are several mechanisms that can achieve it, so to take account of our ignorance one can write a general, gauge invariant, Lorentz invariant, effective Lagrangian;

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} \tag{27}$$

where the first term preserves supersymmetry invariance and the second violates supersymmetry and is given by (dropping the ϵ')

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = \; & \tfrac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{h.c.} \right) \\ & + M_{\tilde{Q}}^2 \tilde{Q}^* \tilde{Q} + M_{\tilde{U}}^2 \tilde{U}^{c*} \tilde{U}^c + M_{\tilde{D}}^2 \tilde{D}^{c*} \tilde{D}^c + M_{\tilde{L}}^2 \tilde{L}^* \tilde{L} + M_{\tilde{E}}^2 \tilde{E}^{c*} \tilde{E}^c \\ & + A_U \lambda_U \tilde{U}^c \tilde{Q} H_2 + A_D \lambda_D \tilde{D}^c \tilde{Q} H_1 + A_E \lambda_E \tilde{E}^c \tilde{L} H_1 + \text{h.c.} \\ & + m_{H_2}^2 H_2^* H_2 + m_{H_1}^2 H_1^* H_1 + (B H_2 H_1 + \text{h.c.}) \end{aligned} \tag{28}$$

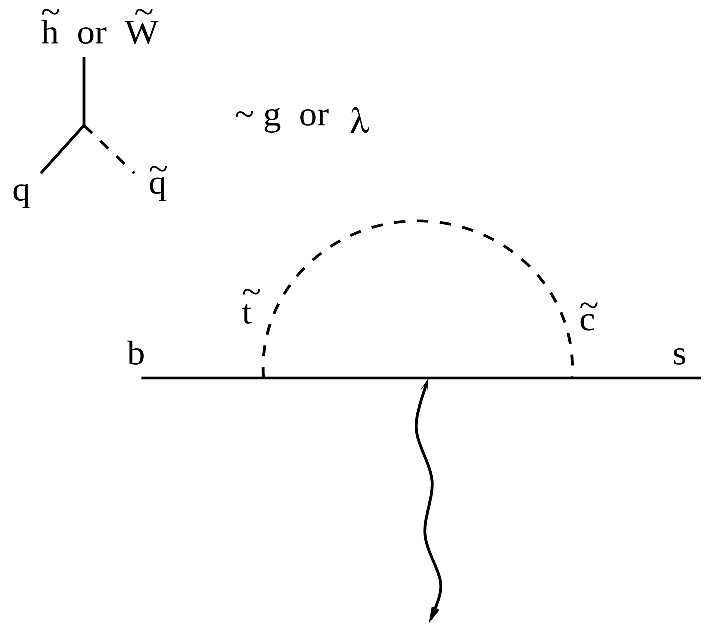
where all generation indices have been suppressed. The most general softly broken MSSM has a total of a 124 physical parameters. The count goes as follows; 6 of the parameters come from gaugino mass terms of the form $M_i = |M_i|\, e^{i\phi_i}$. The squark and slepton masses

[4] $SU(2)$ gauge invariance follows from the fact that $\epsilon' \tau^i = -(\tau^i)^T \epsilon$, so that $(e^{i\lambda.\tau})^T \epsilon' e^{i\lambda.\tau} = \epsilon'$.

are in principle 3 x 3 hermitian matrices with complex matrix elements, contributing $5 \times 6 \times 2 = 60$ parameters. Trilinear couplings between the sfermions and Higgs bosons are arbitrary 3 x 3 complex matrices which constitute $2{\times}9{\times}2 = 36$ parameters. Additional parameters arise due to the μ term, resulting in 33 mass eigenstates, 43 phases and the CKM angle.

3.3 The SUSY Flavour and CP Problems and the Mass Insertion Approximation

Given the most general form of supersymmetry breaking in eq.(28) we can see immediately that there are likely to be phenomenological problems with both flavour changing and CP violating processes. For example using the new supersymmetric contributions to the Lagrangian in eqs.(20,21) we can construct the following diagram with an internal wino and squark, which is one of many possible supersymmetric contributions to $b \to s\gamma$;



The diagram includes a flavour changing mass insertion on the squark line and our lack of knowledge about supersymmetry breaking means that in principle this insertion could be as large as any of the other terms. Likewise, the supersymmetry breaking terms can have arbitrary CP violating phases and it is not difficult to construct diagrams that contribute to electric dipole moments (of which more later).

The appropriate way to treat these problem generally is to diagonalize all the mass matrices and calculate the diagrams with the mass-eigenstates. However there is a useful approximation, the mass-insertion approximation, that can be used when the supersymmetry breaking is approximately generation independent . This type of situation is usually motivated by particular kinds of supersymmetry breaking mediation, for example mediation by gravity resulting in mSUGRA models. In such models the canonical assumption is that the supersymmetry breaking terms are flavour universal at the GUT scale. Thus the most commonly considered scenario has

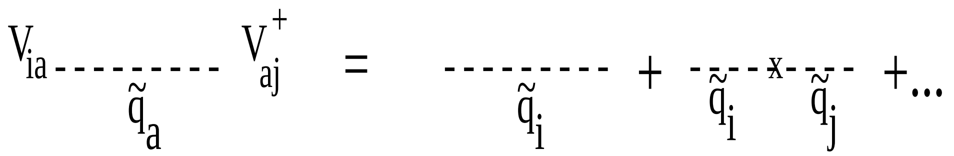
$$
\begin{aligned}
m^2_{\tilde{q}_{ij}} &= \delta_{ij} m_0^2 \\
M_A &= M_{1/2} \\
A_{U_{ij}} = A_{D_{ij}} = A_{E_{ij}} &= A \delta_{ij}\ .
\end{aligned} \tag{29}
$$

Together with the parameters μ and B we then have 5 independent parameters. $|\mu|^2$ can be determined by electroweak symmetry breaking, and traditionally we trade μB for $\tan\beta$, the ratio of Higgs VEVs, leaving us with 4 independent parameters, $m_0, M_{1/2}, A, \tan\beta$. In addition there are 2 physical phases $\phi_A = Arg(AM^*_{1/2})$, $\phi_\mu = Arg(\mu M^*_{1/2})$. All other phases can be rotated away. Typically these phases must be small in order to satisfy all EDM requirements (including that of the mercury EDM [6]). This model, sometimes referred to as the Constrained MSSM, is an example of a Minimal Flavour Violating model (MFV model) defined as models where all flavour and CP violation is attributed to the Yukawa couplings.

Typically, running the renormalization group equation from the GUT to the electroweak scales, will result in flavour non-universality thanks to the contribution from the Yukawa couplings, however they will generally be small. (Note that splittings between left and right-handed scalars mass squareds, and also between squarks and sleptons will generally be large because of their different gauge couplings.) Thus when considering a particular process, it is useful to consider a situation where the squark masses matrices are flavour universal and real, and to find the experimental limits on perturbations. This approach, pioneered by Gabbiani et al in ref.[3], allows one to state the flavour and CP problems more quantitatively since it leads to model independent bounds on mass insertions. One begins by defining perturbations from universal scalar masses, $m^2_{\tilde{f}}$ as follows,

$$
\delta^q_{LR_{ij}} = \frac{(m^2_{\tilde{q}})_{LR_{ij}}}{m^2_{\tilde{f}}},\ \delta^q_{LL_{ij}} = \frac{(m^2_{\tilde{q}})_{LL_{ij}}}{m^2_{\tilde{f}}},\ \delta^q_{RR_{ij}} = \frac{(m^2_{\tilde{q}})_{RR_{ij}}}{m^2_{\tilde{f}}}
$$

It is then fairly straightforward to calculate the leading contributions to various processes by considering the diagrams with a single insertion. The approximation is represented pictorially below. The fully diagonalized quark propagator would have diagonalization matrices for the masses, V_{ia} where i labels flavour and a labels one the 6 squark mass eigenstates, on either end. However, this is equivalent to treating the flavour changing as an interaction vertex (denoted X).



The experimental bounds can then be expressed as limits on the δ's. Thus for example one finds

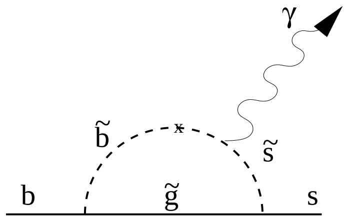
Limits from $\Delta F = 2$ with $\mathbf{M_3(M_W) = m_{\tilde{f}} = 500GeV}$

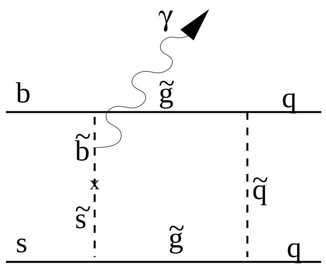
Limits on-	$\sqrt{\lvert Re(\delta^q_{LL_{ij}})^2\rvert}$	$\sqrt{\lvert Re(\delta^q_{LR_{ij}})^2\rvert}$	$\sqrt{\lvert Re(\delta^q_{LL_{ij}})Re(\delta^q_{RR_{ij}})\rvert}$
$ij = sd$	0.04	0.004	0.003
$ij = bd$	0.1	0.03	0.02
$ij = cu$	0.1	0.03	0.02

Limits from $\mathbf{l_i \to l_j \gamma}$ decays with $\mathbf{M_1 = m_{\tilde{f}} = 100GeV}$

Limits on-	$\lvert\delta^e_{LL_{ij}}\rvert$	$\lvert\delta^e_{LR_{ij}}\rvert$
$ij = \mu e$	0.008	2.10^{-6}
$ij = \tau e$	29	0.1
$ij = \tau\mu$	5	0.02

Limits from $\mathbf{b \to s\gamma}$ decays with $\mathbf{m_{\tilde{f}} = 500}$ GeV- main contributions are





$x = M_3^2/m_{\tilde{f}}^2$	$\lvert\delta^d_{LL_{23}}\rvert$	$\lvert\delta^d_{LR_{23}}\rvert$
0.3	4.4	0.013
1	8	0.016
4	26	0.03

CP violating processes constrain imaginary insertions. e.g. for $\mathbf{M_3 = m_{\tilde{f}} = 500}$ GeV we have

ε	$\sqrt{\vert Im(\delta^d_{LL_{12}})^2\vert} < 0.002$	$\sqrt{\vert Im(\delta^d_{LR_{12}})^2} < 0.0006$
ε'/ε	$\vert Im(\delta^d_{LL_{12}})\vert < 0.5$	$\vert Im(\delta^d_{LR_{12}})\vert < 2.10^{-5}$
$d_n < 6.10^{-26}$	$\vert Im(\delta^d_{LR_{11}})\vert < 1.10^{-6}$	$\vert Im(\delta^u_{LR_{11}}) < 3.10^{-6}$

Satisfying experimental bounds/results on flavour changing and CP violating processes obviously prohibits significant perturbations from universality for many of these parameters (although other reasons besides universality have been considered in the literature, and will be mentioned later). This apparent fine tuning is the quantitative form of the flavour and CP problems.

Incidentally, in the case of the mSUGRA, all of these constraints are satisfied. Note that, had this not been the case (i.e. if running the RGE's in mSUGRA had resulted in supersymmetry breaking terms that violated these bounds) there would have been very little hope of ever explaining electroweak phenomena (e.g. symmetry breaking and masses) in terms of Planck or GUT scale physics.

4 The Mass-Spectrum and calculations with generic SUSY breaking

The mass insertion approximation outlined in the previous section is useful for addressing the question, how far can we deviate from universality and still satisfy experimental bounds. However there may be (and often are) other wildly different patterns of soft-supersymmetry breaking which can have a significant degree of non-universality and yet have cancellation of contributions to various processes. (One example is models with Hermitian flavour structure, in which the EDMs are suppressed).

In order to calculate in more general scenarios, it is necessary to diagonalize all mass matrices and find the physical mass-eigenstates. The particle content and diagonalizations go as follows. First we collect the components with the same e.m. charges together into squarks, charginos, neutralinos;

$$\begin{aligned}
&\tilde{q}_{L_{i=1..3}} \oplus \tilde{q}_{R_{i=1..3}} \rightarrow \tilde{q}_{a=1..6} \\
&\tilde{h}_2^+ \oplus \tilde{W}^+ \rightarrow \tilde{\chi}^+ \\
&\tilde{h}_1^- \oplus \tilde{W}^- \rightarrow \tilde{\chi}^- \\
&\tilde{h}_u^0 \oplus \tilde{h}_d^0 \oplus \tilde{W}^0 \oplus \tilde{B} \rightarrow \tilde{\chi}_{a=1..4}
\end{aligned}$$

The sfermion mass-squared terms are given by

$$L_{\tilde{f}-mass-squared} = (\tilde{f}_L^*, \tilde{f}_R^*)_i M^2_{\tilde{f}ij} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}_j$$

where $M^2_{\tilde{f}ij}$ are 6x6 matrices

$$M^2_{\tilde{f}} = \begin{pmatrix} {M_L}^2 + {m_f}^2 + M_z^2(\frac{1}{2} - Q_f \sin^2\theta_W)\cos 2\beta & m_f(A_f - \mu^* R_f) \\ \dagger & M_R^2 + {m_f}^2 + M_z^2 Q_f \sin^2\theta_W \ \cos 2\beta \end{pmatrix},$$

with $R_f = \cot\beta$ ($\tan\beta$) for $I_3 = 1/2$ ($-1/2$) and $\tan\beta = \frac{\langle h_2^0\rangle}{\langle h_1^0\rangle}$. We diagonalize this mass matrix by making a unitary transformation on the squarks

$$M^2_{\tilde{d}_a}\,\delta_{ab} = \left(V^\dagger_{\tilde{d}} M^2_{\tilde{d}} V_{\tilde{d}}\right)_{ab}$$

The Charginos have a mass term

$$L_{charginos} = (\tilde{W}^+, \tilde{h}^+_d)_i M_{C_{ij}} \left(\begin{array}{c} \tilde{W}^- \\ \tilde{h}^-_u \end{array}\right)_j$$

where

$$M_C = \left(\begin{array}{cc} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{array}\right),$$

and we require a biunitary transormation to diagonalize the mass matrix;

$$M_{\tilde{\chi}^\pm} = U^*_C M_C V_C^{-1}$$

For the Neutralinos we have

$$L_{neutralinos} = (\tilde{B}^0, \tilde{W}^0, \tilde{h}^0_d, \tilde{h}^0_u)_i M_{N_{ij}} \left(\begin{array}{c} \tilde{B}^0 \\ \tilde{W}^0 \\ \tilde{h}^0_d \\ \tilde{h}^0_u \end{array}\right)_j$$

where M_N is symmetric;

$$M_N = \left(\begin{array}{cccc} M_1 & 0 & -M_Z\sin\theta_W\cos\beta & M_Z\sin\theta_W\sin\beta \\ T & M_2 & M_Z\cos\theta_W\cos\beta & -M_Z\cos\theta_W\sin\beta \\ T & T & 0 & -\mu \\ T & T & T & 0 \end{array}\right)$$

with

$$M_{\tilde{\chi}^0} = V_N^T M_N V_N$$

The quantities which then enter calculations of FCNC and CP violating processes are then of course the diagonalization matrices, $V_{\tilde{q}}, V_C, U_C, V_N$ which appear on the vertices.

4.1 An example calculation and the SuperGIM mechanism

All calcuations follow more or less the same format, so let us examine just one in detail, the supersymmetric contribution to the EDMs. EDMs are quite complicated beasts, and get contributions from electromagnetic, chromomagnetic as well as purely gluonic contributions shown in fig.(1) [4, 5, 6].

For simplicity I will consider only the electromagnetic contributions to the fermion EDMs.

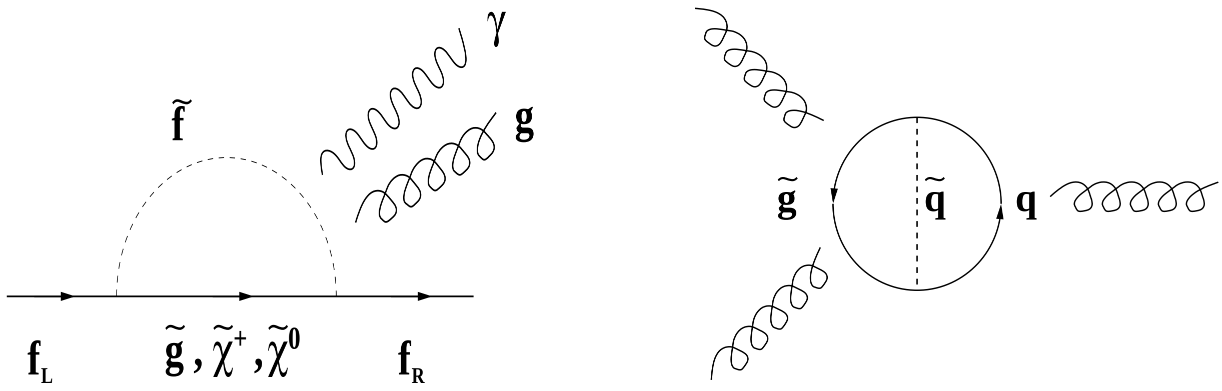


Figure 1. *Leading SUSY contributions to the EDMs. The photon and gluon lines are to be attached to the loop in all possible ways.*

These are given by [5]

$$
\begin{aligned}
d_q^{\tilde{g}\,(E)}/e &= \frac{-2\alpha_s}{3\pi} \sum_{k=1}^{6} \mathrm{Im}(\Gamma_q^{1k}) \frac{m_{\tilde{g}}}{M_{\tilde{q}_k}^2} Q_{\tilde{q}}\ \mathrm{B}\left(\frac{m_{\tilde{g}}^2}{M_{\tilde{q}_k}^2}\right) , \\
d_u^{\chi^+\,(E)}/e &= \frac{-\alpha_{em}}{4\pi \sin^2\theta_W} \sum_{k=1}^{6}\sum_{i=1}^{2} \mathrm{Im}(\Gamma_{uik}) \frac{m_{\chi_i^+}}{M_{\tilde{d}_k}^2} \left[Q_{\tilde{d}}\ \mathrm{B}\left(\frac{m_{\chi_i^+}^2}{M_{\tilde{d}_k}^2}\right) + (Q_u - Q_{\tilde{d}})\ \mathrm{A}\left(\frac{m_{\chi_i^+}^2}{M_{\tilde{d}_k}^2}\right)\right] , \\
d_d^{\chi^+\,(E)}/e &= \frac{-\alpha_{em}}{4\pi \sin^2\theta_W} \sum_{k=1}^{6}\sum_{i=1}^{2} \mathrm{Im}(\Gamma_{dik}) \frac{m_{\chi_i^+}}{M_{\tilde{u}_k}^2} \left[Q_{\tilde{u}}\ \mathrm{B}\left(\frac{m_{\chi_i^+}^2}{M_{\tilde{u}_k}^2}\right) + (Q_d - Q_{\tilde{u}})\ \mathrm{A}\left(\frac{m_{\chi_i^+}^2}{M_{\tilde{u}_k}^2}\right)\right] , \\
d_e^{\chi^+}/e &= \frac{\alpha_{em}}{4\pi \sin^2\theta_W} \sum_{i=1}^{2} \frac{m_{\chi_i^+}}{m_{\tilde{\nu}}^2} \mathrm{Im}(\Gamma_{ei})\ \mathrm{A}\left(\frac{m_{\chi_i^+}^2}{m_{\tilde{\nu}}^2}\right) , \\
d_f^{\chi^0\,(E)}/e &= \frac{\alpha_{em}}{4\pi \sin^2\theta_W} \sum_{k=1}^{6}\sum_{i=1}^{4} \mathrm{Im}(\eta_{fik}) \frac{m_{\chi_i^0}}{M_{\tilde{f}_k}^2} Q_{\tilde{f}}\ \mathrm{B}\left(\frac{m_{\chi_i^0}^2}{M_{\tilde{f}_k}^2}\right) ,
\end{aligned} \tag{30}
$$

where

$$
\Gamma_q^{1k} = e^{-i\phi_3} V_{\tilde{q}4k} V_{\tilde{q}1k}^* , \tag{31}
$$

with ϕ_3 being the gluino phase. The chargino factor Γ_{fik} comes from products of vertices with attached charged higgsino or wino;

$$
\begin{aligned}
\Gamma_{uik} &= \lambda_u V_{Ci2}^* V_{\tilde{d}1k} (U_{Ci1}^* V_{\tilde{d}1k}^* - \lambda_d U_{Ci2}^* V_{\tilde{d}4k}^*) , \\
\Gamma_{dik} &= \lambda_d U_{Ci2}^* V_{\tilde{u}1k} (V_{Ci1}^* V_{\tilde{u}1k}^* - \lambda_u V_{Ci2}^* V_{\tilde{u}4k}^*)
\end{aligned} \tag{32}
$$

and analogously for the electron. The λ_f are the Yukawa couplings

$$
\lambda_u = \frac{m_u}{\sqrt{2} m_W \sin\beta}, \quad \lambda_{d,e} = \frac{m_{d,e}}{\sqrt{2} m_W \cos\beta}. \tag{33}
$$

The neutralino vertex η_{fik} is given by

$$
\begin{aligned}
\eta_{fik} &= \left[-\sqrt{2}\{\tan\theta_W (Q_f - I_{3_f}) V_{N1i} + I_{3_f} V_{N2i}\} V_{\tilde{f}1k}^* - \lambda_f V_{Nbi} V_{\tilde{f}4k}^* \right] \\
&\times \left[\sqrt{2} \tan\theta_W Q_f V_{N1i} V_{\tilde{f}4k} - \lambda_f V_{Nbi} V_{\tilde{f}1k} \right] ,
\end{aligned} \tag{34}
$$

where I_3 is the third component of the isospin, $b = 3$ (4) for $I_3 = -1/2$ (1/2). The loop functions $A(r), B(r)$, and $C(r)$ are defined by

$$\begin{aligned}
A(r) &= \frac{1}{2(1-r)^2}\left(3 - r + \frac{2\ln r}{1-r}\right), \\
B(r) &= \frac{1}{2(r-1)^2}\left(1 + r + \frac{2r\ln r}{1-r}\right), \\
C(r) &= \frac{1}{6(r-1)^2}\left(10r - 26 + \frac{2r\ln r}{1-r} - \frac{18\ln r}{1-r}\right).
\end{aligned} \tag{35}$$

Note that the indices (1) and (4) appear in the above because for the EDMs of the down, up and electron we pick out the left and right chirality fermions of the first generation.

This completes the electromagnetic contribution to the quark and electron EDMs. We can now see in a more general way why flavour universality gives cancellation of contributions. Consider for example the gluino contribution to EDMs. In the event that the squark masses are universal and equal to $M^2_{\tilde{q}}$ this can be written as

$$d_q^{\tilde{g}\,(E)}/e = \frac{-2\alpha_s}{3\pi}\frac{m_{\tilde{g}}}{M^2_{\tilde{q}}} Q_{\tilde{q}}\ \mathrm{B}\left(\frac{m^2_{\tilde{g}}}{M^2_{\tilde{q}}}\right)\sum_{k=1}^{6}\mathrm{Im}(\Gamma_q^{1k}). \tag{36}$$

The last factor in this expression is

$$\mathrm{Im}(\Gamma_q^{1k}) = e^{-i\phi_3}\left(V_{\tilde{q}}V^\dagger_{\tilde{q}}\right)_{41} = 0, \tag{37}$$

and consequently the gluino contribution exactly vanishes because of the unitarity of the diagonalization matrices, $V_{\tilde{q}}$. With exact universality the leading contribution is from the chargino diagram, however these are proportional to $\mathrm{Im}\sum_{k=1}^{6}(\Gamma_{dik})$. Again unitarity means that the main contribution here is from the phase on the μ term (unfortunately the least well understood of all the parameters in the MSSM). Note that even in the CMSSM, unitarity is broken between left and right chiralities at the weak scale because of the different gauge coupling contributions to the mass-squared RGEs. Because of this there is also a small dependence on the phase of the A-term (ϕ_A) even in the CMSSM.

A similar sort of cancellation occurs for flavour changing processes, where it is referred to as the *superGIM* mechanism. For example consider the chargino contributions to ΔM_K shown in fig.(2).

The $K_S - K_L$ mass difference is given by

$$\Delta M_K = 2\mathrm{Re}(M_{12}) \tag{38}$$

where

$$M_{12} = \frac{\alpha_2^2}{24}B_K\eta_K f_K^2\sum_{kk'}^{6}\sum_{ij}^{2}\Lambda_{2ik}\Lambda^*_{1ik'}\Lambda_{2jk'}\Lambda^*_{1jk}\frac{D(M_{\tilde{u}_k}, M_{\tilde{u}_{k'}}, M^2_{\chi_i}, M^2_{\chi_j})}{M^2_{\chi_i}}, \tag{39}$$

where

$$\Lambda_{fik} = V^*_{Ci1}V^*_{\tilde{u}fk} - \sum_{f'=1}^{3}\lambda_{U_{ff'}}V_{Ci2}V^*_{\tilde{u}(f'+3)k} \tag{40}$$

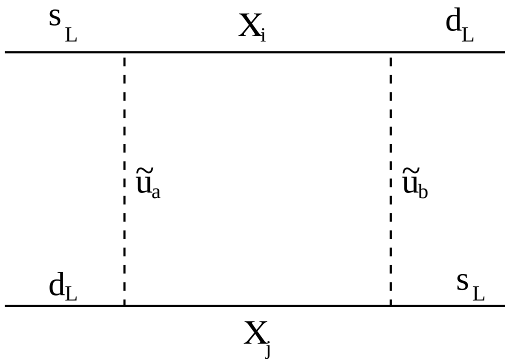


Figure 2. *Chargino contribution to ΔM_K.*

and where f, f' label flavours, and D is a dimensionless one-loop integral which is of order one if the chargino and sqaurk masses are similar. Here the first term in Λ comes from the $\tilde{u}d\tilde{W}$ term and the second from the $\tilde{u}d\tilde{h}$ term in the Lagrangian.

Again we see that degenerate squark masses leads to suppression because of the universality of the diagonalization matrices. Indeed because only left handed external quarks are involved in this particular diagram, the contribution to ΔM_K from the first term in the Λ's is proportional to the mass splitting in the first two generations of left handed up-squarks which in the CMSSM is driven by the renormalization group effects of the Yukawas;

$$\Delta M_K \propto \frac{m_c^2}{m_{\tilde{u}}^2}. \tag{41}$$

In addition to this contribution there are contributions involving cross-terms in the Λ's, but because we need two cross terms (there are 4 Λ's) they will be of the same order of magnitude. This overall suppression is the super-GIM mechanism.

5 Aspects of CMSSM phenomenology

The universality of supersymmetry breaking is found not to hold in most string derived models, and in general supergravity models, and we expect at least some non-universality in the flavour structure of the A terms. However because of the super-GIM mechanism, the CMSSM is the minimal model from the point of view of avoiding nasty FCNC and CP violating contributions. It is important therefore to estimate what the new supersymmetric contributions to these processes are likely to be in the CMSSM, in order to provide a kind of lower bound on the expectations for new physics. These effects come from the running of the renormalization group equations, whence the flavour dependence of the Yukawas enters the supersymmetry beaking. These effects are small and so the mass insertion approximation is valid. However, there is no general way to estimate the resulting size of the δ's induced by the renormalization group, because the RGE's are non-linear and in any case some of the parameters depend on the minimization of the one-loop electroweak potential. Thus a useful approach is to concentrate on the dominant contributions in the CMSSM (and indeed Minimal Flavour models). As we have seen, contributions from diagrams involving gluinos are suppressed in minimal flavour models (this is a general result) and the neutralino contributions are also suppressed in the CMSSM. Hence the dominant contributions are those involving charginos and charged higgses. This greatly reduces the number of parameters and means that one can survey the parameter space rather generally without having to perform the minimization of the one-loop potential.

A study along these lines was carried out in ref.[7] and recently improved and updated in ref.[8] where many more rare decays were also considered. (This reference provides a useful compendium of expressions for calculating various rare processes in SUSY, and summarizes the new operators that are introduced into the basis for H_{eff}. Many of the expressions for the supersymmetric contributions are also found in ref.[9].) Here the discussion will concentrate on the supersymmetric contributions to the Wilson coefficients calculated at $\sim M_W$, so the reader is referred to that reference for more information on how to turn these into the measured parameters.) The later study includes NLO-QCD and QED corrections as well as constraints from electroweak precision studies, exclusion bounds on M_h, and the $B \to X_s\gamma$ decay rate. The input parameters required for such a study are only those that effect the chargino and higgs masses and mixings;

$$m_{\tilde{t}_1},\, m_{\tilde{t}_2},\, \theta_{\tilde{t}},\, m_{\tilde{\chi}_1},\ \tan\beta,\, m_{H^\pm},\, \mu,\, m_{\tilde{l}} \tag{42}$$

where $m_{\tilde{t}_{1,2}}$ are the heavy and light stop masses, $\theta_{\tilde{t}}$ is the stop mixing angle, $m_{\tilde{\chi}_1}$ the lightest chargino mass, and $m_{\tilde{l}}$ the common slepton mass (for the rare decays). Note that there are more parameters here than in the CMSSM (including the breaking of electroweak symmetry), but in performing a study using these parameters one is addressing a wider class of models, including all MSSM models that have Minimal Flavour Violation (for example those in which the scalar masses are flavour independent but not necessarily degenerate). One then scans over reasonable values of these parameters. Again what constitutes 'reasnable' is open to interpretation, however imposing masses less than 1 TeV seems appropriate in order to avoid reintroducing fine tuning. The parameter $|\mu|$ can typically be larger than the range $|\mu| < 500GeV$ chosen in this study, although larger values reduce the mixing in the chargino and therefore its contributions. In addition values of large $\tan\beta > 6$ are excluded in this study. Although the results tend to asymptote for medium $\tan\beta$ there is a region of very high $\tan\beta$ which is interesting from a theoretical point of view; it allows Yukawa unification at the GUT scale. Unfortunately this implies large b-quark Yukawas and hence the top-quark dominance approximation made for the low-mediuma $\tan\beta$ studies is no longer valid.

Before summarizing the results, let us briefly discuss the main effects of supersymmetry. For the kaons, the most interesting effect is that one tends to find a depletion of ε'/ε, unfortunately contrasting with what we would perhaps like given the fact that the measured value is slightly larger than what one would expect from the Standard Model (modulo uncertainties in hadronic matrix elements). The dominant contributions are from Z^0 and photo-penguin diagrams involving charginos and charged higgs. The two contributions come with opposite sign (the chargino being positive), so that there are indeed regions where an enhancement over the SM value can occur, althought his is only of the order of 10%. On the other hand the depletion can be upto 40%. A summary of the most interesting processes is given in the table below;

Process \Ratio w.r.t SM	max	min
ε'/ε	0.53	1.07
$K^+ \to \pi^+\nu\overline{\nu}$	0.65	1.02
$K_L \to \pi^0\nu\overline{\nu}$	0.41	1.03
$K_L \to \pi^0 e^+ e^-$	0.73	1.10
$B \to X_s\mu\overline{\mu}$	0.73	1.34
$B \to \mu\overline{\mu}$	0.68	1.53

Again, the result for ε'/ε is plagued by hadronic uncertainties, whereas the rare decays are theoretically much cleaner places to search for signals of supersymmetry.

One additional interesting feature of Minimal Flavour models is that they predict an absolute lower bound of $\sin 2\beta > 0.42$ [9, 8]. Such a lower bound is possible because one can generalize the expression for the ε hyperbola to include supersymmetric contributions.

$$\overline{\eta}((1-\overline{\rho})A^2\eta_2 F_{tt} + 0.31)A^2\hat{B}_K = 0.266$$

where 0.31 is the charm contribution. From

$$R_t^2 = 1.6\frac{R_0^2}{A^2 F_{tt}}$$

we get

$$\sin 2\beta = \frac{2\overline{\eta}(1-\overline{\rho})}{R_t^2} = \frac{1.6}{R_0^2\eta_2}\left[\frac{0.226}{A^2\hat{B}_K} - 0.31\overline{\eta}(F_{tt})\right]$$

This function may now be plotted as a function of the parameter F_{tt} which is a collection of all SM and SUSY contributions and is typically of order 1 in the CMSSM. (Note that it is not monotonic because the variables are not independent of F_{tt}.) A value of $\sin 2\beta$ less than this value would have been the first indication of flavour structure in soft supersymmetry breaking and arguably an indirect detection of supersymmetry. Unfortunately, at the time of writing, the value of $\sin 2\beta$ coming from $B \to K_S\psi$ measurements at Belle and BaBar indicate higher values.

6 Beyond the CMSSM and MFV

Up to this point we have been looking at the minimal case where all flavour and CP violation in the MSSM is attributed to the Yukawa sector. However, an interesting question to ask is whether there might be other, non-universal scenarios, in which some of these processes are consistent with experimental limits. This is an important question because string models do not generally predict the favourable CMSSM scenario very naturally.

There are a number of ideas for suppressing FCNC and CP processes apart from universality. They are, cancellations of SUSY contributions amongst themselves, flavour alignment, particular flavour structures (e.g. hermitian or off-diagonal phases), decoupling and dynamical (i.e. derived from minimizing a string derived potential). In the next subsection I will discuss cancellations with respect to EDMs and make some general comments. Then I will mention decoupling and in the following section discuss some recent developments in the last of these, FCNC and CP suppression in dynamical string derived patterns of supersymmetry breaking.

6.1 Non-universality and cancellations

When the flavour structure is non-universal at the GUT scale there can be cancellations to various processes. Intuitively it seems unlikely that they will occur simultaneously for different processes, and this indeed turns out to be the case when numerical studies are done as we shall now see.

Non-universality has been studied as a general idea in refs.[10] but more recently there has been a resurgence of interest thanks to 'new' string inspired scenarios, motivated by explicit constructions of SM-like models in for example type I models. These models can lead to flavour non-universality which is relatively 'well-controlled', so naturally there has been some speculation as to whether a particular type of flavour non-universality might lead to cancellations 'across the board' as effectively as universality [11, 12]. The answer to this, at least for those case that have been studied, unfortunately appears to be negative as we shall now see.

The measurables that first received attention in this context were again the EDM's of the neutron, electron, both of which have rather strict experimental bounds. In refs.[11] it was noted that for particular patterns of supersymmetry breaking there can be a simultaneous cancellation of these parameters, even if the CP violating phases on the μ term is large, much larger than what would be allowed in the naive CMSSM with phases. This idea has since attracted some attention and has become known as the 'cancellation mechanism'.

The analysis was based on supersymmetry breaking pattern inspired by so-called D-brane models in which the $SU(2)$ gauge group was associated with a different brane from the $SU(3)$ and $U(1)_Y$ groups of the SM. The result is that the supersymmetry breaking gauginos masses can have different values at the string scale (assumed to be near the GUT scale). On the other hand the A-terms can have a good deal of universality. The overall pattern of supersymmetry breaking can therefore be assumed to be identical to that of the CMSSM with the exception of the gaugino masses which are of the form,

$$\begin{aligned} M_1 = M_3 &= \sqrt{3} m_{3/2} \cos\theta \Theta_1 e^{i\gamma_1} \\ M_2 &= \sqrt{3} m_{3/2} \cos\theta \Theta_2 e^{i\gamma_2} , \end{aligned} \tag{43}$$

where $m_{3/2} \sim M_W$ is the gravitino mass and the angles here are a convenient parameterization of the distribution of supersymmetry breaking amongst the different fields. If we in addition allow an arbitrary phase, ϕ_μ for the μ term, then one finds large values of phases where the neutron and electron EDMs cancel simultaneously as shown in fig.(3a).

Unfortunately when additional experiments are considered the cancellation region begins to be excluded. In this case the experiment in question is the bound on the mercury EDM [6]. This parameter also has a cancellation region, but unfortunately it tends to

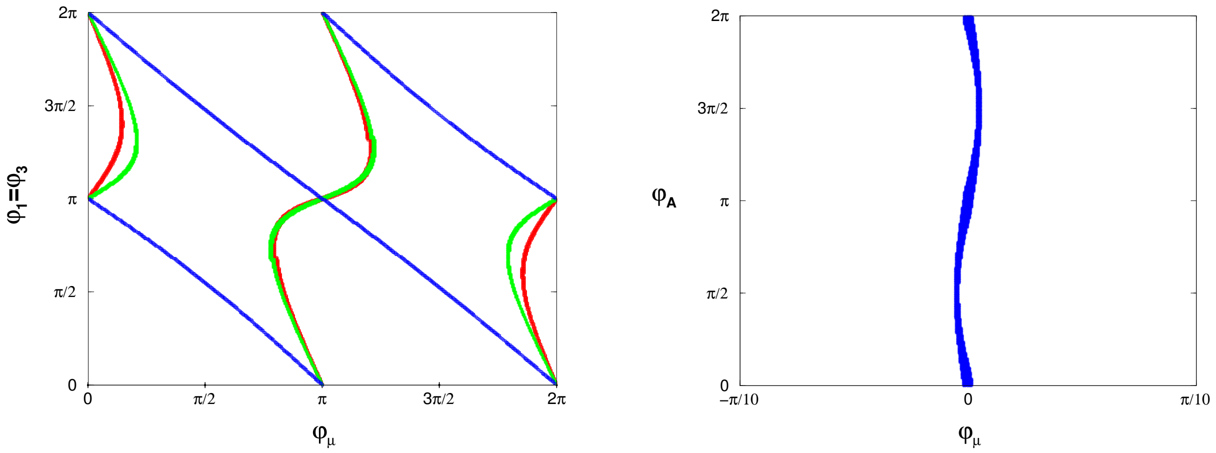


Figure 3. *a) The cancellation regions for EDMs of the electron (red) neutron (green) and mercury atom (blue). The parameters are* $\tan\beta = 2$, $m_{3/2} = 150 GeV$, $\theta = 0.2$ *and* $\Theta_1 = \sqrt{1/2}$. *b) The region of simultaneous n,e,Hg EDM cancellation in mSUGRA for the same* $\tan\beta$ *and* $m_{3/2}$.

cut across the previous cancellation region, and all three EDMs are only consistent with bounds in the trivial small phase region [6].

Note that in contrast, mSUGRA (CMSSM with phases) has a region of cancellation for all three EDMs as shown in fig.(3b) which allows any value of ϕ_A but restricts $\phi_\mu < \pi/100$. (The weak constraint on ϕ_A is a result of the small contribution of ϕ_A to the EDMs (as explained earlier)). Adding a small phase to any of the gauginos separates the regions of small n,e and Hg EDMs and destroys this cancellation. In both cases the cancellation regions decrease in size at more realistic larger $\tan\beta$. Thus for this example the cancellations are not 'across the board'. Indeed given the fact that the cancellation regions shift considerably depending on the adopted model of the neutron (valence-quark or parton) finding a (non-symmetry based) stringy underlying reason for cancellation would in any case be difficult. Although here I have concentrated on EDMs I see no reason why the conclusion would be different with any other CP or FCNC process unless there was some underlying flavour symmetry.

6.2 Decoupling

This possibility is based on the fact that as the supersymmetric particles become heavier their contribution to FCNC and CP processes begins to decouple. For example, even if one allows $\mathcal{O}(1)$ CP violating phases, the contribution to the EDMs will be negligible if the SUSY spectrum is sufficiently heavy [13]. Generally, SUSY fermions are required to be lighter than the SUSY scalars by, for example, cosmological considerations. So the decoupling scenario can be implemented with heavy sfermions only. Note that for FCNCs for example, the contribution is still proportional to

$$\frac{m_{\tilde{i}}^2 - m_{\tilde{j}}^2}{m_{\tilde{i}}^2 + m_{\tilde{j}}^2}, \tag{44}$$

where i and j label different flavours. Thus if for example we put down the suppression of $\Delta M_{K_{SUSY}}$ to large first and second generation masses, we still require a proportionate universality in their values. So this scenario is in a sense 'universality where we need it'.

Here the SUSY contributions to the EDMs are suppressed even with maximal SUSY phases because the squarks in the loop are heavy and the mixing angles are small. In Fig.4 we display the EDMs as functions of the universal scalar mass parameter m_0 for the mSUGRA model with maximal CP-phases $\phi_\mu = \phi_A = \pi/2$ and $m_{1/2} = A = 200$ GeV. We observe that all EDM constraints except for that of the electron require m_0 to be around 5 TeV or more. The mercury constraint is the strongest one and requires

$$(m_0)_{decoupl.} \simeq 10 \text{ TeV} . \tag{45}$$

This leads to a serious fine-tuning problem. Recall that one of the primary motivations for supersymmetry was a solution to the naturalness problem. Certainly this motivation will be entirely lost if a SUSY model reintroduces the same problem in a different sector, i.e. for example a large hierarchy between the scalar mass and the electroweak scale.

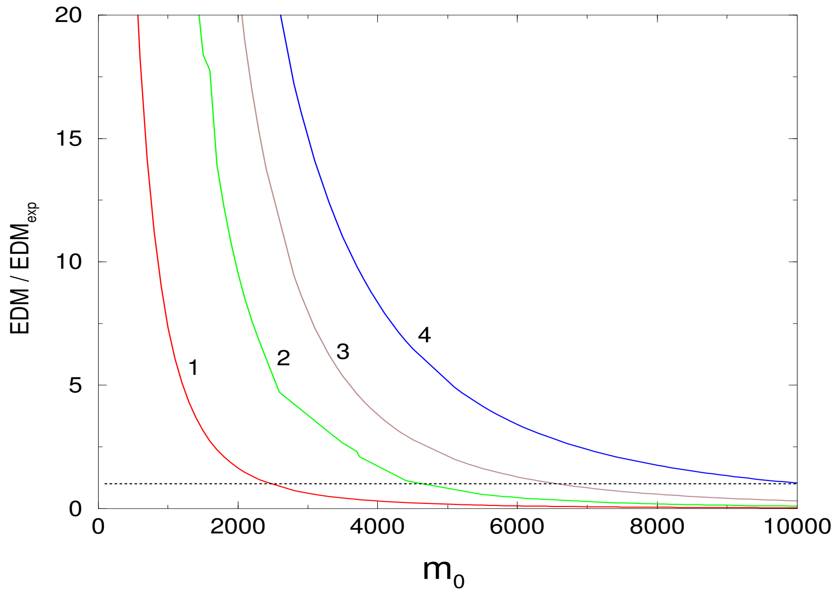


Figure 4. *EDMs as a function of the universal mass parameter m_0. 1 – electron, 2 – neutron (chiral model), 3 – neutron (parton model), 4 – mercury. The experimental limit is given by the horizontal line. Here* $\tan\beta = 3$, *$m_{1/2} = A = 200$ GeV, $\phi_\mu = \phi_A = \pi/2$.*

The degree of fine-tuning can be quantified as follows. The Z boson mass is determined at tree level by

$$\frac{1}{2}m_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 . \tag{46}$$

One can define the sensitivity coefficients [14],[15]

$$c_i \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i^2} \right| , \tag{47}$$

where a_i are the high energy scale input parameters such as $m_{1/2}$, m_0, etc. Note that μ is treated here as an independent input parameter. A value of c_i much greater than one would indicate a large degree of fine-tuning. The Higgs mass parameters are quite sensitive to m_0, so for m_0=10 TeV we find

$$c_{m_0} \sim 5000 \tag{48}$$

for the parameters of Fig.4. For the universal scalar mass of 5 (3) TeV the sensitivity coefficient reduces to 1300 (500). This clearly indicates an unacceptable degree of fine-tuning.

This problem can be mitigated in models with focus point supersymmetry, i.e. when m_{H_2} is insensitive to m_0 [15]. However, this mechanism works for m_0 no greater than 2-3 TeV which is not sufficient to suppress the EDMs. Another interesting possibility is presented by models with a radiatively driven inverted mass hierarchy, i.e. the models in which a large hierarchy between the Higgs and the first two generations scalar masses is created radiatively [16]. However, a successful implementation of this idea is far from trivial [17]. One can also break the scalar mass universality at the high energy scale [18]. In this case, either a mass hierarchy appears already in the soft breaking terms or certain relations among the soft parameters must be imposed (for a review see [19]). These significant complications disfavour the decoupling scenario as a way to solve the SUSY CP problem, yet it remains a possibility.

Concerning the other phenomenological consequences, we remark that the SUSY contributions to the CP-observables involving the first two generations (such as $\varepsilon, \varepsilon'$) are negligible, although those involving the third generation may be considerable. The corresponding CP-phases are constrained through the Weinberg operator contribution to the neutron EDM, which typically prohibits the maximal phase $\phi_{A_{t,b}}\big|_{GUT} = \pi/2$ while still allowing for smaller $\mathcal{O}(1)$ phases.

7 Summary

Supersymmetry breaking is clearly the biggest factor in flavour and CP phenomenology and yet remains the biggest unknown. Ideally one would like to have a picture of CP and flavour from first principles but at the time of writing this seems some way off. Nevertheless there are two hints to do with CP violation that some progress might be possible. The first hint is that in the SM the CP violation is insufficient to account for the observed baryon asymmetry. Secondly, general arguments show that the four dimensional CP is a discrete gauge symmetry in string theory. Thus any viable string scenario must break CP spontaneously. (See ref.[23] for the original observation, and some more recent references on the general connection between supersymmetry breaking and CP.)

In these notes I have given a brief introduction the subject and have omitted many topics (mainly because of my lack of expertise), my main aim being to equip the reader with the techniques rather than to be comprehensive. To finish, I will try to make some ammends by summarizing some of these developments and directing the reader to relevant works. Muon dipole moments have been discussed in ref.[26] where it has been shown that a muon EDM may in the future be at the $10^{-22} ecm$ level and hence measurable. The relations between dipole moment operators have been examined in ref.[27] where it was shown that the muon anomalous magnetic moment is correlated with the electron EDMs.

The question of $(g-2)_\mu$ in mSUGRA with and without phases and universality has been analyzed in ref.[28]. In mSUGRA there are correlations between this quantity and $B_s \to \mu^- \mu^+$ (see Dedes *et al* in the same reference). Alternative approaches to spontaneous CP violation that concentrate instead on CP breaking in extensions of the MSSM with gauge singlet have been considered in ref. [29]. Other possibilities for CP violation are reviewed in ref.[22]. The neutrino sector (to which I have paid scandalously little attention) deserves a review of its own. As well as the direct interest in the flavour structure of the neutrino sector, various patterns of supersymmetry breaking in the neutrino sector may lead to observable effects in rare decays such as $\mu \to e\gamma$ and $\mu \to eee$, as well as interesting possibilities for CP and leptogenesis. The reader is referred to ref.[30] and references therein. I have also been unable to cover R-parity violating models which obviously have interesting implication for CP and flavour. For a review see ref.[33]. The masses of the Higgs sector are strongly dependent on the CP violating phase in the μ term (something that was somewhat skated over in the text). This is discussed in refs.[32]. Finally there are a host of other interesting ideas and I cannot even begin to summarize them. One of many intriguing examples is the idea that all fermion masses are actually the result of radiative corrections and therefore soft. This is discussed in ref.[34].

Appendix A: Conventions

We here briefly recall the essential features of the Weyl notation for the discussion of section 1. In the Weyl representation, a four component, Dirac fermion looks like

$$\psi = \begin{pmatrix} \chi_\alpha \\ \overline{\chi}_c^\beta \end{pmatrix}. \tag{49}$$

where χ and χ_c are two-component spinors (α, $\beta = 1, 2$). The gamma matrices are

$$\gamma^\mu = i \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \tag{50}$$

where $\sigma^\mu = (1, \sigma^i)$, $\overline{\sigma}^\mu = (1, -\sigma^i)$, and σ^i are the Dirac matrices. The γ_5 matrix looks like

$$\gamma_5 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{51}$$

so that the chiral projections (given by $R/L \equiv (1 \pm i\gamma_5)/2$) on the fermions are,

$$\psi_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix} \qquad \psi_R = \begin{pmatrix} 0 \\ \overline{\chi}_c \end{pmatrix} \tag{52}$$

The suffix 'c' refers to charge conjugate, since

$$\psi_c = -\gamma^2 \psi^* = \begin{pmatrix} -i\sigma^2 \overline{\chi}_c^* \\ i\sigma^2 \overline{\chi} \end{pmatrix}, \tag{53}$$

so that $\chi_{c\alpha} = -i\sigma^2_{\alpha\beta} \overline{\chi}_c^{*\beta}$. The matrix

$$\epsilon_{\alpha\beta} = -i\sigma^2_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{54}$$

acts like a metric tensor for the unbarred spinors. For example the invariant mass terms are

$$\begin{aligned} \overline{\psi}\psi &= \overline{\chi}_c^{*\beta} \chi_\beta + \text{h.c.} \\ &= \epsilon^{\beta\alpha} \chi_{c\alpha} \chi_\beta + \text{h.c.} \\ &= (\psi_R^\dagger)^\alpha (\psi_{L_\alpha}) + \text{h.c.} \end{aligned} \tag{55}$$

Because of this we identify the complex conjugate of a field, $\overline{\chi}^{*\beta}$ with χ^β. (The last line we included to make the point that we may identify the two component spinors in terms of the original Dirac ones, so that $(\psi_R^\dagger)^\alpha \equiv \psi_{Lc}^\alpha$. The dagger we shall reserve to indicate an operation on the Dirac spinors.) This means that $(\eta\chi)^\dagger = \overline{\chi\eta}$. Similarly, the complex conjugates form their own representation of the Lorentz algebra, and have their own Lorentz invariant quantities. To avoid confusion the indices of the latter are written with a dot, so that

$$\epsilon_{\dot{\alpha}\dot{\beta}} = -i\sigma^2_{\dot{\alpha}\dot{\beta}}, \tag{56}$$

and in general, Lorentz invariant quantities are written as

$$\chi^\alpha \eta_\alpha \; ; \; \overline{\chi}_{\dot\alpha} \overline{\eta}^{\dot\alpha} \; ; \; \chi^\alpha \sigma^\mu_{\alpha\dot\alpha} \overline{\eta}^{\dot\alpha} \partial_\mu \; ; \; \overline{\chi}_{\dot\alpha} \sigma^{\mu\dot\alpha\alpha} \eta_\alpha \partial_\mu \tag{57}$$

and so on.

Appendix B: Superspace

In the text we saw that the transformations of the $\{\phi, \chi, \mathrm{F}\}$ supermultiplet, could be expressed as the behaviour of a single scalar quantity, the superfield. The eight 'coordinates' $(x, \theta, \overline{\theta})$ are known collectively as *superspace*. Supersymmetry transformations are equivalent to transformations in θ. Transformations in minkowski space are generated by the momentum P_μ, which is realised over space-time coordinates as a differential operator $P_\mu \equiv -i\partial_\mu$. (*i.e.* if $x \to x + a$, then $\exp ia \cdot P$ induces the transformation $\phi(x) \to \phi(x) + a^\mu \partial_\mu \phi(x)$). In fact P_μ is the conserved charge associated with the transformation $x \to x + a$, which, by Noether's theorem is

$$P_\mu = \int \mathrm{d}x^3 \frac{\partial \mathcal{L}}{\partial \dot f} \partial_\mu f - \delta^0_\mu \mathcal{L} \tag{58}$$

where f represents summation over all fields (fermions and bosons) in the Lagrangian. In a similar way, transformations in θ can be written in terms of differential operators acting in the superspace;

$$\delta_\xi = \xi\, Q + \overline{\xi}\, \overline{Q} \tag{59}$$

where

$$\begin{aligned} Q_\alpha &= \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha} \overline{\theta}^{\dot\alpha} \partial_\mu \\ Q^{\dot\alpha} &= \frac{\partial}{\partial \theta_{\dot\alpha}} - i\theta^\alpha \sigma^{\mu\dot\alpha}_{\alpha} \partial_\mu . \end{aligned} \tag{60}$$

Acting (infinitessimally) with these operators on the coordinates gives the following transformations;

$$\begin{aligned} \delta_\xi x &= i\theta\sigma\overline{\xi} - i\xi\sigma\overline{\theta} \\ \delta_\xi \theta &= \xi \\ \delta_\xi \overline{\theta} &= \overline{\xi} \\ \delta_\xi y &= 2i\theta\sigma\overline{\xi}. \end{aligned} \tag{61}$$

It is easy to check that, firstly,

$$\begin{aligned} \Phi(y,\theta) + \delta_\xi \Phi(y,\theta) &= \Phi(y + 2i\theta\sigma\overline{\xi}, \theta + \xi) \\ &= \Phi(y,\theta) + \delta_\xi \phi + \sqrt{2}\theta\delta_\xi \chi + \theta\theta\delta_\xi F \end{aligned} \tag{62}$$

so that the differential operators above do indeed produce the infinitessimal transformations, and secondly, that the momentum is related to the supersymmetry generators as,

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \tag{63}$$

all other anticommutators vanishing. This is one piece of the supersymmetry algebra and leads to eq.(6). The remaining pieces are provided by the Poincaré algebra of normal special relativity (*i.e.* the algebra of commutations of the momentum P_μ and six $M_{\mu\nu}$ generators of the Lorentz group – three boosts M_{i0} and three angular momentum M_{ij} – which represent currents conserved under a Poincaré transformation $x^\mu \to \Lambda^\mu_\nu x^\nu + a^\mu$). The generator Q_α has a current associated with it too, the *super*-current. This is simply the Noether current for the infinitessimal supersymmetry transformations given above;

$$S_{\alpha\mu} = -\sqrt{2}\left(2\chi_{i\alpha}\partial_\mu\phi_i^* - i\frac{\partial W}{\partial\phi_i}(\sigma_\mu\overline{\chi}_i)_\alpha\right) \tag{64}$$

with $Q_\alpha = \int \mathrm{d}x^3 S_{\alpha 0}$.

The algebra that we have described is called $N = 1$ supersymmetry, because there is one generator Q_α. Models with N generators are referred to as having N-*extended* supersymmetry, and their multiplets become increasingly complicated. $N > 8$ is not normally considered because it requires helicities of 5/2 and higher. Besides, we also have more than one graviton for these cases. $N = 4$ is the largest value of N that we see coming from heterotic strings, however $N = 1$ is favoured because it allows chiral generations to exist (in the sense that we may have different representations of left and right handed fields).

Appendix C: Gauge invariant supersymmetry

Using the technology presented in the main text, we are already able to construct gauge invariant interactions such as Yukawa couplings using the superpotential. However gauge invariance also requires some modification of the kinetic terms, as well as a Yang-Mills term. This appendix is devoted to the task of determining gauge invariant expressions for these and is included for completeness.

Gauge invariant kinetic terms

Our first task is to introduce *vector*-superfields [5]. Vector superfields are the other convenient way of deriving supersymmetry invariant Lagrangians. Like the chiral-superfields, they are dimensionless, but obey

$$V(x,\theta,\overline{\theta}) = \overline{V}(x,\theta,\overline{\theta}). \tag{65}$$

In fact (in a roundabout way) we already know how to construct a supersymmetry invariant Lagrangian from such a field; the kinetic energy terms may be derived from the vector superfield

$$K(x,\theta,\overline{\theta}) = \overline{\Phi}\Phi \tag{66}$$

[5] I should point out that the calculations required to produce the results of this appendix become more tedious as we progress. Because of this it is not possible (and would not be very useful) to give their derivation in the same detail that was possible for the superpotential, and the main aim will be to present the flow of the argument.

Clearly this obeys eq.(65), and taylor expanded in thetas, it becomes

$$
\begin{aligned}
K(x,\theta,\overline{\theta}) &= \phi^*\phi + \sqrt{2}(\theta\chi + \overline{\theta}\overline{\chi}) + \theta\theta\phi^* F + \overline{\theta\theta}\phi F^* \\
&\quad -\theta\sigma^\mu\overline{\theta}\left(\chi\sigma\overline{\chi} + i\phi\partial_\mu\phi^* - i\phi^*\partial_\mu\phi\right) \\
&\quad +\theta\theta\overline{\theta}_{\dot\alpha}\left(i/\sqrt{2}\partial_\mu(\overline{\sigma}^{\mu\dot\alpha\alpha}\chi_\alpha\phi^*)\right) \\
&\quad -\overline{\theta\theta}\theta^\alpha\left(i/\sqrt{2}\partial_\mu(\sigma^\mu_{\alpha\dot\alpha}\overline{\chi}^{\dot\alpha}\phi)\right) \\
&\quad +\theta\theta\overline{\theta\theta}\left(\frac{i}{2}(\partial\overline{\chi}\sigma\chi + \partial\chi\sigma\overline{\chi}) - \frac{1}{2}|\partial\phi|^2 + \frac{1}{4}\phi\partial^2\phi^* + \frac{1}{4}\phi^*\partial^2\phi + FF^*\right). \qquad (67)
\end{aligned}
$$

Notice that the series terminates at the $\theta\theta\overline{\theta\theta}$ term, which is the highest possible term. Also notice that, apart from some total derivatives, the final term is the kinetic energy piece of the Lagrangian. Traditionally, the $\theta\theta\overline{\theta\theta}$ term of a vector field is referred to as the *D-term*, so that we may now write the kinetic energy more succinctly as

$$
\mathcal{L}_{KE} = \overline{\Phi}\Phi\Big|_D . \qquad (68)
$$

As we have seen this term transforms into a total derivative under a supersymmetry transformation. In fact this is a general property of D-terms, which is perhaps not so surprising when we realise that D-terms, like F-terms, are auxilliary fields which do not propagate.

In order to show this we need to find the supersymmetry transformations for the components of an arbitrary vector superfield, which we can write in its taylor expanded form as,

$$
\begin{aligned}
V(x,\theta,\overline{\theta}) &= C(x) + \theta\zeta(x) + \overline{\theta\zeta}(x) + \theta\theta G(x) + \overline{\theta\theta}G^*(x) \\
&\quad -\theta\sigma^\mu\overline{\theta}v_\mu(x) \\
&\quad +i\theta\theta\overline{\theta}_{\dot\alpha}\left(\overline{\lambda}(x)^{\dot\alpha} + \overline{\sigma}^{\mu\dot\alpha\alpha}\partial_\mu\zeta_\alpha\right) \\
&\quad -i\overline{\theta\theta}\theta^\alpha\left(\lambda_\alpha(x) - \sigma^\mu_{\alpha\dot\alpha}\partial_\mu\overline{\zeta}^{\dot\alpha}\right) \\
&\quad +\theta\theta\overline{\theta\theta}\left(\frac{1}{2}D(x) + \frac{1}{4}\partial^2 C(x)\right). \qquad (69)
\end{aligned}
$$

The spin one component v^μ lends its name to the entire vector superfield. In gauge superfields, the components λ, $\overline{\lambda}$ and v^μ will be the gaugino, anti-gaugino and gauge boson respectively. There are a number of ways to determine the supersymmetry transformations of the components in this multiplet. We could perform the same operation as in the text for the chiral supermutliplet (*i.e.* begin with $\delta_\xi C(x) = \xi\zeta + \overline{\xi\zeta}$, and continue up through the multiplet, checking at each stage that the supersymmetry algebra operates correctly on each component), or we could deduce it by comparison with $K(x,\theta,\overline{\theta})$, or most simply, we could extract the components of

$$
V(x + i\theta\sigma\overline{\xi} - i\xi\sigma\overline{\theta}, \theta + \xi, \overline{\theta} + \overline{\xi}) = V(x,\theta,\overline{\theta}) + \delta_\xi V = V(x,\theta,\overline{\theta}) + (\xi Q + \overline{\xi Q})V \qquad (70)
$$

where $Q, \overline{Q}$ are the differential operators defined in the introduction. The result for the last three components is

$$
\begin{aligned}
\delta_\xi\lambda^A_\alpha(x) &= i\xi_\alpha D^A - \frac{1}{2}(\sigma^\mu\overline{\sigma}^\nu)_\alpha^\beta\xi_\beta F^A_{\mu\nu} \\
\delta_\xi v^A_\mu(x) &= -i\left(\xi\sigma^\mu\overline{\lambda}^A + \lambda^A\sigma^\mu\overline{xi}\right) - \partial_\mu\left(\xi\sigma^\mu\zeta^A + \overline{\xi\zeta}^A\right) \\
\delta_\xi D^A(x) &= \left(\xi\sigma^\mu\overline{\partial}_\mu\lambda^A + \partial_\mu\lambda^A\sigma^\mu\overline{\xi}\right). \qquad (71)
\end{aligned}
$$

The D-term transforms into a total derivative as required. Having determined this fact, let us now try to introduce gauge invariance into our kinetic term. Under a gauge transformation, we wish our supermultiplets to retain all their properties, so that the natural extension of the transformation

$$\phi_i \to \phi_i' = \left(e^{i\lambda}\right)_{ij} \phi_j \tag{72}$$

(different λ, with $\lambda_{ij} = \lambda^A T^A_{ij}$, and A, i, j as group indices) is

$$\Phi_i \to \Phi_i' = \left(e^{i\Lambda}\right)_{ij} \Phi_j \tag{73}$$

where now Λ is a chiral superfield (remember that the product of two chiral superfields is another chiral superfield). From now on we shall drop the explicit gauge indices. The first thing to notice is that our kinetic term

$$K'(x, \theta, \overline{\theta}) = \overline{\Phi} e^{-i\overline{\Lambda}} e^{i\Lambda} \Phi \tag{74}$$

is not gauge invariant, since the two exponentials do not cancel. It can be made gauge invariant by sandwiching another superfield with the correct transformation properties, between the chiral superfields. This field is the *gauge*–superfield, and since the kinetic term must transform as a vector superfield, it is also a vector superfield. Conventionally the gauge superfield is defined as $gV_{ij} = g_A V_A T^A_{ij}$, such that the kinetic term is

$$K(x, \theta, \overline{\theta}) = \overline{\Phi} e^{2gV} \Phi. \tag{75}$$

The constant g will turn out to be the conventional coupling constant. For gauge invariance, under a gauge transformation V must transform so that it cancels the troublesome phases in Eq.(74). That is we must have

$$e^{2gV} \to e^{i\overline{\Lambda}} e^{2gV} e^{-i\Lambda}. \tag{76}$$

We now have nearly all the ingredients we need for our Lagrangian. The kinetic energy term is found, as before, by extracting the D-term which transforms as a total derivative, but is now gauge invariant too;

$$\mathcal{L}_{KE} = \overline{\Phi} e^{2gV} \Phi \Big|_D \tag{77}$$

In order to extract the D-term, we need to expand the exponential. To do so, we first make a gauge transformation such that $C, \zeta, \overline{\zeta}, G = 0$. As we shall see below, by examining infinitessimal gauge transformations, one finds that this gauge, the *Wess-Zumino gauge*, is always possible. In the Wess-Zumino gauge, things are particularly simple since,

$$\begin{aligned} V_A(x, \theta, \overline{\theta}) &= -\theta\sigma^\mu\overline{\theta} v_\mu(x) + i\theta\theta\overline{\theta}\overline{\lambda}(x) - i\overline{\theta}\overline{\theta}\theta\lambda + \frac{1}{2}\theta\theta\overline{\theta}\overline{\theta} D(x), \\ V_A V_B &= -\frac{1}{2}\theta\theta\overline{\theta}\overline{\theta} v^\mu_A v_{A\mu}, \\ V_A V_B V_C &= 0 \end{aligned} \tag{78}$$

so that $e^{2gV} = 1 + V + V^2/2$. After some lengthy manipulations and Fierz-rearranging, the kinetic terms in the Lagrangian are found to be

$$\begin{aligned} \mathcal{L}_{KE} &= -i\chi\sigma^\mu\overline{\mathcal{D}}_\mu\overline{\chi} - (\mathcal{D}_\mu\phi)^\dagger\mathcal{D}^\mu\phi + F^*F \\ &\quad + i\sqrt{2}g\left(\phi^*\lambda\chi - \overline{\chi}\overline{\lambda_A}\phi\right) + g\phi D\phi^*. \end{aligned} \tag{79}$$

Note that there are suppressed indices floating around here. For example $g\phi D\phi^* \equiv \sum_{Aij} g_A \phi_i D_A T^A_{ij} \phi^*_j$ and $g\phi^*\lambda\chi \equiv \sum_{Aij} g_A \phi^*_i T^A_{ij} \lambda^\alpha_A \chi_{i\alpha}$ where g_A allows for group factors and $U(1)$ charges. The derivatives are the usual covariant ones;

$$
\begin{aligned}
\mathcal{D}_{\mu ij} &= \delta_{ij}\partial_\mu + i g_A T^A_{ij} v_{A\mu} \\
\mathcal{D}_{\mu AB} &= \delta_{AB}\partial_\mu + g_A f^A_{BC} v^{C\mu}.
\end{aligned}
\tag{80}
$$

In order to maintain the Wess-Zumino gauge, supersymmetry transformations must be accompanied by the required gauge transformation. That is one performs an infinitessimal supersymmetry transformation on the gauge superfield. This generates new $C, \zeta, \overline{\zeta}, G$ terms which are found to cancel for a certain additional infinitessimal gauge transformation on V. Thus the purpose of Wess Zumino gauge is to make explicit all the physical degrees of freedom. (Explicitly, the supersymmetry transformations give $\delta_\xi C = 0$, $\delta_\xi \zeta = -i\sigma^\mu \overline{\xi} v_\mu$, $\delta_\xi G = 2\overline{\xi}\overline{\lambda}$, and the gauge transformation which cancels it is given by $\Lambda = \{0, \frac{i}{\sqrt{2}}\sigma^\mu \overline{\xi} v_\mu, -\overline{\xi}\overline{\lambda}\}$.) The combined transformation we shall refer to as a *supergauge* transformation δ^{SG}_ξ. Roughly speaking, this changes the derivatives in the supersymmetry transformations into covariant ones. That is, for the chiral superfields we have

$$
\begin{aligned}
\delta^{SG}_\xi \phi &= \sqrt{2}\xi\chi \\
\delta^{SG}_\xi \chi &= i\sqrt{2}\sigma^\mu \overline{\xi} \mathcal{D}_\mu \phi + \sqrt{2}\xi F \\
\delta^{SG}_\xi F &= i\sqrt{2}\overline{\xi}\overline{\sigma}^\mu \mathcal{D}_\mu \chi + 2g\phi\overline{\lambda}\overline{\xi},
\end{aligned}
\tag{81}
$$

And for the gauge boson and gauginos and D-field we have,

$$
\begin{aligned}
\delta^{SG}_\xi \lambda^A_\alpha(x) &= i\xi_\alpha D^A - \frac{1}{2}(\sigma^\mu \overline{\sigma}^\nu)^\beta_\alpha \xi_\beta F^A_{\mu\nu} \\
\delta^{SG}_\xi v^A_\mu(x) &= -i\left(\xi\sigma^\mu \overline{\lambda}^A - \lambda^A \sigma^\mu \overline{xi}\right) \\
\delta^{SG}_\xi D^A(x) &= \left(\xi\sigma^\mu \overline{\mathcal{D}}^A_B \mu \lambda^B + \mathcal{D}^A_B \lambda^B \sigma^\mu \overline{\xi}\right).
\end{aligned}
\tag{82}
$$

The Yang Mills Terms

This completes the kinetic terms for the matter fields. The final terms that we need are the Yang-Mills terms. These can be found as in the text by forming an additional chiral superfield $\mathcal{W}_\alpha(y,\theta)$ out of the three physical fields, λ^A_α, $F^A_{\mu\nu}$, and D^A. Since the gaugino is the component with lowest dimension, we begin by defining;

$$
\mathcal{W}_\alpha(x,0,0) = -i\lambda_\alpha(x).
\tag{83}
$$

This chiral superfield is a function with dimension 3/2 and helicity 1/2, and is the *field-strength superfield*. Its general form may be found by repeating what we did in the main text but this time for finite supergauge transformations. Making an arbitrary supergauge transformation, and using the transformation rules for the components above we find,

$$
\begin{aligned}
\mathcal{W}^A_\alpha(y,\theta) &= \delta^{SG}_\theta \mathcal{W}^A_\alpha(x,0) \\
&= -i\lambda^A_\alpha(y) + \theta_\beta\left(\delta^\beta_\alpha D^A + \frac{i}{2}(\sigma^\mu\overline{\sigma}^\nu)^\beta_\alpha F^A_{\mu\nu}\right) + \theta\theta\sigma^\mu_{\alpha\dot{\alpha}} \mathcal{D}^{AC}_\mu \overline{\lambda}^{\dot{\alpha}}_C
\end{aligned}
\tag{84}
$$

where

$$
F^A_{\mu\nu} = \partial_\mu v^A_\nu - \partial_\nu v^A_\mu - g_A f^A_{BC} v^A_\mu v^B_\nu .
$$

The required, supergauge invariant, Lagrangian can be formed in the usual way by taking the F-component of a combination of these superfields; it is given by

$$\begin{aligned}\mathcal{L}_{YM} &= \frac{1}{4}\mathrm{tr}\left(\mathcal{W}^{\alpha}_{A}\mathcal{W}^{A}_{\alpha}\Big|_{F} + \overline{\mathcal{W}}^{A}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha}_{A}\Big|_{F^*}\right) \\ &= -\frac{1}{4}F^{A}_{\mu\nu}F^{\mu\nu}_{A} - i\lambda^{A}\sigma^{\mu}\mathcal{D}^{C}_{\mu A}\overline{\lambda}_{C} + \frac{1}{2}D^{A}D_{A} \end{aligned} \tag{85}$$

in the Wess-Zumino gauge.

The full $N = 1$ supersymmetric, gauge invariant Lagrangian is given by the sum of Eqs.(77,15,85);

$$\mathcal{L} = \mathcal{L}_{KE} + \mathcal{L}_{int} + \mathcal{L}_{YM} \tag{86}$$

This is the main result of this appendix. Those who would rather avoid any discussion of superfields at all, could simply verify that this lagrangian is indeed invariant under the general supergauge transformations given above. Notice again, that the auxilliary D field may be easily removed by using its equations of motion;

$$D^{A} = g_{A}\phi^{*}_{i}T^{A}_{ij}\phi_{j}. \tag{87}$$

The remainder of this appendix expands a little on the role played by the chiral $\mathcal{W}_{\alpha}$ field; it may be skipped at the reader's discretion.

In standard gauge theory, the gauge field is the connection required for covariant derivatives. These are quantities which, under a gauge transformation

$$\phi_i \to \phi'_i = \left(e^{i\lambda}\right)_{ij}\phi_j \tag{88}$$

transform covariantly,

$$\mathcal{D}_{\mu}\phi_i \to (\mathcal{D}_{\mu}\phi_i)' = \left(e^{i\lambda}\right)_{ij}\mathcal{D}_{\mu}\phi_j. \tag{89}$$

The connection is everything other than the derivative in $\mathcal{D}_{\mu} = \partial_{\mu} - igv_{\mu}$, and the transformation of v_{μ} is therefore defined so that

$$\mathcal{D}'_{\mu} = \left(e^{i\lambda}\right)_{ij}\mathcal{D}_{\mu}\left(e^{-i\lambda}\right)_{ij}. \tag{90}$$

Quite generally, there is only one non-trivial tensor that can be made from the the connection, by taking its covariant derivative. This is the curvature, which is given by

$$F_{\mu\nu} = \mathcal{D}_{\mu}A_{\nu} - \mathcal{D}_{\nu}A_{\mu}. \tag{91}$$

So much for standard Yang-Mills theory, but what about its supersymmetric counterpart? The field strength superfield $\mathcal{W}_{\alpha}$, may also be related to the connection in a super/gauge covariant derivative (just as the usual field strength $F_{\mu\nu}$ is related to the gauge bosons). We begin with the derivatives which anti-commute with Q_{α} and $\overline{Q}^{\dot\alpha}$ which we shall attempt to make gauge covariant; these are

$$\begin{aligned} D_{\alpha} &= \frac{\partial}{\partial\theta^{\alpha}} + i\sigma^{\mu}_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_{\mu} \\ D^{\dot\alpha} &= -\frac{\partial}{\partial\theta_{\dot\alpha}} - i\theta^{\alpha}\sigma^{\mu\dot\alpha}_{\alpha}\partial_{\mu}. \end{aligned} \tag{92}$$

We can easily verify that $\{D_\alpha, Q_\alpha\} = \{D_\alpha, \overline{Q}_{\dot\alpha}\} = \{\overline{D}_{\dot\alpha}, \overline{Q}_{\dot\alpha}\} = \{\overline{D}_{\dot\alpha}, Q_\alpha\} = 0$. In addition $\overline{D}_{\dot\alpha} y = 0$. This means that an alternative (and in fact the usual) definition of a chiral superfield is $\overline{D}_{\dot\alpha}\Phi(y,\theta) = 0$. The fact that $\overline{D}_{\dot\alpha}\overline{D}^2 = 0$ gives us a useful *chiral projection operator*,

$$P = -\frac{1}{4}\overline{D}^2, \tag{93}$$

which, when applied any function of $x, \theta, \overline{\theta}$ turns it into a chiral superfield (that is a function of y and θ only). Now consider the derivative

$$D_\alpha \Phi. \tag{94}$$

This derivative is not gauge covariant. A covariant derivative is one which transforms as follows under gauge transformations

$$\begin{aligned} \nabla'_\alpha \Phi' &= \nabla'_\alpha e^{i\Lambda}\Phi \\ &= e^{i\Lambda}(\nabla_\alpha \Phi). \end{aligned} \tag{95}$$

Thus

$$\nabla'_\alpha = e^{i\Lambda}\nabla_\alpha e^{-i\Lambda}. \tag{96}$$

By examining the gauge transformations of V in Eq.(76), we see that

$$\nabla_\alpha = e^{-2gV} D_\alpha e^{2gV} \tag{97}$$

transforms in the correct way, where D_α acts on everything to its right. The connection superfield is defined as the difference between this term and D_α. *i.e.*

$$\Gamma_\alpha = e^{-2gV}(D_\alpha e^{2gV}). \tag{98}$$

Finally the chiral field $\mathcal{W}_\alpha$ which we derived earlier is the chiral projection of this;

$$\mathcal{W}_\alpha = P\Gamma_\alpha. \tag{99}$$

The gauge invariance of the Yang-Mills term is now manifest.

8 Appendix D: R-symmetry and R-parity

As well as the terms in the superpotential of the MSSM there are many other Yukawa couplings that we could have included but didn't. For example,

$$W_{problem} = \mu' L^T \epsilon' H_2 + \lambda' E L^T \epsilon' L + \lambda'' D L^T \epsilon' Q + \lambda''' \epsilon_{abc} U^a D^b D^c, \tag{100}$$

where a, b, c are colour indices, are all invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$. Including *all* of the these terms would lead to disastrously fast proton decay (supressed by only the inverse squark mass squared). Thus, counting over generation indices, we have to explain the absence of 30 possible couplings in the MSSM superpotential. This can be done by the introduction of a global $U(1)_R$ symmetry known as *R–symmetry.* An R–transformation may be defined as a transformation of the spinor coordinates;

$$\begin{aligned} \theta &\to e^{i\alpha}\theta \\ \overline{\theta} &\to e^{-i\alpha}\overline{\theta}. \end{aligned} \tag{101}$$

Under this transformation we may assign a charge, the *R–charge*, to each of the chiral superfields

$$\phi_i'(x,\theta',\overline{\theta}') = e^{i\alpha R_i}\phi_i'(x,\theta,\overline{\theta}). \tag{102}$$

For the elements of the supermultiplet this means

$$\begin{aligned} \{\phi_i,\chi_i,F_i\} &\rightarrow e^{i\alpha R_i}\{\phi_i, e^{-i\alpha}\chi_i, e^{-2i\alpha}F_i\} \\ \{\phi_i^*,\overline{\chi}_i,F_i^*\} &\rightarrow e^{-i\alpha R_i}\{\phi_i^*, e^{-i\alpha}\overline{\chi}_i, e^{2i\alpha}F_i^*\}. \end{aligned} \tag{103}$$

Since the elements transform differently this also means that R transformations do not commute with supersymmetry transformations (*i.e.* in order to be consistent the supersymmetry generators in appendix A must obey $Q \rightarrow e^{-i\alpha}Q$ which may easily be checked). Now let us examine which of the terms in the Lagrangian is invariant under R–transformations. The ϕ kinetic term is invariant only if we assign $R_v = 0$ to the gauge vector, since $R_\phi = -R_{\overline{\phi}}$. Examining the kinetic terms in for example eq.(21), we find $\lambda \rightarrow e^{i\alpha}\lambda$. The supersymmetric Yang-Mills lagrangian (shown in appendix C) is also invariant under this transformation. This leaves the interaction term which looks like

$$\begin{aligned} \mathcal{L}_{int} &= W(\Phi)|_F + \text{h.c.} \\ &= \lambda_{ijk}\Phi_i\Phi_j\Phi_k|_F + \text{h.c.} \end{aligned} \tag{104}$$

By looking at the transformations in Eq.(103), the interaction lagrangian transforms as

$$\lambda_{ijk}\Phi_i\Phi_j\Phi_k|_F \rightarrow e^{i\alpha(\sum_i R_i - 2)}\,\lambda_{ijk}\Phi_i\Phi_j\Phi_k|_F \tag{105}$$

and therefore only the terms with

$$\sum_i R_i = 2 \tag{106}$$

survive in the R–invariant superpotential. A choice of charges which results in the MSSM is

$$R_i = 3B_i + L_i \tag{107}$$

where B_i and L_i are the baryon number and lepton number of Φ_i. Clearly Eq.(106) is not satisfied by any of the 30 couplings in Eq.(100). When supersymmetry is broken (for example by a majorana mass for the gauginos), a residual *discrete* symmetry remains in the lagrangian, given by

$$f_i \rightarrow (-1)^{2S_i+3B_i+L_i} f_i \tag{108}$$

where f_i is an arbitrary field of spin S_i. This particular choice of discrete symmetry is called *R–parity* or *matter–parity*. Other (equally valid) choices are said to be R–parity breaking. For example, one particularly interesting choice, called *baryon–parity* is [35],

$$f_i \rightarrow (-1)^{2S_i+3B_i} f_i \tag{109}$$

In this case only the first two terms in Eq.(100) survive, but nucleon decay is absent. This model is interesting because this discrete symmetry can arise in certain string models (for more details see [35, 36]).

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