Reaction-diffusion models on infinite graphs.

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Outline

Introduction
- Context
- Result
- Scheme

Construction
- Finite IC
- compact IC
- Liggett-Spitzer extension

A Family of IPS
- Tighness
- Martingales
- Gaussian representation
- Convergence

The project

The project

Joint with:

Bernardo da Costa

Daniel Valesin

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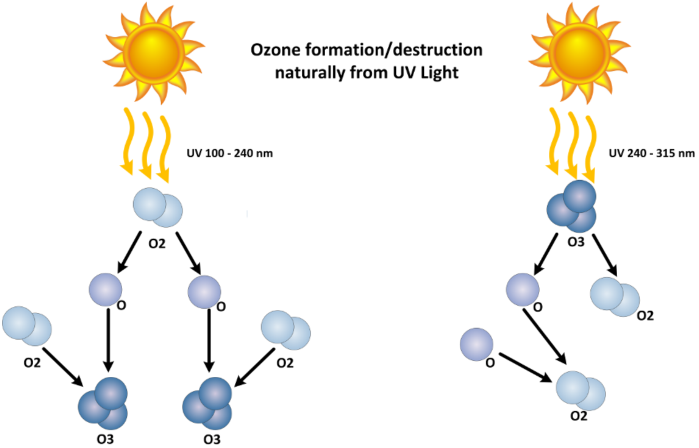
Bernardo da Costa

Daniel Valesin

Federal University of Rio de Janeiro, IMPA
Leiden University, Groningen University
Durham University.

Context



Ozone formation/destruction
naturally from UV Light
UV 100 - 240 nm
O2
O
O
O2
O2
O3
O3
UV 240 - 315 nm
O3
O
O2
O
O2

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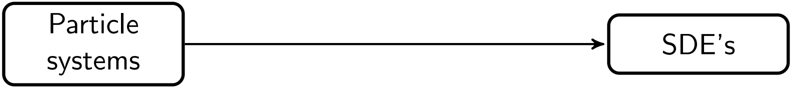

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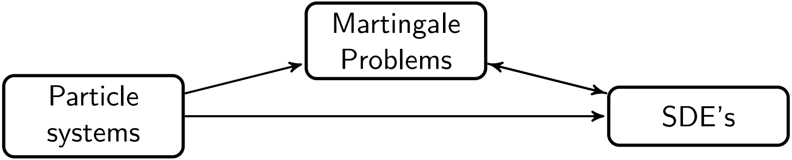

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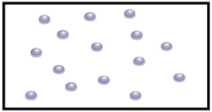

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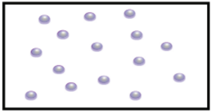

$$\Delta u + F(u) = \partial_t u$$

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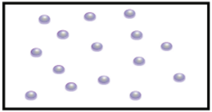

$$\Delta u + F(u) = 0$$

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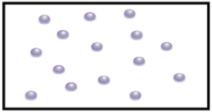

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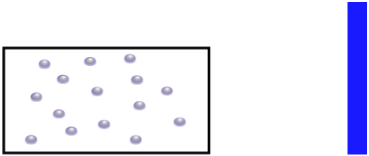

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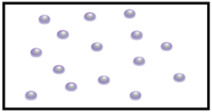

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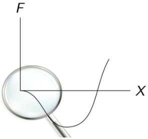

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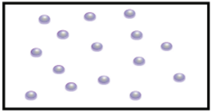

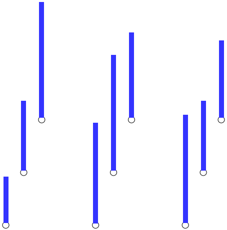

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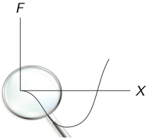

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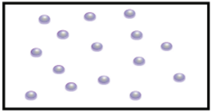

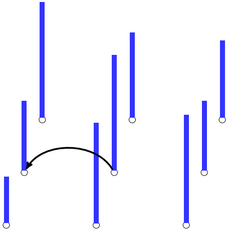

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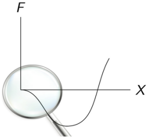

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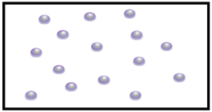

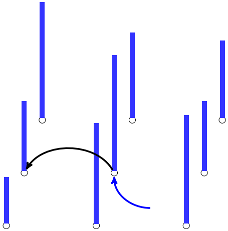

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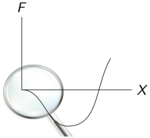

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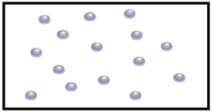

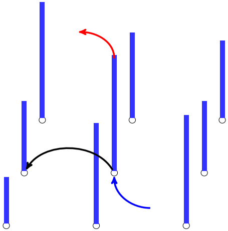

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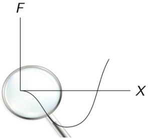

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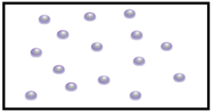

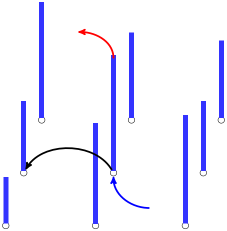

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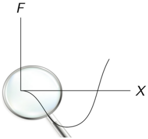

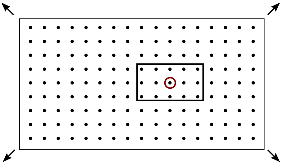

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Fix k, ℓ positive integers.

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$$\begin{cases} d\xi_t(x) = \left[\Delta_p \xi_t(x) - (\xi_t(x))^k\right] dt + \sqrt{(\xi_t(x))^\ell}\, dB_t^x \\ \xi_0(x) = \rho_0(x) \end{cases}$$

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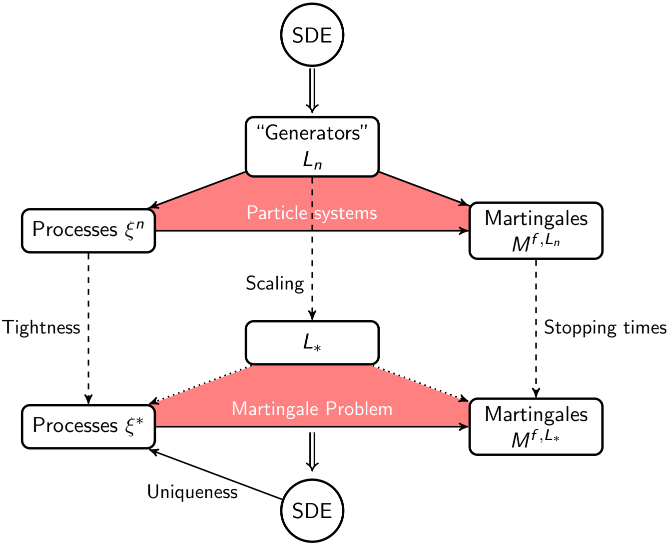
Can be solved as the limit of interacting particle systems.

Scheme


SDE
"Generators"
L_n
Particle systems
Processes ξ^n
Martingales
M^{f,L_n}
Scaling
Tightness
Stopping times
L_*
Martingale Problem
Processes ξ^*
Martingales
M^{f,L_*}
Uniqueness
SDE

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$$\begin{aligned} \Gamma^{x,+}(\eta) &:= \eta + \delta_x, \\ \Gamma^{x,-}(\eta) &:= \eta - \delta_x, \\ \Gamma_x^{(x,y)}(\eta) &:= \eta - \delta_x + \delta_y \end{aligned}$$

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$$0 \le F^+ \le F^-, \quad F^-(0) = 0; \quad F^+ - F^- \text{ is decreasing}$$

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State spaces:

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$$\eta^k = \Gamma^{\mathsf{a}^k}(\eta^{k-1}) \text{ where } \mathsf{a}^k \in \mathcal{M}^k.$$

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$$s^k := \inf\{s \in (s^{k-1}, t] : \exists\, \mathrm{a} \in \mathcal{M}, u > 0, (s,u) \in \mathcal{X}^{\mathrm{a}},\ R^{\mathrm{a}}(\eta^{k-1}) > u\},$$

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almost surely there exists $k_\star$ such that $s^{k_\star} < t$ and $s^{k_\star+1} = \infty$;

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Lemma For $\eta \in E_0 \cap E^m$

(1) **monotonicity-** $m \mapsto \Phi_t^m(\eta)$ is increasing.

(2) **convergence-** For any $\eta \in E_0$, almost surely there exists $m_0 = m_0(\omega, \eta)$ such that

$$[\Phi^m(\omega)]_t(\eta) = [\Phi(\omega)]_t(\eta) \quad \text{for all } m \geq m_0,\ t \geq 0.$$

Liggett-Spitzer extension

Localization function $\qquad E_\alpha = \{\eta \in \mathbb{N}^V : \|\eta\| < \infty\}$

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Extension: $\eta_n \uparrow \eta \in E_\alpha, \qquad \Phi_t(\eta_n) \uparrow \Phi^*_t(\eta) \in E_\alpha$

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$$\begin{aligned} L_n f(\eta) = \sum_{x,y \in V} \eta(x) p(x,y) \left[f(\eta^{x,y}) - f(\eta) \right] + \sum_{x \in V} F_n^+(\eta(x)) \left[f(\eta^{x,+}) - f(\eta) \right] \\ + \sum_{x \in V} F_n^-(\eta(x)) \left[f(\eta^{x,-}) - f(\eta) \right] \end{aligned}$$

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forall $A > 0$,

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$$Lf(\eta) := \sum_{x \in \mathbb{V}} b_x(\eta) \partial_x f(\eta) + \frac{1}{2} \sum_{x,y \in \mathbb{V}} a_{xy} \partial_x \partial_y f(\eta)$$

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Definition: A probability measure P on $C[0,\infty)^d$, under which

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$$M_t^{x,y,L_*} = \zeta_t^*(x)\zeta_t^*(y) - \zeta_0^*(x)\zeta_0^*(y) - \int_0^t L_*(\zeta_s^*(x)\zeta_s^*(y))\, ds$$

Martingale Problem

$$L_* f(\zeta) = \sum_x \partial_x f(\zeta)(\Delta_p \zeta(x) - [\zeta(x)]^k) + \frac{1}{2}\sum_{x,y} \partial_x \partial_y f(\zeta) 1_{x=y} [\zeta(x)]^\ell$$

$L_n \to L_*$: $\qquad f \in \{ f_x, f_{x,y} \mid x, y \in \mathbb{V} \}, \qquad A > 0,$

$$\lim_n \sup_{\|\zeta\|<A} \|\mathcal{L}^n f(\zeta) - \mathcal{L}^* f(\zeta)\| = 0.$$

$$\langle M^{x,L_*}, M^{y,L_*} \rangle_t = 1_{x=y} \int_0^t (\zeta_s^*(x))^\ell \, ds$$

Constructing a Gaussian family

By the Martingale representation Theorem we can construct a family of independent Brownian motions $\{ B^x \}_{x \in V}$ such that almost surely

$$M_t^{x,L_*} = \int_0^t \sqrt{(\zeta_s^*(x))^\ell} \, dB_s^x.$$

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We have found a solution to the SDE on the infinite Graph.

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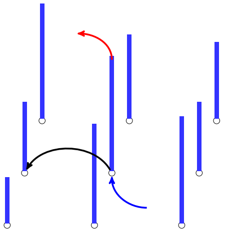

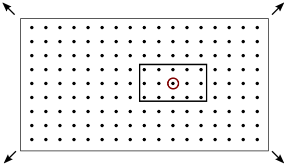

Thank you!

$$\mathrm{P}\left(\sup_{0\le t\le T}\|\eta_t-\eta'_t\|>A\right)\le\frac{e^{(C+1)T}\cdot\|\eta_0-\eta'_0\|}{A}.$$

$$\begin{array}{ccc}\zeta_\cdot^{n,r} & \to & \zeta_\cdot^{*,r}\\ \updownarrow & \le\varepsilon & \\ \zeta_\cdot^{n,R} & \to & \zeta_\cdot^{*,R}\end{array}$$