1 String Phenomenology Overview

It is often said that string theory has had many successes. That is a theorists assessment. What is meant is that string theory is a surprisingly consistent theory and looks 'right', and more to the point looks like a viable theory of quantum gravity.

Unfortunately string theory is too beautiful and symmetric to look like the real world; one could (and many people do) argue that it has been unsuccessful in the sense that it has not yet made a single prediction that can be or has been tested by experiment. At the time of writing therefore, "string phenomenology" is an embryonic subject. It is merely the process of identifying how we have to hit string theory to make it as ugly as the world we live. We then see if any predictions might emerge, and see if they are likely to match up with our requirements. In other words, we try to guess what string theory will tell us, and then decide if we think it is interesting.

I will take the second question first and outline what predictions would be interesting from a phenomenological point of view. This will also allow us to collect some results that we will need for the rest of the course. In the second part of this lecture I will give a broad outline of some of the approaches that have been followed in an attempt to make string theory look something like the Standard Model. For this 2nd lecture will take the total immersion approach; it will be more seminar style and so will almost certainly be difficult for you to follow. You should think of the second lecture as being mainly for cultural development. Whilst you may not end up knowing what a Calabi-Yau manifold is, for example, you will at least know that such a thing exists, and that it is used to get $N=1$ supersymmetry in 4 dimensions.

The rest of the course is much more pedagogical and is laid out as follows. In the 3rd and 4th lectures I will discuss in detail the fermionic and orbifold constructions, including the derivation of N=1 supersymmetry in 4 dimensional compactifications of heterotic strings. In letures 5/6 I will turn to more recent constructions involving D-branes and type I theories. Lectures 7/8 will be devoted to more phenomenological aspects, including the effective low energy theory and supersymmetry breaking. I will probably leave the more technical aspects of C-Y manifolds until later/never.

1.1 What do we want from String theory?

1.1.1 A consistent low energy field theory

String theory is a consistent theory par excellence. However it is consistent in ways that are sometimes suprising, so it is not always obvious that it will lead to a consistent low energy field theory. As an example consider anomalies. Anomaly cancellation is a requirement of a consistent quantum theory. However, string theory cancels anomalies in a non-field theoretic way - known as the Green-Schwarz mechanism (to be discussed in chapter XXX). This can result in a field theory whose particle content is anomalous. However the anomaly cancellation makes sense at the field theory level if one includes shifts in the dilaton field under gauge transformations. Many string formulations lead to models that can be written in a consistent perturbative framework.

There are several additional requirements for consistency. For example the theory should lead to a low energy perturbative version of the Standard Model (otherwise all links between string theory and phenomenology are lost) and this perturbative theory should have Lorentz invariance. It should be able to describe fermions and gauge bosons, and in addition we require that the theory is stable - i.e. there are no tachyons. (Note that supersymmetry implies absence of tachyons but the converse need not be true and indeed there are tachyon free non-supersymmetric string theories.)

1.1.2 N=1 Supersymmetry

At the moment this is currently the best guess for how Nature avoids the so called hierarchy problem (for an introduction to supersymmetry and the MSSM, see SUSY primer in SUSY primer `http://www.dur.ac.uk/`). This can be understood as follows. Consider a GUT theory that has a GUT breaking scale of say $10^{16}GeV$ and a weak one that is of course around the 10^2GeV mark. These breakings are driven by higgs fields that I will call Φ and h which, respectively, break the GUT gauge group down to the Standard Model one, and the SM one down to $SU(3)_c \times U(1)_{em}$. Phenomenology requires $\langle\Phi\rangle \sim 10^{16}GeV$ (in order to avoid rapid proton decay) and $\langle h\rangle \sim 10^2GeV$. To arrange this we might imagine that the higgs potential looks something like

$$V = -\frac{M_\Phi^2}{2}\Phi^2 + \frac{\lambda_\Phi}{4}\Phi^4 - \frac{m_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\kappa}{2}\Phi^2 h^2$$

This potential has mass terms for both the GUT and electroweak higgses which ideally would be given to us by the underlying fundamental theory, in addition to interactions to make the potential bounded from below. Since we eventually need $\langle h\rangle \ll \langle\Phi\rangle$, the VEV of Φ is

$$\langle\Phi\rangle \approx \sqrt{\frac{M_\Phi^2}{\lambda_\Phi}}$$

With the Φ field at its VEV value we can integrate out its fluctuations and replace it everywhere by $\langle\Phi\rangle$ giving an electroweak VEV of

$$\langle h\rangle = \sqrt{\frac{M_h^2 - \kappa\Phi^2}{\lambda_h}} \approx \sqrt{\frac{M_h^2 - \frac{\kappa}{\lambda_\Phi}M_\Phi^2}{\lambda_h}}$$

We now have to tune M_h^2 and κ very precisely so that they cancel to one part in 10^{26} if we want to get the electroweak higgs VEV. Note that we cannot avoid the problem by arranging for $M_h \sim 10^2GeV$ and setting $\kappa = 0$, since the latter will appear at the quantum level anyway. Indeed, since h and Φ appear in GUT

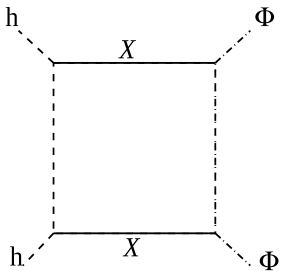


Figure 1: Interactions induced in GUT theories

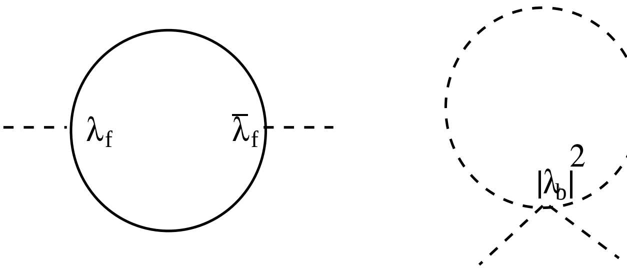


Figure 2: Cancellation of divergences in SUSY

representations, one can always construct the diagram in figure 1 where X are GUT gauge bosons;

Supersymmetry provides superpartner fermions that cancel this interaction. The cancellation is easier to formulate at the level of the effective low energy theory. The potential for the electroweak higgs has divergences coming from the one-loop bubble diagram in figure 2 where $\lambda_b^2 \equiv \lambda_h$. In a supersymmetric theory, there are additional Yukawa interactions of strength λ_f involving fermionic superpartners. The one-loop divergences is of the form

$$\Lambda^2(|\lambda_b|^2 - |\lambda_f|^2)$$

and they cancel because supersymmetry ensures $\lambda_f = \lambda_b$.

$N = 1$ supersymmetry is required because it is the only supersymmetry that allows chiral supermultiplets. Interactions in global supersymmetry are described by the superpotential, W

1.1.3 N=1 supergravity

As well as the gauge interactions, string theory should also provide gravitational interactions. The framework for this is $N = 1$ supergravity in which the kinetic terms and interactions are described by the Kahler potential often denoted

$$G = K(\overline{\Phi}, \Phi) + \log |W|^2$$

and the gauge terms are described by the gauge kinetic function, denoted $f_{ab}(\Phi)$. The derivation is rather complicated (see Appendix A for a summary of supergravity), but in the global supersymmetry limit the Lagrangian can be written

$$L = \int \mathrm{d}\theta^2 \mathrm{d}\overline{\theta}^2 \left(3e^{-K(\overline{\Phi}e^{2gV},\Phi)/3}\right) + \left(\int \mathrm{d}\theta^2 W(\Phi) + f_{ab}W^a W^b + h.c.\right)$$

where V represents general vector superfields that make the expressions gauge invariant, and W^a are the field strength superfields. The fact that everything depends only on the function G becomes evident in the supergravity theory by making a superweyl rescaling of the vielbein. In the above you can think of this as rescaling the θ^2 by a factor $e^{-\log W}$. The scalar part of the Lagrangian contains the usual Einstein-Hilbert action plus a part that depends on the scalar fields

$$L_{scalar} = -e^G(-3 + G_i G^{i\overline{j}} G_{\overline{j}}) \tag{1}$$

where subscript-i denotes differentiation with respect to Φ_i. Meanwhile the gravitino mass is given by

$$m_{3/2} = e^{G/2}$$

and the gauge kinetic function leads to the gauge terms

$$L_{gauge} = -\frac{1}{4}Re(f_{ab})F^{a\,\mu\nu}F^b_{\mu\nu} - \frac{i}{4}Im(f_{ab})F^{a\,\mu\nu}\tilde{F}^b_{\mu\nu}. \tag{2}$$

Note that the imaginary part of f_{ab} is a source of axions, and hence it is important to determine the form of f_{ab} in order to see if it could lead to a possible Peccei-Quinn solution to the strong CP problem.

1.1.4 Soft SUSY breaking

Supersymmetry breaking is obviously a requirement of the eventual phenomenology. This is the most phenomenologically difficult aspect of the MSSM but unfortunately as we shall see string theory only has a limited amount to say about it. The general scale of supersymmetry breaking in supergravity models is given by

$$F^i = e^{G/2} G^{i\overline{j}} G_{\overline{j}}.$$

Phenomenology is more favourable if we assume that the source of supersymmetry breaking does not couple directly to the standard model. That is we assume that there is some "hidden" sector in which the supersymmetry breaking occurs and that all the F_i of the Standard Model fields are zero to first order.

The supersymmetry breaking in the visible sector can then be communicated by gravity. For example, in the canonical case have $G_i \sim \phi_i + W_i/W$. The scalar part of the lagrangian therefore includes a supersymmetry breaking mass-squared term for the scalars of the form$-e^G|\phi_i|^2$; thus in models where the supersymmetry breaking is communicated by gravity the gravitino mass and the mass splitting in the visible sector are similar in magnitude and should both be of order $M_W \sim 100\,GeV$. Phenomenology depends greatly on how the supersymmetry breaking is distributed amongst the different fields. If it is dominated by for example the dilaton, F_S, then the supersymmetry breaking terms in the visible sector are universal and phenomenology is very favourable (see the SUSY primer). At the time of writing it has not been possible to engineer this. Usually other fields, such as moduli fields, are found to dominate, and the resultant flavour dependent supersymmetry breaking leads to large flavour changing processes such as the $K_L - K_S$ mass difference, and neutron electric dipole moments. These are all phenomenologically unnacceptable and explaining how supersymmetry breaking avoids them remains a major challenges.

1.1.5 The MSSM

Particle physics is currently described by the Standard Model whose gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$. Thus string theory should give a model that has either this group or a unified group that can be broken down to this one. There are a many different ways to break down from a unified group, some of the most favourable being

$$\begin{aligned} E_6 &\rightarrow SU(5) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1) \\ SO(10) &\rightarrow SU(4) \times SU(2) \times SU(2) \rightarrow SU(3) \times SU(2) \times U(1) \\ E_6 &\rightarrow SU(3) \times SU(3) \times SU(3) \rightarrow SU(3) \times SU(2) \times U(1) \end{aligned}$$

and so on.

In addition to the gauge requirements there MSSM also has three generations. Note that what we mean here is *nett* number of generations. One important aspect of a theory like the Standard Model that has broken parity, is that one is unable to write mass terms for the light fermions. On the other hand if pairs of multiplets with opposite chiralities are expected to get large masses. In string theory (and more generally in theories compactified from higher dimensions) the nett number of generations is a topological invariant as we shall see.

Apart from these aspects there is also the particle content of the MSSM and the Yukawa couplings in the superpotential. The supepotential looks like

$$W_{MSSM} = \lambda_{U_{ij}} Q_i H_2 U_j^c + \lambda_{D_{ij}} Q_i H_1 D_j^c + \lambda_{E_{ij}} L_i H_1 E_j^c + \mu H_1 H_2,$$

where $i, j = 1..3$ are generation indices. (Consult the SUSY primer for a summary of the MSSM.) In addition here we include a parameter $\mu \sim M_W$. This parameter is required in order to give the correct higgs breaking but note that it is a part of the superpotential and has nothing to do with the soft-supersymmetry breaking which must separately be of the same order. This double fine-tuning looks very unnatural and is commonly referred to as the μ problem. One possible way to evade it is to include a singlet field N that appears as follows;

$$W_N = \lambda N H_1 H_2 + \kappa N^3.$$

A superpotential like this can lead to VEVs of order M_W for the singlet N and the two higgs fields, and so the μ term is generated dynamically. On the down side one has to explain the absence of N^2 terms which would drive the VEVs to much larger values. A Z_3 symmetry is often invoked to do this but it leads to cosmologically unnacceptable electroweak domain walls.

The Yukawa couplings are a particular problem for string theory because they are so hierarchical. It is hard to imagine any fundamental theory being able to explain them convincingly. There are a number of field theoretical possibilities for generating Yukawa hierarchies. One that is commonly considered is the Froggatt-Nielsen mechanism. One imagines that the underlying theory has additional scalars which are singlets with respect to the Standard Model group but that have some U(1) charge. Lets call the fields ϕ and their charges q_ϕ. If the visible fields also have charges, q_{Q_i}, q_{U_j} etc, then Yukawa couplings can only appear with ϕ fields raised to the appropriate power to make them invariant under the additional U(1), so that for example, the effective up quark Yukawa becomes

$$\lambda_{U_{ij}} \sim \left(\frac{\langle\phi\rangle}{M_P}\right)^{-\frac{q_{Q_i}+q_{U_j}+q_{H_2}}{q_\phi}}.$$

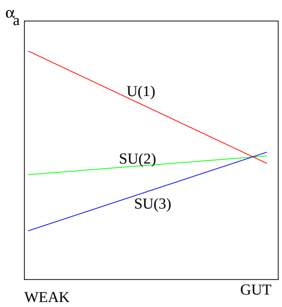
If ϕ gets a VEV of say $0.1M_P$, it is possible to generate a generation hierarchy in Yukawas with a suitable choice of U(1) charges for the different generations.

1.1.6 Gauge unification

Experiment suggests that the gauge couplings run with energy scale as determined by the renormalization group equations. The renormalization group equations of the MSSM take a particularly simple form. At one loop they can be written

$$\frac{dg_a}{dt} = \frac{\beta_a}{16\pi^2} g_a^3$$

where the beta-functions are (-11,-1,3) respectively and t is the log of the renormalization scale. Typically one needs to use the two loop expressions for these and the Yukawa couplings in order to get a sufficiently accurate estimate of the GUT scale quantities from the weak ones. The input is of course experimental and consists of determinations of the couplings at various scales (from the weak mixing angle, the weak boson masses, deep inelastic scattering and so on). Extrapolating up from M_W gives the famous unification of couplings shown schematically below.



The predicted unification scale is around $2 \times 10^{16} GeV$, substantially below the Planck scale expected string scale. However such an apparent unification does not occur in the Standard Model so this is a significant success for the MSSM.

1.1.7 Discrete symmetries

There are a number of discrete symmetries that are of interest. One is CP symmetry which rather general arguments tell us should be a discrete gauge symmetry in string theory. A second discrete symmetry is so-called R-parity, usually invoked in order to avoid fast proton decay. The latter is a danger since, as well as the terms in the superpotential of the MSSM there are many other Yukawa couplings that we could have included but didn't. For example,

$$W_{problem} = \mu' L^T \epsilon' H_2 + \lambda' E L^T \epsilon' L + \lambda'' D L^T \epsilon' Q + \lambda''' \epsilon_{abc} U^a D^b D^c, \quad (3)$$

where a, b, c are colour indices, are all invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$. (The $\epsilon' = i\sigma_2$ contraction indicates the proper contraction between doublets of SU(2); the contractions in the MSSM superpotential are of this form as well.) Including *all* of the these terms would lead to disastrously fast proton decay (supressed by only the inverse squark mass squared). Thus, counting over generation indices, we have to explain the absence of 30 possible couplings in the MSSM superpotential. This can be done by the introduction of a discrete symmetry that reverses the sign of some of the fields given by

$$f_i \to (-1)^{2S_i + 3B_i + L_i} f_i \quad (4)$$

where f_i is an arbitrary field of spin S_i and B_i and L_i are its baryon and lepton number (e.g. for a quark, 1/3 and 0 respectively). This particular choice of discrete symmetry is called *R–parity* or *matter–parity*, but other (equally valid) choices are possible. They are said to be R–parity breaking. For example, one particularly interesting choice, called *baryon–parity* is [7],

$$f_i \to (-1)^{2S_i + 3B_i} f_i \quad (5)$$

In this case only the first two terms in Eq.(3) survive, but nucleon decay is absent. This model is interesting because this discrete symmetry can arise in certain string models (for more details see [7, 8]).

1.2 An overview of how we can make String Theory look 'realistic'

In the previous section we wrote down a rather long wish-list for string theory. However the areas where string theory has any bearing are limited. Throughout the course I will deal with some examples in detail but for the remainder of this lecture I want to put the various developments of string theory in historical context, in order to give a more general overview. At the same time I will outline the main concepts with very broad brush, in order to give some sort of overall framework. In later lectures I will fill topics in detail but inevitably will not have time to cover everything. Because of this I will use this section as an opportunity to highlight where there has been some success in addressing the issues on our wish list. Finally you can also refer back to this section as a glossary. Again I stress that you are not expected to understand all the points below in detail.

1.2.1 Superstrings

The first major success of string theory was the discovery of superstrings. These consist of theories that have world-sheet supersymmetry. Despite this, in the first instance the space-time theory is not supersymmetric and has tachyons. However these faults are removed (and in any case consistency requires) a projection on the states, known as the GSO projection, after which we indeed have absence of tachyons and supersymmetry albeit in 10 space time dimensions. There are two type of set-up that look favourable from a phenomenological point of view.

1.2.2 Heterotic strings

There are a number of different type of strings, but before there was a proper understanding of the non-perturbative objects (such as D-branes) that can exist in them, the likeliest set-up for phenomenology seemed to be the heterotic string. The hetereotic string is a closed string with the unusual feature that it treats the compactification of left and right movering degrees of freedom separately. The left moving sector on the worldsheet is completely bosonic and hence 26 dimensional. The right moving sector is supersymmetric and absence of conformal anomalies requires it to be 10 dimensional. Thus the world sheet degrees of freedom can be summarized by the following table;

Left Movers	Right Movers
$X^i(\tau+\sigma)$; $i=0..9$	$X^i(\tau-\sigma)$
$X^I(\tau+\sigma)$; $I=1..16$	$S^a(\tau-\sigma)$

The additional 16 bosonic degrees of freedom in the left moving sector can be compactified on a lattice. Various considerations (see later, but for example, equal masses in left and right moving sectors) restrict this lattice to be the root lattice of $E_8 \times E_8$ or $Spin(32)/Z_2$. Thus heterotic string theory was initially favoured from a phenomenological point of view because it was the theory with only fundamental strings that simultaneously incorporated a large gauge group and supergravity.

1.2.3 Type I and type IIB constructions with D-branes

In the 3rd and 4th lectures I will discuss in detail the construction of type I and type II models involving D-branes. Models with D-branes have open strings and the interesting aspect of them is that their ends carry gauge indices known as Chan-Paton indices. These lead to gauge groups of either $U(N)$ or $USp(N)$ associated with stacks of N branes. Stringy excitations then come in bi-fundamental representations (just like the (3,2) left handed quarks of the Standard Model) due to the gauge index at either end of the string. General constructions involving type IIB orientifolds (gauging of world-sheet parity transformations, Ω) were developed by Gimon and Polchinski who combined them with Z_2 orbifolds to get N=2 theories in 4-dimensions. Berkooz and Leigh and later Aldazabal et al constructed more general $N = 1$ models with simple orbifolds. I will leave the details of the constructions to the lectures.

The different constructions are summarize in the two figures below. The heterotic construction involves only closed strings in 6 internal dimensions.
On the other hand type I and II strings include open strings that are attached in all possible ways to branes of differing dimension.
An interesting aspect of the latter construction is that the gauge groups do not feel the whole of the compact space. This liberates the string scale from the Planck scale, since the gravitational modes are closed strings that feel the whole space and gravity is therefore diluted by the volume.

The effective 4-D theory generally has too much supersymmetry (phenomenologically we need N=1 because N=2 and N=4 don't have chiral fermions - so we can't treat left handed and right handed fermions differently). We can see this immediately by considering the number of gravitinos. The 4-D gravitinos just come from the dimensional reduction of the single 10-D gravitino. But a 10-D fermion has far more degrees of freedom that a 4-D one (in D space-time dimensions there are $2^{(D-2)/2}$ elements in a Majorana fermion). All those extra degrees of freedom have to be removed otherwise, from the 4-D point of view, it looks like we have more gravitinos and hence too many supersymmetries.

The 6-dimensional compact space has to have special properties if it is to remove all but one of the gravitinos that wind up in the 4-D theory. There are number of approaches that we can use and here I will outline two of them, concentrating on the heterotic string.

1.2.4 How to get N=1 – difficult: Calabi-Yau spaces

The 4 dimensional effective theory arises on compactification. The reasonable assumption to make is that the 10 dimensional universe is compactified down to a product of a 4 dimensional Minkowski space and an internal 6 dimensional compact manifold;

$$M_{10} \to M_4 \times K_6.$$

It is the properties of K_6 that will determine how many light gravitinos we see in the 4-D model. K_6 has a structure that is given by the VEVs of the fields describing the space, i.e. the internal metric, and equivalently the corresponding spin connection, ω, which describes how fermions transform as they are transported around the internal space. As specified in the previous subsection we want the compactification to preserve N=1 supersymmetry.

In order to achieve $N = 1$ the spinors will need to transform non-trivially when parallel transported around a closed path on the compactification manifold. We can see this as follows. Fermion transformations under parallel transport can be written as

$$\psi \to U\psi.$$

On a small path with area tensor given by Δ^{IJ} we have $U = 1+\Delta^{IJ}[D_I, D_J]+...$, where I, J label the 6 internal coordinates of the manifold, and D_I are the covariant derivatives of spinors;

$$D_I = \partial_I + \omega_I^{JK} M^{JK}$$

where M^{JK} the Lorentz generators in matrix form. The group generated by the matrices U is called the *holonomy* group of the manifold. Now, ψ begins life as a 10-D spinor (i.e. it has $2^{8/2} = 16$ elements that can be split into a product of 4-D x 6-D spinors – $4 \times 2^{(6-2)/2}$) so U is a matrix that can act on all the spinor indices, but for a sensible conclusion we must restrict it to transform only the internal K_6 elements of the fermion. From a 4-D point of view the 10-D gravitino looks like a normal 4-D one with an internal spinor index, i.e. the spinor representation of $SO(6)$ or equivalently the 4 of the isomorphic group, $SU(4) \equiv SO(6)$. Thus, if the compactification has trivial holonomy (i.e. K_6 is the six dimensional torus T_6) we have $U = 1$ (by definition) and we don't remove any gravitino degrees of freedom. We then find 4 gravitinos in the 4-D theory, and hence N=4 supersymmetry.

On the other hand if we just allow the most general manifold, all 4 components of the fermions will be transformed by parallel transport and all supersymmetries will be broken by the manifold (heuristically, we have effectively a non-zero VEV for $\langle \omega_I^{JK} \rangle$ which looks like a mass term for the gravitinos and they become heavy. A similar phenomenon can happen for the gauge fields, called Wilson line breaking, which we will dicuss shortly.)

To end up with $N = 1$ supersymmetry then, the required holonomy has to leave one 4-D fermion invariant. To proceed note that any 4 of $SU(4)$ can be transformed by a rotation into;

$$\varepsilon = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \varepsilon_0 \end{pmatrix}.$$

In this form it is obvious that the required holonomy has to be the group of $SU(3)$ rotations in the 3x3 submatrix acting on the upper three entries. Thus only the 4th component from any 4 of $SU(4)$ satisfies $\varepsilon = U_{SU(3)}\varepsilon$. Conversely, for any 4 of $SU(4)$ there is a manifold of $SU(3)$ holonomy that leaves it invariant.

Finding manifolds of $SU(3)$ holonomy is non-trivial. However progress can be made by examining the implications of having a covariantly constant spinor. For example $[D_I, D_J]\varepsilon = 0$ implies $R_{IJ} = 0$; i.e. the manifold is Ricci flat. In addition it turns out that it has to be $Kahler$ (i.e. a complex manifold with complex coordinates z_i whose metric is given by $g_{i\bar{j}} = \partial_i \overline{\partial_j} \phi$ where ϕ is a function called the Kahler potential) and have vanishing first Chern class (a topological property that need not condern us). Given this information, one can then utilize a theorem due to Calabi and Yau; A Kahler manifold of vanishing first Chern class admits a Kahler metric of $SU(3)$ holonomy. This allows us to generate many candidates.

Believe it or not, this has rather immediate implications for phenomenology. The ten dimensional theory has a Bianchi identity which arises as a result of the Jacobi identity on covariant derivatives

$$[D_{[\mu}, [D_\nu, D_{\lambda]}] = 0.$$

Because the covariant derivative contains both spin connections and gauge connections the Bianchi identity connects the two sectors. Under certain assumptions we can satisfy it by having

$$Tr(R \wedge R) = \frac{1}{30} Tr(F \wedge F).$$

The factor $1/30$ arises because of the different representations. Although the Bianchi identity adds no new physics, it must be satisfied for consistency and this provides a constraint on the theory. The simple way to satisfy it is known as the standard embedding in which the $SU(3)$ holonomy on the gravity side is embedded in the gauge side. That is we set part of the $E_8 \times E_8$ gauge fields equal to the spin connection, ω, so that schematically the gauge fields have a VEV

$$A = \begin{pmatrix} 0 & 0 \\ 0 & \omega \end{pmatrix}.$$

Thus we need to find an SU(3) subgroup of one of the E_8's for the embedding. In the simplest case the gauge group is broken to

$$E_8 \times E_8 \to SU(3) \times E_6 \times E_8.$$

This looks promising because E_6 allows different numbers of generations of different chirality, whereas E_8 is non-chiral - i.e. it has equal numbers of modes of each chirality. An assumption that is commonly made for heterotic phenomenology is that the hidden sector (where supersymmetry breaking occurs) is in the 2nd E_8 factor. The generations of E_6 appear in 27 and $\overline{27}$ representations. In the $SO(10) \times U(1)$ decomposition, the 27 is built from $(16, -1) + (10, 0) + (1, 1)$. It also has a nicely symmetric decomposition under $SU(3) \times SU(3) \times SU(3)$; $27 = (3, 3, 1) + (3, 1, 3) + (1, 3, 3)$ which is an possible route for further gauge breaking.

There is a second interesting phenomenological outcome which arises from a relation between the topological properties of the compactification manifold and the number of generations. We can see this by looking at the dimensional reduction of the 10-D Klein Gordon equation. Massless modes satisfy

$$\triangle_{10}\psi = (\triangle_4 + \triangle_6)\psi = 0.$$

In the 4-D theory this is the Klein-Gordon equation for a particle with mass given by

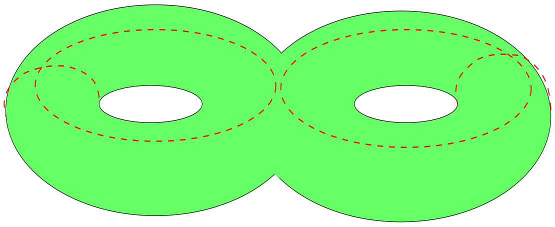
$$m^2\psi = \triangle_6\psi.$$

Thus the massless sector is given by the zero eigenvalues of $\triangle_6$, i.e. the number of independent harmonic forms we can write down for the compactification manifold K_6. But the number of harmonic forms of degree-p is given by the $p - th$ Betti number, b_p, the number of independent p-dimensional cycles on a manifold. For example the object in figure 3 has $b_1 = 4$ (shown) and $b_0 = b_2 = 1$ (trivial). This gives us the number of massless generations plus anti-generations. However we would like to know the nett number of generations, $n_+ - n_-$ (i.e. the effective number of low energy generations). It turns out that this is also given by a topological quantity - the Euler number χ (equal to $2 - \textit{the genus}$, where the genus is the number of holes or handles);

$$n_+ - n_- = \frac{\chi(K_6)}{2}.$$

The object in figure 3 has genus 2 and therefore $\chi = 0$. The simplest Calibi-Yau manifolds are hypersurfaces of complex projective spaces. These unfortunately have very high Euler number (-200, -176 , -144 or -128). More reasonable values can be obtained by dividing the manifold by a discrete symmetry group that acts with no fixed points (i.e. freely, unlike orbifolds which we will discuss presently where there *are* fixed points). The resulting manifolds are now non-simply connected, but the generation numbers are unfortunately always even.

Figure 3: There are 4 independent 1-cycles on a dog-chew. A doughnut has 2 and a pretzel has 6. (There seem to be no foodstuffs with more than 6 independent 1-cycles.)



1.2.5 How to get N=1 – Less difficult: Orbifolds

The Calabi-Yau models are difficult to construct (as you will by now appreciate) and extremely difficult to do any meaningful phenomenology with, and we would certainly prefer simpler flat space solutions. As we have seen those based on torus compactifications have $N = 4$ supergravity in 4 dimensions. The proposal of Dixon et al [2, 3] was to compactify on orbifolds, spaces that are projected out with some non-freely acting discrete symmetry (i.e. with fixed points).

The simplest example of such a space is a cone. We can make a cone by beginning with the two dimensional plane and identify points under a rotation. In complex coordinates, we identify $z = e^{2\pi i/n}z$. This divides the complex plane into n identical segments. A single segment has the edges identified as in figure 4a. The second orbifold is T_2/Z_2. The torus is defined by identifying

$$\begin{aligned} z &= z + i \\ z &= z + 1, \end{aligned}$$

and the Z_2 projection is the identification $z = -z$. Finally we show the torus defined by

$$\begin{aligned} z &= z + e^{\pi i/3} \\ z &= z + 1, \end{aligned}$$

divided by the Z_3 projection $z = e^{2\pi i/3}z$.

The Z_3 case is interesting for phenomenology because one can directly construct an $N = 1$ model. First complexify the six internal coordinates to three complex z_i defining T_6;

$$\begin{aligned} z_i &= z_i + e^{\pi i/3} \\ z_i &= z_i + 1. \end{aligned}$$

Then project it out with the single Z_3 identification

$$\begin{aligned} z_1 &\to e^{2\pi i/3}z_1 \\ z_2 &\to e^{2\pi i/3}z_2 \\ z_3 &\to e^{-4\pi i/3}z_3 \end{aligned}$$

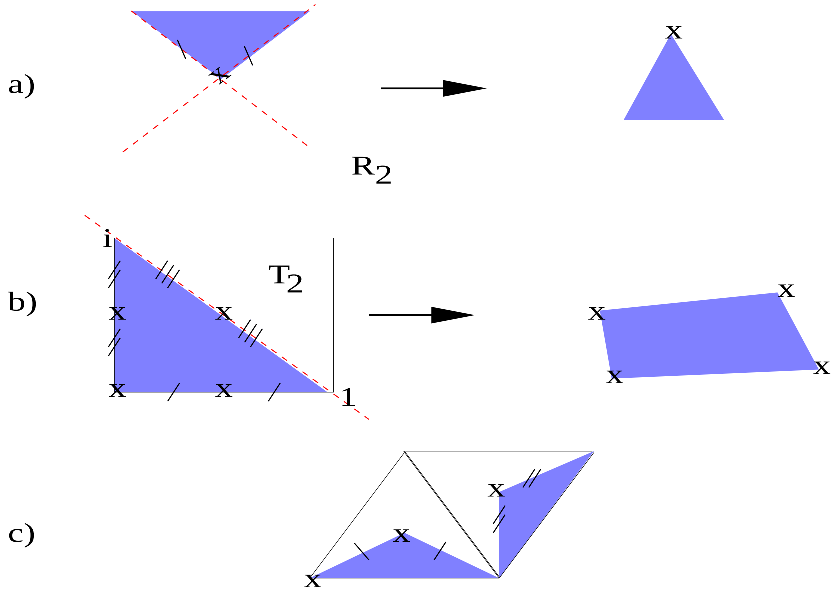


Figure 4: Some common orbifolds; R_2/Z_4 which is a cone with deficit angle 270^o at the tip; T_2/Z_2 which is a cushion with four corners; and T_2/Z_3. The edges with equal strokes are identified.

(all at the same time) which is usually written more succinctly as

$$z_i = e^{2\pi i v_i} z_i \ \ ; \ \ v = (\frac{1}{3}, \frac{1}{3}, \frac{-2}{3}).$$

The Z_3 acts slightly differently on one of the internal degrees freedom in order to satisfy modular invariance constraints (i.e. for consistency of one-loop amplitudes). Modular invariance also constrains how the left-handed (gauge side) behaves. In fact the constraints can be satisfied by again embedding the Z_3 projection in the gauge side (there are other possibilities). That is we separate out 3 bosonic coordinates from the gauge side and fermionize them into complex fermions with the same CFT, $\tilde{\psi}_{i=1..3} = e^{iX_i}$. We then apply the Z_3 projection simultaneously on both sides. That is the projection becomes

$$(z, \psi, \tilde{\psi}) \rightarrow e^{2\pi i(v,v,\tilde{v})}(z, \psi, \tilde{\psi}).$$

This projects out some of the gauge bosons and results in the breaking $E_8 \times E_8 \rightarrow SU(3) \times E_6 \times E_8$. Indeed this is evident from the fact that the Z_3 rotation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

is one of the Cartan generators of $SU(3)$ and hence commutes with it. We will see this in detail later, but we are already able to confirm the fact that only one gravitino remains in the 4-D theory. Note that on the supersymmetric side the fermions transform in the same way as the bosons,

$$\psi_i = e^{2\pi i v_i}\psi_i \ ; \ \ v = (\frac{1}{3}, \frac{1}{3}, \frac{-2}{3}).$$

The $N = 4$ gravitino multiplet comes from the states in the R-NS sector (for now just take this to mean zero vacuum energy for the left movers). Thus massless states involve only the zeroth mode in the normal-mode expansion of the fermions; they can be written

$$\eta = \psi_{0,0}^{n_1}\psi_{1,0}^{n_2}\psi_{2,0}^{n_3}\psi_{3,0}^{n_4}\,|0\rangle_R \times \alpha_{-1}^{\mu}|0\rangle_L = |a\rangle_R \times \alpha_{-1}^{\mu}|0\rangle,$$

where the excitations can be $n_i = 0, 1$ and ψ_0 is the transverse zeroth mode of the 4-D space time degrees of freedom, and $\psi_{1,2,3}$ correspond to the 3 complex internal dimensions (6-real). Initially we have $2^4 = 16$ gravitinos, but after the GSO projection we have only an even number of $\psi's$ so this becomes 8. We can then interpret this as four 4-D gravitinos in the spinor representation of $SO(6)_{K_6}$. Thus the $N = 1$ gravitino of the 10-D theory gives us 4 gravitinos in the 4-D theory. However applying the Z_3 projection removes some of these. Indeed the α_{-1} does not transform under the Z_3 so that the only Z_3 invariant states have $n_2 = n_3 = n_4 = 0, 1$ and $n_0 = 0, 1$ leaving a single 4-D gravitino.

1.2.6 Further gauge breaking Wilson lines

Once we have achieved a theory with chiral fermions, the next task is to break the gauge group down to the Standard Model one. A particularly useful trick that one can use on non-simply connected manifolds is Wilson line breaking. Since it will crop up from time to time I will give a brief outline.

Consider the gauge group E_6. In general, when we construct the path-ordered product of gauge elements around a loop, C, we have a gauge rotation

$$U \sim Pe^{\int_C A_\mu dx^\mu}$$

where U is matrix valued. This quantity is gauge invariant and consquently should be independent of the path C. However, on a small path the Wilson loop becomes (by Stokes' law)

$$U \to e^{F_{\mu\nu}\Delta^{\mu\nu}}$$

where $\Delta^{\mu\nu}$ is the area tensor of the closed path. Thus, on simply connected manifolds, when the curvature vanishes, we expect Wilson loops to be unity since we can always contract the path to a point. This is not the case on non-simply connected manifolds (c.f. the dotted lines in figure 3), where we can have vanishing curvature tensors, but Wilson loops that are not equal to unity for paths that are homotopically equivalent to non-trivial cycles. The remaining gauge group is the subgroup that commutes with U.

The situation is precisely the same as when a cosmic string is formed. Far away from the string the gauge field strength vanishes. However the gauge fields themselves go though a gauge rotation if we take a test particle on a path around the cosmic string. The vacuum manifold is consequently not simply connected, the obstruction being the cosmic string itself (i.e. we cannot continuously contract the path to zero through the cosmic string).

Since the Wilson lines are in the internal (K_6) space, we can identify the higgs fields that are responsible for the breaking from the 4-D point of view. Schematically they arise as follows. The Yang-Mills terms are functions of the covariant derivatives

$$(\delta_b^c \partial^\mu - f_{ab}^c A^{a\mu}) A_\nu^b,$$

where the indices run over all ten space time indices. Let there be a Wilson line along internal direction y such that $A_\mu^a(y) = e^{i\lambda^b f_{bc}^a y} A_\mu^c(0)$ (i.e. $A_y^b = i\lambda^b$). Thus $\partial^y A_\nu^a \propto i\lambda^b f_{bc}^a A_\nu^c \neq 0$ for the 6-internal dimensions. The 4-D Lagrangian now gets a mass-squared term

$$\left(i\lambda^b f_{bc}^a A_\nu^c\right)^2$$

which is non-zero for the gauge fields that do not commute with U. The masses of the broken gauge bosons are proportional to the VEV of the component of the gauge field along the Wilson line (in this case A_y^b) which is playing the role of the higgs field.

As an example, let us see how Wilson lines can break the E_6 gauge group down to $SU(3)_c \times SU(3)_L \times SU(3)_R$ and its subgroups. We can specify the Wilson line element in terms of the $SU(3)^3$ subgroup;

$$U_0 = (\alpha) \times \begin{pmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta^{-2} \end{pmatrix} \times \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \gamma^{-1}\delta^{-1} \end{pmatrix}.$$

We choose them to be of this form because at the very least we need to keep the Standard Model gauge group, $SU(3)_c \times SU(2)_L \times U(1)_Y$. When α, β, γ, δ are all cube roots of unity, the three factors clearly all commute with the three $SU(3)$ subgroups and we have $SU(3)_c \times SU(3)_L \times SU(3)_R = E_6/Z_3$. If we instead choose only α to be a cube root of unity (which is an element of $SU(3)$) but γ, β δ to be n'th roots of unity we find the gauge group $SU(3)_c \times SU(2)_L \times U(1) \times U(1) \times U(1)$. Note that the Wilson lines have not reduced the *rank* of the group which contains superfluous $U(1)$ factors. This is a common problem with heterotic theories and they must be eliminated with some other mechanism. In addition they may be anomalous. Of course one might also try to use them to one's advantage by for example trying to implement some kind of Froggatt-Nielsen mechanism for the Yukawa couplings.

1.3 Outline of the course

In lecture 3 I will describe the fermionic approach to the heterotic string and various realistic models. Lecture 4 will deal with orbifolds and Wilson line breaking in particular examples. Lectures 5 will be devoted to general constructions in type I and type II. Lecture 6 will consider some examples. Lecture 7 will look at the effective supergravity theory in both heterotic and type I/II models. Lecture 8 will discuss axions and the Green Schwarz mechanism, and supersymmetry breaking via gaugino condensation, the Sherk-Schwarz mechanism.

Appendix A: N=1 Supergravity

A summary will appear here when I get around to it.

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