1 General Information

Welcome to second year! About 1,200 undergraduates take modules provided by the Department. This booklet provides information on second-year modules offered by the department. It also contains summary information on key policies related to assessment and academic progress.

Full details of the department’s policies and procedures are available in the departmental degree programme handbooks at http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/, which also contains an on-line version of the course descriptions contained in this booklet.

Information concerning general University regulations, examination procedures etc., are contained in the Faculty Handbooks (www.dur.ac.uk/faculty.handbook/) and the University Calendar, which provide the definitive versions of University policy.

The Teaching and Learning Handbook (www.dur.ac.uk/teachingandlearning.handbook/) contains information about assessment procedures, amongst other things.

You should keep this booklet for future reference. For instance, prospective employers might find it of interest. It is usual for the second year to be regarded as harder than the first year. Time spent during the vacation in catching up on first-year work which was not understood, or in browsing through some of the recommended books and following the suggestions made at the end of each module description, will be well rewarded. Have a good vacation!

1.1 Useful Contacts

The first point of contact for issues referring to a particular course or module should be the relevant lecturer. For more general questions or difficulties you are welcome to consult the Course Director, your Adviser (if you have one) or Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk).

For issues involving University registration for mathematics modules, please see the Registration Co-ordinator.

Head of Department:
Prof P. Mansfield (CM207, p.r.w.mansfield@durham.ac.uk)
Director of Undergraduate Studies:
Prof S. Ross (s.f.ross@durham.ac.uk)
Director of Support Teaching:
Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Director of Registrations:
Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Department Disability Representative (DDR):
Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Chair of Staff Student Consultation Committee A (SSCCA):
Prof. S. F. Ross (CM218, s.f.ross@durham.ac.uk)
Chair of Staff Student Consultation Committee B (SSCCB):
Dr F. Tari (CM 315, farid.tari@durham.ac.uk)
Chair of Board of Examiners: Prof J. Parker (CM220, j.r.parker@durham.ac.uk)
Deputy Chair of Board of Examiners: Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Secretary of Board of Examiners: Dr H. Gangl (CM108, herbert.gangl@durham.ac.uk)
The Course Directors for students are determined by their programme and level of study as follows:

Students on Mathematics programmes at level one:
Dr P. Bowcock (CM307, peter.bowcock@durham.ac.uk)

Students on Mathematics programmes at level two:
Dr N. Peyerimhoff (CM320, norbert.peyerimhoff@durham.ac.uk)

Students on Mathematics programmes at levels three and four:
Prof. C. S. Chu (CM305, chong-sun.chu@durham.ac.uk)

Students on Natural Sciences and Combined Honours programmes at all levels:
Dr D. Wooff (CM326, d.a.wooff@durham.ac.uk)

Students on programmes other than Mathematics and Natural Sciences and Combined Honours at all levels:
Dr Farid Tari (CM315, farid.tari@durham.ac.uk)

For each Joint Honours degree there is a designated member of staff from each participating department whom you may contact if you wish to discuss any aspect of your joint degree course. The relevant contacts in the Department are as follows:

Joint degrees with Physics:
Dr F. Tari (CM315, farid.tari@durham.ac.uk)

Joint degree with Chemistry:
Dr D. J. Smith (CM231a, douglas.smith@durham.ac.uk)

Joint degree with Education:
Dr V. E. Hubeny (CM306, veronika.hubeny@durham.ac.uk)

We may also wish to contact you! Please keep the Mathematics Office informed of your current term-time residential address and e-mail address.

1.2 Registration for 2H

You will register for the required number of modules in June. You may attend additional modules during the first few weeks of the Michaelmas Term. If you then decide that you want to change one or more of your modules you must see Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk) Any such change must be completed during the first four weeks of the Michaelmas Term.

1.3 Course Information

Term time in Durham is Michaelmas (10 weeks), Epiphany (9 weeks) and Easter (9 weeks). There are 22 teaching weeks, and the last seven weeks are dedicated to private revision, examinations and registration for the subsequent academic year.

Timetables giving details of places and times of your commitments are available on Departmental web pages and noticeboards in the first floor corridor of the Department. It is assumed that you read them!

You may access your own Maths timetable at www.maths.dur.ac.uk/teaching/ and then clicking on the ‘My Maths timetable’ link.

Also, teaching staff often send you important information by e-mail to your local ‘@durham.ac.uk’ address, and so you should scan your mailbox regularly. Note that in October it takes time to sort out groups for tutorials and practicals, and so these classes start in week 2.
1.4 Assessment

Full details of the University procedures for Examinations and Assessment may be found in Section 6 of the Learning and Teaching Handbook, http://www.dur.ac.uk/learningandteaching.handbook/. The Department’s policies and procedures are described in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/. The Department follows the marking guidelines set out by the University Senate:

<table>
<thead>
<tr>
<th>Degree Class</th>
<th>Marking Range(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>70 – 80</td>
</tr>
<tr>
<td>II(i)</td>
<td>60 – 69</td>
</tr>
<tr>
<td>II(ii)</td>
<td>50 - 59</td>
</tr>
<tr>
<td>III</td>
<td>40 – 49</td>
</tr>
<tr>
<td>Fail</td>
<td>0 – 39</td>
</tr>
</tbody>
</table>

With the exception of Numerical Analysis II (MATH 2051), assessment for second year modules is by written examination. For Numerical Analysis II (MATH 2051), 20% of the assessment is based on summative electronic assignments and 80% is based on a written examination. All courses include either summative or formative assessed work, with assignments being set on a regular basis in lecture-based courses.

The purpose of formative and summative assessment of coursework is to provide feedback to you on your progress and to encourage effort all year long.

Regular assignments are marked A-E to the following conventions:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Equivalent Mark</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\geq 80%$</td>
<td>Essentially complete and correct work</td>
</tr>
<tr>
<td>B</td>
<td>60%—79%</td>
<td>Shows understanding, but contains a small number of errors or gaps</td>
</tr>
<tr>
<td>C</td>
<td>40%—59%</td>
<td>Clear evidence of a serious attempt at the work, showing some understanding, but with important gaps</td>
</tr>
<tr>
<td>D</td>
<td>20%—39%</td>
<td>Scrappy work, bare evidence of understanding or significant work omitted</td>
</tr>
<tr>
<td>E</td>
<td>&lt;20%</td>
<td>No understanding or little real attempt made</td>
</tr>
</tbody>
</table>

1.5 Academic progress

The Department is responsible for ensuring that students are coping with the courses and meeting their academic commitments.

For 2nd year modules you are required:
- to attend tutorials
- to submit summative or formative assessed work on time to a satisfactory standard.

Assessed work which is graded D or E is counted as being of an unsatisfactory standard.

Attendance and submission of work is monitored through a database. It is your responsibility to ensure that your attendance is recorded by signing the relevant attendance sheets.

Students who are not keeping up with their commitments will be contacted by course directors to help get them back on track.
Persistent default will result in a formal written warning, which may be followed by the initiation of Faculty procedures.

Full details of academic progress requirements for the department are available in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/.

1.6 Durham University Mathematical Society

MathSoc: Necessary and Sufficient

Durham University Mathematical Society, affectionately known as MathSoc, provides an opportunity for maths students (or anyone with an interest in maths) to meet away from lectures.

We arrange a variety of events throughout the year, such as bar crawls, invited speakers, a Christmas meal, film nights and the highlight of the year – a trip to see Countdown being filmed! So there’s something for everyone.

MathSoc also helps the Maths Department to arrange the Undergraduate Colloquia, where departmental and external lecturers give talks on their current research. These cover a wide range of mathematical topics with previous titles including ‘Dot-dots, zig-zags and plank-planks’ and ‘Defects of integrable field theory’. These are at a level such that anyone with an interest in maths can enjoy them and they aim to inspire an interest in a part of maths you may not have seen before.

We have our own website (www.durham.ac.uk/mathematical.society), where you will find all the most up-to-date information about the society. Here you will also find our second-hand book list, which has many of the books needed for courses for much cheaper than you will find them in the shops. Last year people saved up to £50 by using this service!

If you would like any more information about either the society itself, or advice on any other aspect of the maths course for example module choices for next year, feel free to get in touch with any of the exec listed below or via the society email address (mathematical.society@durham.ac.uk).

To join:

Come and see our stand at the freshers’ fair, or email at any time: it costs only £6 for life membership, or £3 for a year.

This year’s Exec is:

| **President** | Matthew Palmer (m.i.palmer@durham.ac.uk) |
| **Secretary** | Jacqui Rhodes (j.m.rhodes@durham.ac.uk) |
| **Treasurer** | Tom Sargeant (tom.sargeant@durham.ac.uk) |
| **Social Secretary** | Daniel Rowbottom (daniel.rowbottom@durham.ac.uk) |
| **Publicity Officer** | Peter Clayton (peter.clayton@durham.ac.uk) |
1.7 Disclaimer

The information in this booklet is correct at the time of going to press in May 2011. The University, however, reserves the right to make changes without notice to regulations, programmes and syllabuses. The most up-to-date details of all undergraduate modules can be found in the Faculty Handbook on-line at [www.dur.ac.uk/faculty.handbook/](http://www.dur.ac.uk/faculty.handbook/).

1.8 Booklists and Descriptions of Courses

The following pages contain brief descriptions of the Level 2 modules available to you. Each module description is followed by a list of recommended books and a syllabus. For some modules you are advised to buy a particular book, indicated by an asterisk; for others a choice of titles is offered or no specific recommendation is given. There are also suggestions for preliminary reading and some time spent on this during the summer vacation may well pay dividends in the following years.

Syllabuses, timetables, handbooks, exam information, and much more may be found at [www.maths.dur.ac.uk/teaching/](http://www.maths.dur.ac.uk/teaching/) or by following the link ‘teaching’ from the Department’s home page ([www.maths.dur.ac.uk](http://www.maths.dur.ac.uk)). The syllabuses are intended as guides to the modules. No guarantee is given that additional material will not be included and examined nor that all topics mentioned will be treated.
1.8.1 Algebra II – MATH2581 (38 lectures)

Dr D. Badziahin / Dr H. Gangl

Algebra forms the basis for much of mathematics and is in fact the prerequisite of several third and fourth year modules such as Algebraic Geometry, Galois Theory, Geometry, Number Theory, Representation Theory, and Topology.

The first part of the module is an introduction to the theory of rings, and we look at various examples of rings and fields (integers, rationals, complex numbers, integers modulo $n$, polynomials...). Among the topics covered are various properties of polynomials (factorizability, for example).

In the second part we return to the group theory introduced in Core A with some revision and some new examples, and more details on general properties of groups. An important theme is that of homomorphisms, normal subgroups and quotient groups. One central idea is that of a group acting on, or permuting, the elements of a set. For example, the group of rotational symmetries of a cube acts on the set of edges of the cube (each rotation permutes the edges). By exploiting this idea we can obtain results in group theory (a partial converse to Lagrange’s theorem), geometry (the classification of the finite rotation groups) and combinatorics (there are 9,099 essentially different ways of labelling the faces of a dodecahedron using three labels). We then turn to the description of finitely generated abelian groups as quotients of $\mathbb{Z}/n\mathbb{Z}$. We complete the discussion of finitely generated abelian groups showing that each is isomorphic to just one direct product of cyclic groups of a particular form.

In the last few lectures we take up some more advanced topics in linear algebra, in particular we revisit the Jordan decomposition from a more abstract point of view.

Recommended Books


The book by Allenby covers most of the main topics. It contains many historical references and potted biographies of famous mathematicians which you may well find interesting. Complementary material on group theory may be found in the book by Armstrong. Herstein covers similar ground to Allenby but is somewhat leaner and less comprehensive. The books by Blyth and Robertson contain many examples and problems. Chapter 2 in Cameron is nicely written and very clear.

Preliminary Reading Read Chapter 1 of Allenby and, if time allows, the first half of Chapter 3. Dip into the Prologue as well. Alternatively, read Chapter 1.3-1.4 and the beginning of Chapter 2 in Cameron.
Review your first year group theory by reading the relevant sections in Allenby or Armstrong (above).
**Outline of course**

**Aim**: To introduce further concepts in abstract algebra and linear algebra, develop their theory, and apply them to solve problems in number theory and other areas.

**Term 1** (20 lectures)

**Rings and fields** (16 lectures)
Definitions and examples of rings e.g. \( \mathbb{Z}/n\mathbb{Z} \), matrix rings, quaternions, polynomial rings.
Homomorphism of rings, integral domains and fields, units of a ring. Polynomials over a field, greatest common divisor of polynomials, division algorithm, Euclidean algorithm, irreducibility: Gauss’s lemma, Eisenstein’s criterion.
Ideals and Quotient Rings: Ideals e.g. kernels of ring homomorphisms, quotient rings, first isomorphism theorem for rings, Chinese remainder theorem for rings, prime/maximal ideals and their characterisation in terms of quotient rings.

**Group theory I** (4 lectures)
Review of formal definition of group and elementary manipulations. Notion of subgroup, order of an element of a group, cosets of a group, Lagrange’s Theorem.
Examples of groups: Cyclic and dihedral groups, rotational symmetry groups of regular polyhedra, matrix groups, symmetric and alternating groups.
Group homomorphism, kernel and image, isomorphism.
Operations on groups: Direct product, product of cyclic groups. Isomorphisms between various groups including groups of platonic solids, Cayley’s theorem.

**Term 2** (18 lectures)

**Group theory II** (12 lectures)
Homomorphisms and Quotient Groups: group homomorphism, Normal subgroups. First isomorphism theorem for groups and applications. Automorphisms, inner automorphisms. Simple groups: \( A_n \) for \( n \geq 5 \). Commutator subgroup.
Group actions: Action of a group on a set. Orbits, stabilisers and the orbit-stabiliser theorem. Cauchy’s theorem. Conjugacy, conjugacy classes in \( S_n \). Centre, centre of a \( p \)-group, groups of order \( p^2 \). Sylow theorems. Finite subgroups of \( O(2) \) and \( SO(3) \).
Finitely generated abelian groups: Classification, uniqueness of rank and torsion coefficients. Systems of linear equations in integers, recognition of groups from a finite presentation.

**Topics in linear algebra** (6 lectures)
Vector spaces over arbitrary fields.
Dual spaces and bases, quotient vector spaces, isomorphism theorems.
Introduction to modules, structure theorem for finitely generated modules over a PID (statement), Jordan decomposition revisited.
The description of most natural phenomena is based on models that involve functions of several variables. The highlight of this module is the study of two prominent partial differential equations, namely Laplace’s equation and the heat diffusion equation. These equations relate two or more partial derivatives of an unknown function of several variables, and play an enormous role in science. The methods to solve these partial differential equations are cornerstones of applied mathematics.

The first quarter of the module deals with functions depending on \( n \) real variables. The mathematical properties and procedures are simply the natural extensions of those for the one-variable case you are already familiar with. We then move on to study ordered triples of functions of 3 real variables and their generalisations, which are called vector functions or vector fields. Vector algebra is so prodigiously rich in applications that it plays a crucial role in many areas of science. But vector calculus goes much beyond vector algebra. Differential and integral vector calculus opens the door to three great integration theorems: Green’s theorem, Gauss’s theorem (commonly known as the divergence theorem) and Stokes’s theorem. All three theorems can be cast in the same general form: an integral over a region \( S \) is equal to a related integral over the boundary of \( S \). Vector calculus was actually invented to provide an elegant formulation of the laws of electrostatics but it has applications in many other scientific contexts. When you master it, you will be fully prepared for many courses in year 3 and 4, such as Electromagnetism, Continuum Mechanics and Quantum Mechanics.

**Recommended Books**

Most topics in this module are covered in:


A useful source of worked examples for the first and the second terms of the module, with summaries of the theory, are respectively

Outline of course

Analysis in Many Variables II

**Aim**: To provide an understanding of calculus in more than one dimension, together with an understanding of and facility with the methods of vector calculus. To understand the application of these ideas to a range of forms of integration and to solutions of a range of classical partial differential equations.

**Term 1** (20 lectures)

**Functions of many variables.** Limits and continuity of functions. Partial derivatives and linear approximations. Tangent planes and normal lines to surfaces. Differentiable functions of many variables. Chain rules in many variables. Implicit function theorems.

**Vector functions and vector fields.** The gradient and directional derivatives of a function, The differential and the Jacobian of a vector field. Line integrals of functions and vector fields. The divergence and the curl of a vector field.

**Extrema of functions of two variables.** Local and global extrema of functions. Extrema of functions with constraints.

**Term 2** (18 lectures)

**Vector Calculus: Integral Theorems**


**Solution of Poisson’s and Laplace’s Equations**: Uniqueness of solution of Laplace’s and Poisson’s equations. General solution of Poisson’s equation. Green’s function. Simple examples of solution of Laplace’s equation by separation of variables.

**Fourier Transform, Heat Kernels and Green’s Functions**: Fourier transform and inverse, convolution theorem. Solution of heat equation on $\mathbb{R}^n$ using Fourier transform and construction of heat kernel. Connection between heat kernel and Green’s function.
Term 1 - Codes:

Error-correcting codes use very abstract mathematics in very concrete applications: in essence we will do the mathematics that is behind the capability of a CD-player to cope with a scratch on a CD. But error-correcting codes are also used more widely, e.g., for data transmission over noisy channels.

We start with the basics of error-correcting codes, working with vectors and matrices with coefficients in $\mathbb{Z}_p$ ($p$ prime), and discuss the error-detection and error-correction capabilities of various codes. In order to understand the special error-correcting capabilities of the Reed-Solomon code used on CDs we have to introduce finite field extensions towards the end.

We shall use a good deal of the Linear Algebra from the first term of Core A; the ideas about fields also link to Algebra II.

Recommended Books

The course is based on ‘A First Course in Coding Theory’ by Raymond Hill (Oxford University Press, ISBN 0198538030, about £25), but ‘Coding theory: a first course’ by San Ling and Chaoping Xing (Cambridge University Press, ISBN 0521529239, about £24) also has large parts in common with it. Only the very last topic will be taken from ‘Error-Correcting Codes and Finite Fields’ by Oliver Pretzel (Oxford University Press, ISBN 0192690671, about £28), which as a whole is more substantial than the first book.

Term 2 - Actuarial Mathematics:

This course provides an introduction to applications of mathematics in actuarial work, focussing on aspects of life insurance. The emphasis is on mathematics of compound interest and probabilistic models for lifetime, including the use of life tables. As insurance is a topic of great practical interest, this course will be of benefit not only to those who may opt for a career in insurance, but to all students as many of the concepts introduced are quite common in every day life. Students taking this course should have a good understanding of basic probability theory.

Recommended Books

Outline of course

**Aim**: To study two separate topics in mathematic at least one of which will demonstrate how mathematics can be applied to real world situations.

**Term 1** (20 lectures)

Block codes: Hamming distance, Error-detection and correction: procedures, capabilities, probabilities. Sphere-packing bound, Singleton bound, notion of perfect code, equivalence.

Modular arithmetic, in particular $\mathbb{F}_p = \mathbb{Z}_p$ with $p$ prime. Matrices and basic linear algebra over $\mathbb{F}_p$ (span, linear independence, basis, dimension, determinants).


Hamming codes, BCH codes over $\mathbb{F}_p$; decoding algorithms.

The ring $\mathbb{F}_p[x]$, working modulo $f(x)$, finite fields $\mathbb{F}_{p^r}$. Linear codes over such fields; in particular, Reed-Solomon codes.

Burst-errors and ways to fix them: interleaving, binRS codes.

**Term 2** (18 lectures)

**Introduction to Life Insurance**: What is life insurance; the role of mathematics, in particular probability and statistics.

**The Mathematics of Compound Interest**: Effective and nominal interest rates; continuous payments; perpetuities; annuities; repayment of a debt.

**The Future Lifetime**: Models and notation; force of mortality; analytical distributions; curtate future lifetime; life tables.

**Life Insurance**: Elementary insurance types, including whole life and term insurance and endowments; more general types of life insurance.

**Life Annuities**: Elementary life annuities; variable life annuities.

**Net Premiums**: Random total loss to insurer; equivalence principle; net premiums for elementary forms of insurance; exponential utility.

**Further Topics**: There is a wide variety of possible further topics, a selection of which can be included. Examples are: stochastic interest; net premium reserves; multiple decrements; multiple life insurance; portfolios; expense loading; estimating probabilities of death; commutation functions, family income insurance.
1.8.4 Codes and Geometric Topology II – MATH2141 (38 lectures)

Dr S. K. Darwin / Prof. J. R. Parker

Term 1 - Codes:

Error-correcting codes use very abstract mathematics in very concrete applications: in essence we will do the mathematics that is behind the capability of a CD-player to cope with a scratch on a CD. But error-correcting codes are also used more widely, e.g., for data transmission over noisy channels.

We start with the basics of error-correcting codes, working with vectors and matrices with coefficients in $\mathbb{Z}_p$ ($p$ prime), and discuss the error-detection and error-correction capabilities of various codes. In order to understand the special error-correcting capabilities of the Reed-Solomon code used on CDs we have to introduce finite field extensions towards the end.

We shall use a good deal of the Linear Algebra from the first term of Core A; the ideas about fields also link to Algebra II.

Recommended Books

The course is based on ‘A First Course in Coding Theory’ by Raymond Hill (Oxford University Press, ISBN 0198538030, about £25), but ‘Coding theory: a first course’ by San Ling and Chaoping Xing (Cambridge University Press, ISBN 0521529239, about £24) also has large parts in common with it. Only the very last topic will be taken from ‘Error-Correcting Codes and Finite Fields’ by Oliver Pretzel (Oxford University Press, ISBN 0192690671, about £28), which as a whole is more substantial than the first book.

Term 2 - Geometric Topology:

This course gives an introduction to topology in an intuitive, visual way by studying knots, links and surfaces. Apart from everyday life, knot theory has applications in many sciences (e.g. in quantum physics or molecular biology) and in various branches of mathematics. Knots and links give rise to exciting geometric objects and the course provides tools to study them. Starting with combinatorial moves we introduce sophisticated invariants, like the Conway- and Jones-polynomials, which are effectively computable and carry important topological information.

Surfaces are also very intuitive objects appearing in many branches of mathematics, and we will show applications to knot theory, and study properties of vector fields on surfaces.

Recommended Books


**Outline of course**

**Aim**: To study two separate topics in mathematics at least one of which will demonstrate how mathematics can be applied to real world situations.

**Term 1** (20 lectures)

Block codes: Hamming distance, Error-detection and -correction: procedures, capabilities, probabilities. Sphere-packing bound, Singleton bound, notion of perfect code, equivalence.

Modular arithmetic, in particular $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$ with $p$ prime. Matrices and basic linear algebra over $\mathbb{F}_p$ (span, linear independence, basis, dimension, determinants).


Hamming codes, BCH codes over $\mathbb{F}_p$; decoding algorithms.

The ring $\mathbb{F}_p[x]$, working modulo $f(x)$, finite fields $\mathbb{F}_{p^r}$. Linear codes over such fields; in particular, Reed-Solomon codes.

Burst-errors and ways to fix them: interleaving, binRS codes.

**Term 2** (18 lectures)

Knots, equivalence, knot diagrams, Reidemeister moves, 3-colouring, writhe and linking numbers, alternating knots.

Knot polynomials, Jones polynomial, trefoil not isotopic to mirror image, Alexander-Conway polynomial, absolute polynomial.

Surfaces, triangulation of surfaces, Euler characteristic, classification of surfaces, Seifert surfaces, genus of knots.

Plane vector fields, index round a closed curve, non-zero index means field vanishes somewhere inside curve. Brouwer Fixed Point Theorem. Degree of map of circle into punctured plane, vector fields on surfaces, Poincare-Hopf theorem.
1.8.5 COMPLEX ANALYSIS II – MATH2011 (38 lectures)

Dr J. Bolton / Dr A. Stasinski

Term 1:

The module develops the theory of functions of a single complex variable. In your first year you will have already met several examples of complex functions such as polynomials in $z$; $e^z$; $\cos z$ and $\sin z$. Functions of a complex variable behave in a very different way from their real counterparts. For example, a complex function which is differentiable once everywhere is differentiable any number of times; even more striking, if $f(z)$ is differentiable everywhere and bounded, then it must be constant.

Term 1: We begin by building up a library of complex functions, and investigating their geometrical properties. For instance, such functions turn out to be angle-preserving (or conformal) at all points in the plane where the complex derivative is non-zero.

We then consider different types of convergence for sequences and series of functions, concentrating on uniform convergence because this has (most of) the nice properties you would hope were true!

Term 2: In this term we shall investigate the properties of complex differentiable functions, show how to integrate them along curves in the complex plane, and apply our results to produce simple algorithms for the evaluation of real integrals of the form $\int_{-\infty}^{\infty} f(x) \, dx$. We shall also consider other applications and prove, for example, that any polynomial of degree $n$ with coefficients in $\mathbb{C}$ has $n$ roots (not necessarily distinct).

Recommended Books

L. Ahlfors, Complex Analysis, Tata McGraw Hill.
G.J.O. Jameson, A First Course on Complex Functions, Chapman and Hall.
D.O. Tall, Functions of a Complex Variable, (Parts I & II), Routledge and Kegan Paul.
M.R. Spiegel, Complex Variables, Schaum (for worked examples).

Preliminary Reading

Find out as much as you can about

(i) complex functions
(ii) the Cauchy-Riemann equations

by reading the relevant sections in one of the above references.
Outline of course

**Aim**: To introduce the student to the theory of complex analysis.

**Term 1** (20 lectures)

**Complex differentiation** (6 lectures)
Complex functions, differentiation, Cauchy-Riemann equations.
Elementary functions: \( e^z \), \( \log(z) \), \( z^a \), \( \cos(z) \), \( \cosh(z) \), etc. Cuts and branches

**Conformal mappings** (8 lectures)
Harmonic functions and holomorphic maps. Holomorphic maps and conformality. The geometry of conformal maps, with examples.
Group of Möbius transformations, point at infinity, Riemann sphere. Three points determine a Möbius transformation. Möbius transformations send circles to circles, preservation of cross ratio. Möbius transformations of unit circle to itself.
Statement of Riemann mapping theorem illustrated by mappings of half-disc and quarter-disc to the unit disc. Application to finding solutions of the Laplace equation in two dimensions with given boundary conditions (with mention of fluid flow, electrostatic potential, heat flow).

**Uniform convergence** (6 lectures)
Term-by-term integration and differentiation of sequences. Uniform convergence of series. Weierstrass \( M \)-test. Power series: Circle of convergence, term by term differentiation, Taylor series. Application to power series of \( e^z \), \( \sin(z) \) etc.

**Term 2** (18 lectures)

**Contour integrals** (7 lectures)

**Calculus of residues** (4 lectures)
Evaluation of integrals (and series) by calculus of residues. Jordan’s lemma.

**Applications** (7 lectures)
1.8.6 Elementary Number Theory & Cryptography – MATH2591 (38 lectures)

Dr H. Gangl / Dr D. A. Badziahin

Term 1: The first part discusses the basic properties of the integers, such as the Euclidean algorithm and the fundamental theorem of arithmetic, that is, the unique factorisation into prime numbers. Then we discuss congruences and modular arithmetic culminating in the classic quadratic reciprocity law which relates the question when a prime $p$ is a square modulo another prime $q$ to the reciprocal question when $q$ is a square modulo $p$. This problem and the search for generalisations have been driving number theory for the last 200 years. The second half of the term deals with two fundamental applications, namely the question of factoring integers into primes and the ever increasing importance of number theory in cryptography. In particular, we discuss public key cryptography which is fundamental in internet commerce and online banking.

Term 2: At the beginning we consider some very classical Diophantine problems such as when a prime number is the sum of two squares or to find integral solutions for Pell’s equation $x^2 - dy^2 = \pm 1$ for $d$ a square-free positive integer. We then discuss elliptic curves which are solutions to certain cubic equations in the plane. Elliptic curves enjoy a remarkable group law and also have a very rich arithmetic structure. We discuss in particular the application of elliptic curves in cryptography which has been developed during the last twenty years. Finally, we turn to the problem of counting prime numbers. We give an overview to the prime number theorem which gives an approximation to the number of primes less than a given bound. We close with the Riemann hypothesis, “the holy grail of Number Theory”, which can be rephrased in terms of a refinement of the prime number theorem.

Recommended Books

G.H.Hardy, E.M. Wright, An Introduction to the Theory of Numbers, OUP.

J. Silverman, A Friendly Introduction to Number Theory, Prentice Hall.


**Outline of course**

**Elementary Number Theory & Cryptography II**

**Aim:** To introduce fundamental topics in elementary number theory and demonstrate applications in cryptography.

**Term 1** (20 lectures)

**Basic properties of integers** (4 lectures)
Divisibility, greatest common divisor, prime number, factorisation, fundamental theorem of arithmetic.
Linear equations and divisibility, Euclidean algorithm.

**Congruences and modular arithmetic** (4 lectures)
Residues, \( \mathbb{Z}/n\mathbb{Z} \), \((\mathbb{Z}/n\mathbb{Z})^\times\), Euler-Fermat, Wilson, primitive roots, Chinese remainder theorem, Euler’s \( \phi \)-function, modular exponentiation.

**Quadratic reciprocity** (3 lectures)
Statement and elementary proof.

**Applications to cryptography** (9 lectures)
Carmichael numbers and primality testing.
Quadratic residues/square roots mod \( n \) and factorisation, quadratic sieve.
RSA. Discrete Logarithm problem and application to cryptography (Diffie-Hellman Key Exchange, ElGamal Public Key Cryptosystem).

**Term 2** (18 lectures)

**Diophantine equations** (8 lectures)
Pythagoras and the equation \( X^4 + Y^4 = Z^4 \): descent method.
Gaussian integers and sums of two squares, Lagrange 4-square theorem. Continued fractions, Pell’s equation. Diophantine approximation, irrationality questions.

**Elliptic curves** (5 lectures)
Elliptic curves modulo \( p \) and the group law. Applications in cryptography.

**Counting prime numbers** (5 lectures)
Arithmetic functions and Möbius inversion.
Existence of infinitely many primes, of infinitely many primes of the form \( \equiv 3(4) \), statement of prime number theorem.
Bernoulli numbers, Zeta function, Riemann Hypothesis.
The concept of symmetry is the basis of many modern developments in fundamental physics. This module explores the power of symmetry in Special Relativity and revisits Classical Dynamics using the Lagrangian/Hamiltonian formalism, which is designed with symmetry in mind. It provides the necessary foundations for a sound understanding of recent developments in Mathematical Physics where more abstract symmetries are encountered in Nature’s modelling.

**Term 1:** In the first part, we make use of the calculus of variations and Hamilton’s Principle to set up an elegant and powerful formulation of classical mechanics. This allows us to quickly analyse the motion of a wide range of systems, which would appear to be complicated by using the more Newtonian ‘forces-on-a-body’ approach. This formulation also allows us to identify the symmetries of a system quite readily, and we discover the link between symmetries and more advanced algebraic approaches like group theory through the concept of ‘Poisson Brackets’. Other important applications we consider are small oscillations of systems with many degrees of freedom. This naturally leads us to the second part, where we study a familiar system of an oscillator with an infinite number of degrees of freedom, a stretched string. We therefore revisit the wave equation, using our new language, and study some of its properties and related physical phenomena such as dispersion, reflection, refraction, and transmission of energy.

**Recommended Books**

**Preliminary Reading:** The introductory chapter and Appendix A of the book by Kibble and relevant chapters of your favourite physics textbook.

**Term 2:** The astounding physical consequences of Special Relativity are worked out on Einstein’s profound appreciation of the power of symmetry: mass is equivalent to energy and time is married to space. We show how many concepts based on common sense must be jettisoned in Special Relativity, like the absolute character of time, which is deeply rooted in us. We introduce the mathematical setting appropriate for the description of the kinematics and the dynamics of Special Relativity, and make an incursion in the world of 4-dimensional vectors and tensors. Once we have done this, it is easy to see for instance that the famous equation $E = mc^2$ is just the consequence of the invariant length of a vector in four dimensions. The term concludes with a brief discussion of collision processes among elementary particles.

**Recommended Books**
**Aim**: To appreciate the conceptual framework of classical physics, both discrete and continuous, as well as the mathematical foundation of Special Relativity.

**Term 1.** (20 lectures)


**Small oscillations of systems of particles**: Positions of equilibrium and stability. Normal modes of oscillation and normal coordinates. Stationary properties of frequencies of systems with constraints.

**Waves**: Review one-dimensional wave equation. Energy, energy density, energy carried by wave. Boundaries and junctions: reflection of a monochromatic wave at a fixed boundary, reflection and transmission at a junction, energy flow. Wave equation in two or more dimensions. Reflection and refraction at a boundary.

**Term 2. Special Relativity** (18 lectures)


**Systems of free particles**: Conservation of 4-momentum. Centre of mass frame. Collision processes.
It is easy to find the roots of the equation $x^2 - 3x + 2 = 0$. We also think of the evaluation of $\int_0^1 \sin x \, dx$ as an easy problem because we can express it as $1 - \cos 1$, and a calculator can give a numerical value, to a certain accuracy. Such simplicity is unusual. For example, it is not possible to express the integral $\int_0^1 \frac{\sin x}{x} \, dx$ in terms of a finite number of elementary functions, and there is no simple formula for the root of the equation $x - \cos x = 0$; numerical methods allow us to solve these, and other more interesting problems, to any required accuracy.

The ability to compute numerical answers to mathematical problems has always been an important part of mathematics. For example, an effective method for evaluating square roots was discovered more than 3700 years ago. Now numerical methods are more important than ever because mathematical models are widely used in science, engineering, finance and other areas, to formulate theories, to interpret data and to make predictions. Indeed, modern areas of mathematics such as Mathematical Biology hinge on numerical techniques. While one has to develop a realistic model consistent with the Biology, the equations which arise are rarely solvable and without numerical techniques are of very little use. Numerical Analysis is concerned with the development of numerical methods, and the “Analysis” in the title refers to the study of the accuracy, reliability and efficiency of the resulting algorithms.

Numerical Analysis shares some of the attractions of both pure and applied mathematics. For its derivations and analysis it draws on many areas of pure mathematics, yet its objective is practical — to produce reliable numerical approximations, and to do so efficiently. Practical experience of numerical computation is essential for a full understanding of the successes and failures of particular numerical methods. The practical sessions for this course form part of the assessment and are designed to allow you to experiment with a variety of numerical methods through the software package Maple. The computer will do all the calculations for you, leaving you free to investigate and to use your knowledge of the theory to explain and interpret the numerical results.

Recommended Books

*R.L. Burden and J.D. Faires, Numerical Analysis (Brooks Cole)*


R. Plato, *Concise Numerical Mathematics* (AMS)


E. Süli and D. Mayers, *An Introduction to Numerical Analysis* (CUP)

All of the recommended books cover the necessary material for this module. Burden and Faires will be used extensively in classes and covers the majority of material for future numerical analysis modules. It is less mathematical than Stoer and Bulirsch but is more mathematical than the excellent book by Cheney and Kincaid.

**Preliminary Reading:** Burden and Faires, Chapter 2.
Outline of course

Aim: To introduce the basic framework of the subject, enabling the student to solve a variety of problems and laying the foundation for further investigation of particular areas in the Levels 3 and 4.

Term 1 (20 lectures)

Introduction (1): The need for numerical methods. Statement of some problems which can be solved by techniques described in this course. What is Numerical Analysis?


Term 2 (18 lectures)


Practical Sessions: Each student will have a weekly one-hour practical session in a computer classroom. The Maple package will be used to implement the numerical methods introduced in the lectures.
A Markov chain is a process in which the next state depends only on the current state but not on the past. Such processes were first introduced by A. Markov in the early XXth century in an attempt to extend the validity of the law of large numbers to dependent variables. As an interesting application, A. Markov used such processes to describe the distribution of letters in Pushkin’s novel Eugene Onegin. Since then the theory of Markov chains became one of the most important areas of contemporary probability theory, providing the foundation for understanding, explaining and predicting phenomena in diverse areas including biology, chemistry, physics, economics, finance to name just a few. Markov chains are also used in computer science and statistics (Markov Chain Monte Carlo is one of the most popular method of simulation) as well as in many everyday applications (e.g., Google ranks the webpages by using a particular Markov chain).

In addition to Markov Chains and random walks, we will discuss other important topics including generating functions, convergence, and elements of integration.

Recommended Books


**Markov Chains** by J.R.Norris [CUP, 1997; ISBN:0-521-48181-3] provides a more advanced treatment of discrete and continuous time Markov chains; it also contains various applications and shall be useful if you decide to follow Stochastic Processes III/IV.

**Probability and random processes** by G.R.Grimmett and D.Stirzaker [3rd ed., OUP, 2001; ISBN:0-19-857223-9] is a more advanced text covering many different topics of contemporary probability; it can be useful if you decide to take Probability III/IV and/or Stochastic Processes III/IV.

**Term 2 - Actuarial Mathematics:**

This course provides an introduction to applications of mathematics in actuarial work, focussing on aspects of life insurance. The emphasis is on mathematics of compound interest and probabilistic models for lifetime, including the use of life tables. As insurance is a topic of great practical interest, this course will be of benefit not only to those who may opt for a career in insurance, but to all students as many of the concepts introduced are quite common in every day life. Students taking this course should have a good understanding of basic probability theory.

**Recommended Books**

Outline of course Probability and Actuarial Mathematics II

Aim: To study two separate topics in mathematics at least one of which will demonstrate how mathematics can be applied to real world situations.

Term 1 (20 lectures)

Revision of Core A Probability, main notions and results.

Generating functions, their properties, some applications.

Markov chains: Markov property, classification of states, hitting times, convergence to equilibrium.

Infinite collections of events, Borel-Cantelli lemmas.

Introduction to convergence and integration modes of convergence, definition of the Lebesgue integral via simple functions, statement of limit theorems, applications.

Random walks, some properties.

Topics chosen from continuity and inversion theorems, weak forms of LLN & CLT, branching processes, asymptotic analysis for combinatorics.

Term 2 (18 lectures)

Introduction to Life Insurance: What is life insurance; the role of mathematics, in particular probability and statistics.

The Mathematics of Compound Interest: Effective and nominal interest rates; continuous payments; perpetuities; annuities; repayment of a debt.

The Future Lifetime: Models and notation; force of mortality; analytical distributions; curtate future lifetime; life tables.

Life Insurance: Elementary insurance types, including whole life and term insurance and endowments; more general types of life insurance.

Life Annuities: Elementary life annuities; variable life annuities.

Net Premiums: Random total loss to insurer; equivalence principle; net premiums for elementary forms of insurance; exponential utility.

Further Topics: There is a wide variety of possible further topics, a selection of which can be included. Examples are: stochastic interest; net premium reserves; multiple decrements; multiple life insurance; portfolios; expense loading; estimating probabilities of death; commutation functions, family income insurance.
Term 1 - Probability:

A Markov chain is a process in which the next state depends only on the current state but not on the past. Such processes were first introduced by A. Markov in the early 20th century in an attempt to extend the validity of the law of large numbers to dependent variables. As an interesting application, A. Markov used such processes to describe the distribution of letters in Pushkin’s novel Eugene Onegin. Since then the theory of Markov chains became one of the most important areas of contemporary probability theory, providing the foundation for understanding, explaining and predicting phenomena in diverse areas including biology, chemistry, physics, economics, finance to name just a few. Markov chains are also used in computer science and statistics (Markov Chain Monte Carlo is one of the most popular methods of simulation) as well as in many everyday applications (e.g., Google ranks the webpages by using a particular Markov chain).

In addition to Markov Chains and random walks, we will discuss other important topics including generating functions, convergence, and elements of integration.

Recommended Books


**Markov Chains** by J.R.Norris [CUP, 1997; ISBN:0-521-48181-3] provides a more advanced treatment of discrete and continuous time Markov chains; it also contains various applications and shall be useful if you decide to follow Stochastic Processes III/IV.

**Probability and random processes** by G.R.Grimmett and D.Stirzaker [3rd ed., OUP, 2001; ISBN:0-19-857223-9] is a more advanced text covering many different topics of contemporary probability; it can be useful if you decide to take Probability III/IV and/or Stochastic Processes III/IV.

Term 2 - Geometric Topology:

This course gives an introduction to topology in an intuitive, visual way by studying knots, links and surfaces. Apart from everyday life, knot theory has applications in many sciences (e.g. in quantum physics or molecular biology) and in various branches of mathematics. Knots and links give rise to exciting geometric objects and the course provides tools to study them. Starting with combinatorial moves we introduce sophisticated invariants, like the Conway- and Jones-polynomials, which are effectively computable and carry important topological information.

Surfaces are also very intuitive objects appearing in many branches of mathematics, and we will show applications to knot theory, and study properties of vector fields on surfaces.

Recommended Books

Outline of course Probability and Geometric Topology II

Aim: To study two separate topics in mathematics at least one of which will demonstrate how mathematics can be applied to real world situations.

Term 1 (20 lectures)

Revision of Core A Probability, main notions and results.

Generating functions, their properties, some applications.

Markov chains: Markov property, classification of states, hitting times, convergence to equilibrium.

Infinite collections of events, Borel-Cantelli lemmas.

Introduction to convergence and integration modes of convergence, definition of the Lebesgue integral via simple functions, statement of limit theorems, applications.

Random walks, some properties.

Topics chosen from continuity and inversion theorems, weak forms of LLN & CLT, branching processes, asymptotic analysis for combinatorics.

Term 2 (18 lectures)

Knots, equivalence, knot diagrams, Reidemeister moves, 3-colouring, writhe and linking numbers, alternating knots.

Knot polynomials, Jones polynomial, trefoil not isotopic to mirror image, Alexander-Conway polynomial, absolute polynomial.

Surfaces, triangulation of surfaces, Euler characteristic, classification of surfaces, Seifert surfaces, genus of knots.

Plane vector fields, index round a closed curve, non-zero index means field vanishes somewhere inside curve. Brouwer Fixed Point Theorem. Degree of map of circle into punctured plane, vector fields on surfaces, Poincare-Hopf theorem.
Anyone who collects information must decide how to draw useful conclusions from it. For example, can an opinion poll, involving maybe 1000 people, be trusted to give an accurate picture of everyone else’s opinions? Answering this question requires that we combine our knowledge of probability theory with our understanding of how opinion polls are performed. The kind of reasoning that results is called statistical inference.

Other areas of popular debate where statistical problems arise include understanding the effects of food additives, interpreting the results of clinical trials of medical treatments, the reliability of electrical and other products and the incidence of leukaemia near nuclear power stations. A knowledge of statistics is essential not only to those who specialise in studying such phenomena but also to anyone who wishes to develop informed opinions about them. The module will introduce you to some basic ideas of statistical inference and develop solutions to some standard problems. There are two schools of thought about the fundamental principles of statistics, the Bayesians and the frequentists. The module will cover both viewpoints but the majority of methods presented will be the more widely used frequentist ones.

Practical computing sessions, using the freely available statistical package R, will be held throughout the year. They serve three purposes: to bring the module closer to the real world of applied statistics, to provide additional insight into the lectured material and to introduce you to an excellent piece of statistical software.

**Recommended Books**


The book by Rice covers most of the whole module, and you are advised to have ready access to a copy. The books by De Groot and Schervish and by Berry give better coverage of Bayesian statistics than Rice.

**Preliminary Reading**

Begin by making sure you understand the most important parts of the 1H probability module, and read the summary sheets you were given! Read some of the early chapters on statistical ideas in at least one of the books listed above. The basic ideas of statistics may be unfamiliar. If you can get the ideas in place, the mathematics should be relatively straightforward.
**Outline of course**

**Aim:** To introduce the main ideas, methods of statistics and statistical computing, including a comparison of the Bayesian and frequentist approaches.

**Exploring Data:** Summary statistics: mean, median, standard deviation, inter-quartile range, correlation. Ideas of location, scale and association. Displays: dot-plot, histogram, stem-and-leaf plot, boxplot and scatterplot. Exploration for model building.


**Bayesian Inference:** Inference using Bayes theorem. Prior, likelihood and posterior. Conjugate prior distributions for binomial, Poisson and mean of Gaussian samples. Posterior mean as weighted average of prior mean and sample average. Posterior probability intervals. Prediction.


**Goodness of Fit and Diagnostics:** Importance of model validation. Likelihood based goodness of fit tests. Quantile-quantile plots. Use of residuals for linear models.

**Non-Parametric Inference:** Order statistics, ranks and sample quantiles. Sign test. Confidence interval for median. Mann-Whitney and Kruskal-Wallis tests.
A  Details of Modules and Programmes

All mathematics modules are open, except for

MATH3121 (Mathematics Teaching III) — tied to G100, G101, G103, G104, CFG0, FGC0, QRV0, QRVA and X1G1.
MATH3131 (Communicating Mathematics III) — tied to G100, G104, CFG0 and QRV0.
MATH3161 (Independent Study III) — tied to G100, G103 and G104

G100: B.Sc. in Mathematics
G101: Master of Mathematics (European Studies)
G103: Master of Mathematics
G104: B.Sc. in Mathematics (European Studies)

• If you do MATH3131 (Communicating Mathematics III), you cannot take another project module in another department.

• If you do MATH3121 (Mathematics Teaching III), you cannot take another into schools module in another department.

• The double module Project IV is open to students doing an M.Sc. in Natural Sciences, provided the minimal requirements for taking a Level 4 mathematics module are met, provided the corequisite of one other Level 4 mathematics module and provided no Level 4 Project is taken in another department.
### Level 1

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Course Title</th>
<th>Pre-requisites</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH1012</td>
<td>Core Mathematics A</td>
<td>[P: AA*; C:-; EC: SM*]</td>
</tr>
<tr>
<td>MATH1051</td>
<td>Core Mathematics B1</td>
<td>[P: AA*; C: 1012; EC: SM*]</td>
</tr>
<tr>
<td>MATH1041</td>
<td>Core Mathematics B2</td>
<td>[P: AA*; C: 1012 &amp; 1051; EC: SM*]</td>
</tr>
<tr>
<td>MATH1711</td>
<td>Data Analysis, Modelling &amp; Simulation</td>
<td>[P: AC*; C:-; EC: 1541]</td>
</tr>
<tr>
<td>MATH1031</td>
<td>Discrete Mathematics</td>
<td>[P: AC*; C:-; EC:-]</td>
</tr>
<tr>
<td>MATH1551</td>
<td>Maths for Engineers &amp; Scientists</td>
<td>[P: AC*; C:-; EC: 1012, 1561 &amp; 1571]</td>
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<tr>
<td>MATH1561</td>
<td>Single Mathematics A</td>
<td>[P: AC*; C:-; EC: 1012 &amp; 1551]</td>
</tr>
<tr>
<td>MATH1571</td>
<td>Single Mathematics B</td>
<td>[P: AC*; C:1561; EC: 1551]</td>
</tr>
<tr>
<td>MATH1541</td>
<td>Statistics</td>
<td>[P: AC*; C:-; EC: 1711]</td>
</tr>
</tbody>
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### Level 2

<table>
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<tr>
<th>Course Code</th>
<th>Course Title</th>
<th>Pre-requisites</th>
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<tbody>
<tr>
<td>MATH2581</td>
<td>Algebra II</td>
<td>[P: 1012 ; C: -; EC: SM*]</td>
</tr>
<tr>
<td>MATH2031</td>
<td>Analysis in Many Variables II</td>
<td>[P: 1012; P/C: 1051; EC: SM*]</td>
</tr>
<tr>
<td>MATH2131</td>
<td>Codes &amp; Actuarial Mathematics II</td>
<td>[P: 1012 ; C:-; EC: 2141, 2161, 2171 &amp; 2571]</td>
</tr>
<tr>
<td>MATH2141</td>
<td>Codes &amp; Geometric Topology II</td>
<td>[P: 1012 ; C:-; EC: 2131,2151 &amp; 2571]</td>
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<tr>
<td>MATH2011</td>
<td>Complex Analysis II</td>
<td>[P: 1012; P/C: 1051; EC: SM*]</td>
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<tr>
<td>MATH2591</td>
<td>Elementary Number Theory &amp; Cryptography II</td>
<td>[P: 1012; C:-; EC: SM*]</td>
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<tr>
<td>MATH2071</td>
<td>Mathematical Physics II</td>
<td>[P: 1012; P/C1041 or PHYS1*; EC: SM*]</td>
</tr>
<tr>
<td>MATH2051</td>
<td>Numerical Analysis II</td>
<td>[P: 1012; P/C: 1051; EC: SM*]</td>
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<tr>
<td>MATH2161</td>
<td>Probability &amp; Actuarial Mathematics II</td>
<td>[P: 1012 ; C:-; EC: 2131,2151 &amp; 2171]</td>
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<tr>
<td>MATH2151</td>
<td>Probability &amp; Geometric Topology II</td>
<td>[P: 1012 ; C:-; EC: 2141 &amp; 2161]</td>
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<tr>
<td>MATH2041</td>
<td>Statistical Concepts II</td>
<td>[P: 1012 ; C:-; EC: SM*]</td>
</tr>
</tbody>
</table>

AA*: A level Mathematics at grade A.
AC*: A level Mathematics at grade C or above.
SM*: 1551, 1561 & 1571

Prob‡: 2151 or 2161 or 2561 or 2571
Act‡: 2131 or 2161

P: Pre-requisite; C: Co-requisite; P/C: Pre- or Co-requisite; EC: Excluded combination.

CHEM1012: Core 1A Chemistry;
PHYS1*: PHYS1111 (Fundamental Physics A) or PHYS1122 (Foundations of Physics I).
## Level 3

**MATH3131** Communicating Mathematics III \((A_3)\) [P: At least 3 Maths modules taken in 2nd year, at least two of which are at Level 2; C: At least two other Level 3 maths module; EC: 3121 or Level 3 project modules other Depts.]  

**MATH3121** Mathematics Teaching III \((A_3)\) [P: At least 3 Maths modules taken in 2nd year, at least two of which are at Level 2; C: At least two other Level 3 maths modules; EC: 3131, GEOL3251, PSYC3191 & PHYS3611]

### List A

<table>
<thead>
<tr>
<th>Module Code</th>
<th>Module Title</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>MATH3321</td>
<td>Algebraic Geometry III ((A_1))</td>
<td>P: 2011 &amp; 2581; C: ; EC: 4011</td>
</tr>
<tr>
<td>MATH3011</td>
<td>Analysis III ((A_1))</td>
<td>P: 2011 &amp; 2031; C: ; EC: 4201</td>
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<tr>
<td>MATH3081</td>
<td>Approx. Th. &amp; Solns to ODEs III ((A_2))</td>
<td>P: 2051 and (1051* or 1*); C: ; EC: 4221</td>
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<td>MATH3341</td>
<td>Bayesian Statistics III ((A_1))</td>
<td>P: 2041; C: ; EC: 4031</td>
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<tr>
<td>MATH3101</td>
<td>Continuum Mechanics III ((A_1))</td>
<td>P: 2031 &amp; (1* &amp; (1041 or PHYS1*)) or (1041 &amp; 1051*); C: ; EC: 4081</td>
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<tr>
<td>MATH3071</td>
<td>Decision Theory III ((A_3))</td>
<td>P: 1012 C: ; EC: -</td>
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<td>MATH3021</td>
<td>Differential Geometry III ((A_3))</td>
<td>P: 2031; C: ; EC: -</td>
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<td>MATH3091</td>
<td>Dynamical Systems III ((A_3))</td>
<td>P: 2031 &amp; 2011; C: ; EC: -</td>
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<td>MATH3181</td>
<td>Electromagnetism III ((A_3))</td>
<td>P: 2031 &amp; (2071 or PHYS2*); C: ; EC: -</td>
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<td>MATH3221</td>
<td>Elliptic Functions III ((A_2))</td>
<td>P: 2011 &amp; (1051* or 1*); C: ; EC: 4151</td>
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<td>MATH3041</td>
<td>Galois Theory III ((A_3))</td>
<td>P: 2581 C: ; EC: -</td>
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<td>MATH3331</td>
<td>General Relativity III ((A_1))</td>
<td>P: 2031 &amp; (2071 or PHYS2*); C: ; EC: 4051</td>
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<td>MATH3201</td>
<td>Geometry III ((A_2))</td>
<td>P: 2581, 2031 &amp; 2011; C: ; EC: 4141</td>
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<td>MATH3161</td>
<td>Independent Study III ((A_3))</td>
<td>P: 2H Honours Mathematics; C: ; EC: -</td>
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<td>MATH3171</td>
<td>Mathematical Biology III ((A_3))</td>
<td>P: 2031; C: ; EC: -</td>
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<td>MATH3301</td>
<td>Mathematical Finance III ((A_3))</td>
<td>P: (1051* and 1*) or 2* &amp; Prob(^\dagger); C: ; EC: 4181</td>
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<td>MATH3031</td>
<td>Number Theory III ((A_2))</td>
<td>P: 2581; C: ; EC: 4211</td>
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<td>MATH3141</td>
<td>Operations Research III ((A_3))</td>
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<td>MATH3291</td>
<td>Partial Differential Equations III ((A_3))</td>
<td>P: 2031 &amp; (1051* or 1*); C: ; EC: 4041</td>
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<td>MATH3211</td>
<td>Probability III ((A_2))</td>
<td>P: 2031, 2011 &amp; Prob(^\dagger); C: ; EC: 4131</td>
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<td>MATH3111</td>
<td>Quantum Mechanics III ((A_3))</td>
<td>P: 2031 &amp; 2071; C: ; EC: -</td>
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<td>MATH3371</td>
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<td>MATH3231</td>
<td>Solitons III ((A_2))</td>
<td>P: 2031 &amp; 2011; C: ; EC: 4121</td>
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<tr>
<td>MATH3351</td>
<td>Statistical Mechanics III ((A_2))</td>
<td>P: 2031 &amp; (1051* or 1*); C: ; EC: 4231</td>
</tr>
<tr>
<td>MATH3051</td>
<td>Statistical Methods III ((A_3))</td>
<td>P: &amp; 2041 ; C: ; EC: -</td>
</tr>
<tr>
<td>MATH3251</td>
<td>Stochastic Processes III ((A_1))</td>
<td>P: 2031 &amp; Prob(^\dagger); C: ; EC: 4091</td>
</tr>
<tr>
<td>MATH3361</td>
<td>Topics in Statistics III ((A_2))</td>
<td>P/C:3051; C: ; EC: 4071</td>
</tr>
<tr>
<td>MATH3281</td>
<td>Topology III ((A_3))</td>
<td>P: 2581, 2031 &amp; 2011; C: ; EC: 4021</td>
</tr>
</tbody>
</table>

\(^*\) If taken in Year 2.  \(^1\) One Level 2 mathematics module.  
\(^*\) Two Level 2 mathematics modules.  \(^i\) One Level 3 mathematics module.  
Prob\(^\dagger\) 2151 or 2161 or 2561 or 2571  
PHYS\(^1\): PHYS1111 (Fundamental Physics A) or PHYS1122 (Foundations of Physics I).  
PHYS\(^2\): PHYS2511 (Foundations of Physics II).
Level 4

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Course Title</th>
<th>Level (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH4072</td>
<td>Project IV (B3)</td>
<td></td>
</tr>
</tbody>
</table>

List B

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Course Title</th>
<th>Level (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH4061</td>
<td>Advanced Qntm. Th. IV (B3)</td>
<td></td>
</tr>
<tr>
<td>MATH4011</td>
<td>Algebraic Geometry IV (B1)</td>
<td></td>
</tr>
<tr>
<td>MATH4161</td>
<td>Algebraic Topology IV (B3)</td>
<td></td>
</tr>
<tr>
<td>MATH4201</td>
<td>Analysis IV (B1)</td>
<td></td>
</tr>
<tr>
<td>MATH4221</td>
<td>Approx. Th. &amp; Solns to ODEs IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4031</td>
<td>Bayesian Statistics IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4081</td>
<td>Continuum Mechanics IV (B1)</td>
<td></td>
</tr>
<tr>
<td>MATH4151</td>
<td>Elliptic Functions IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4051</td>
<td>General Relativity IV (B1)</td>
<td></td>
</tr>
<tr>
<td>MATH4141</td>
<td>Geometry IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4181</td>
<td>Mathematical Finance IV (B3)</td>
<td></td>
</tr>
<tr>
<td>MATH4211</td>
<td>Number Theory IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4041</td>
<td>Partial Diff. Eqns IV (B3)</td>
<td></td>
</tr>
<tr>
<td>MATH4131</td>
<td>Probability IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4241</td>
<td>Representation Theory IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4171</td>
<td>Riemannian Geometry IV (B1)</td>
<td></td>
</tr>
<tr>
<td>MATH4121</td>
<td>Solitons IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4231</td>
<td>Statistical Mechanics IV (B2)</td>
<td></td>
</tr>
<tr>
<td>MATH4091</td>
<td>Stochastic Processes IV (B1)</td>
<td></td>
</tr>
<tr>
<td>MATH4071</td>
<td>Topics in Statistics IV (B2)</td>
<td></td>
</tr>
</tbody>
</table>

[P: Pre-requisite; C: Co-requisite; P/C: Pre- or Co-requisite; EC: Excluded combination.]

3*: See Level 3 module.
† See appendix A.
2† in addition, a minimum of two maths modules at Level 3.
3† Three maths modules in Years 2 and 3, with at least one module at Level 3
4† Four maths modules in Years 2 and 3, with at least one module at Level 3
4‡ Four maths modules in Years 2 and 3, with at least two modules at Level 3
5† Five maths modules in Years 2 and 3, with at least two modules at Level 3
5‡ Five or more maths modules in Years 2 and 3, with at least two modules at Level 3

**Table 1: Masters degrees in Mathematics***

<table>
<thead>
<tr>
<th>Course</th>
<th>Year 1 Details</th>
<th>Year 2 Details</th>
<th>Year 3 Details</th>
<th>Year 4 Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Master of Mathematics (G103)</strong></td>
<td>Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies.</td>
<td>Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.</td>
<td>Modules to the value of 120 credits chosen from Mathematics Teaching III and List A.</td>
<td>Project IV and either modules to the value of 80 credits chosen from List B or modules to the value of 60 credits chosen List B AND an open 20 credit module (at level 4) chosen from those offered by any other Board of Studies.</td>
</tr>
<tr>
<td><strong>Master of Mathematics (European Studies) (G101)</strong></td>
<td>Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies.</td>
<td>Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.</td>
<td>Students must study and be assessed in a mathematics programme (together, possibly, with other topics) in a European university under the ERASMUS/SOCRATES Programme, and submit an essay on a mathematical topic at the end of the year. The results obtained will count fully towards the award of the degree.</td>
<td>Project IV and either modules to the value of 80 credits chosen from List B or modules to the value of 60 credits chosen List B AND an open 20 credit module (at level 4) chosen from those offered by any other Board of Studies.</td>
</tr>
</tbody>
</table>

*The Ordinary degree regulations are the same as the Honours degree regulations.*
Table 2: Bachelors degrees in Mathematics*

<table>
<thead>
<tr>
<th><strong>B.Sc. Mathematics (G100)</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1: Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies.</td>
<td></td>
</tr>
<tr>
<td>Year 2: Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.</td>
<td></td>
</tr>
<tr>
<td>Year 3: Communicating Mathematics III or Mathematics Teaching III, and <em>either</em> modules to the value of 100 credits chosen from List A <em>or</em> modules to the value of 80 credits chosen from List A AND one open 20 credit module chosen from those offered by any other Board of Studies.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>BSc Mathematics (European Studies) (G104)</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1: Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies, of which at least 20 credits must be an appropriate language module. The language requirement does not apply to students spending the year abroad at Trinity College, Dublin.</td>
<td></td>
</tr>
<tr>
<td>Year 2: Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.</td>
<td></td>
</tr>
<tr>
<td>Year 3: Students must study and be assessed in a mathematics programme (together, possibly, with other topics) in a European university under the ERASMUS/SOCRATES Programme.</td>
<td></td>
</tr>
<tr>
<td>Year 4: Communicating Mathematics III or Mathematics Teaching III, and <em>either</em> modules to the value of 100 credits chosen from List A <em>or</em> modules to the value of 80 credits chosen from List A AND an open 20 credit module chosen from those offered by any other Board of Studies.</td>
<td></td>
</tr>
</tbody>
</table>

*The Ordinary degree regulations are the same as the Honours degree regulations.*
Table 3: Example routes for NS students who took Core Mathematics A (MATH1012) and Core Mathematics B1 (MATH1051), but not Core Mathematics B2 (MATH1041), during 1H.

<table>
<thead>
<tr>
<th>During 2H, take:</th>
<th>During 3H, choose from:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. AMV II (2031)</td>
<td>DT III (3071), DG III (3021), DS III (3091), MB III (3171), OR III (3141), PDE III (3291), Top III (3281), ANL III (3011)*, EF III (3221)†, Sol III (3231)†, possible further choices dependent on **.</td>
</tr>
<tr>
<td>AMV II (2031)</td>
<td>DT III (3071), DG III (3021), MB III (3171), OR III (3141), PDE III (3291), SM III (3051), BM III (3311)*, TS III (3361)†, possible further choices dependent on **.</td>
</tr>
<tr>
<td>3. AMV II (2031)</td>
<td>DG III (3021), EF III (3181), MB III (3171), PDE III (3291), CM III (3101)*, SMech III (3351)†, DT III, OR III.</td>
</tr>
<tr>
<td>4. AMV II (2031)</td>
<td>DT III (3071), DG III (3021), GT III (3041), MB III (3171), OR III (3141), PDE III (3291), RTM III (3191)*, NT III (3031)†, possible further choices dependent on **.</td>
</tr>
<tr>
<td>5. AMV II (2031)</td>
<td>DG III (3021), DT III (3071), MB III (3171), PDE III (3291), BM III (3311)*, AT III (3081)†, OR III† possible further choices dependent on **.</td>
</tr>
<tr>
<td>6. CA II (2011)</td>
<td>DT III (3071), GT III (3041), OR III (3141), MF III (3301)* RTM III (3191)*, EF III (3221)†, NT III (3031)†, possible further choices dependent on **.</td>
</tr>
<tr>
<td>7. AMV II (2031)</td>
<td>DT III (3071), DG III (3021), OR III (3141), MF III (3301)* MB III (3171)*, PDE III (3291)†, SM III (3051)†, possible further choices dependent on **.</td>
</tr>
</tbody>
</table>

*: available in 2011 - 2012 and every other year thereafter.  
†: available in 2012 - 2013 and every other year thereafter.
Table 4: Example routes for NS students who took Core Mathematics A (MATH1012) during 1H and take Core Mathematics B1 (MATH1051) during 2H.

<table>
<thead>
<tr>
<th>During 2H, take:</th>
<th>During 3H, choose from:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. AMV II (2031) Core B1 (1051) Plus one other level 2 module**</td>
<td>PDE III (3291), DT III (3071), DG III (3021), MB III (3171), OR III (3141), SP III (3251)*, possible further choices dependent on **.</td>
</tr>
<tr>
<td>2. AMV II (2031) CA II (2011) Core B1 (1051)</td>
<td>DG III (3021), DS III (3091), MB III (3171), PDE III (3291), AN III (3011)*, EF III (3221)†, PR III (3211)†, DT III (3071), OR III (3141).</td>
</tr>
<tr>
<td>3. Core B1 (1051) Prob II (2151 or 2161) SC II (2041)</td>
<td>DT III (3071), OR III (3141), SM III (3051), BM III (3311)<em>, MF (3301)</em>, TS III (3361)†,</td>
</tr>
<tr>
<td>4. AMV II (2031) Core B1 (1051) SC II (2041)</td>
<td>DT III (3071), DG III (3021), MB III (3171), PDE III (3291), BM III (3311)*, TS III (3361)†, SM III (3051), OR III (3141).</td>
</tr>
<tr>
<td>5. ALG II (2581) Core B1 (1051) Plus one other level 2 module**</td>
<td>DT III (3071), GT III (3041), OR III (3141), RTM III (3191)*, NT III (3031)†, possible further choices dependent on **.</td>
</tr>
</tbody>
</table>

†: available in 2011 - 2012 and every other year thereafter.
*: available in 2012 - 2013 and every other year thereafter.
Table 5: Mathematics module content of the BSc/MSci Mathematics and Physics, and MSci Chemistry and Mathematics Programmes. Joint Honours students also take modules prescribed by the partner department. Of course, you must have the prerequisites for any module you choose.

<table>
<thead>
<tr>
<th>Programme</th>
<th>Year 2:</th>
<th>Year 3:</th>
<th>Year 4:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B.Sc. JH Natural Sciences (Mathematics and Physics) (CFG0)</strong></td>
<td>Analysis in Many Variables II, Complex Analysis II and one other level 2 mathematics module.</td>
<td>Level 3 mathematics modules to the value of 40 credits, and possibly an extra 20 credit Level 3 mathematics module.</td>
<td></td>
</tr>
<tr>
<td><strong>MSci JH Natural Sciences (Mathematics and Physics) (NatSci3)</strong></td>
<td>Analysis in Many Variables II, Complex Analysis II and one other level 2 mathematics module.</td>
<td>Level 3 mathematics modules to the value of 60.</td>
<td>Level 4 mathematics modules to the value of 40 credits from list B and Project IV (optional).</td>
</tr>
<tr>
<td><strong>MSci JH Natural Sciences (Chemistry and Mathematics) (NatSci1)</strong></td>
<td>Analysis in Many Variables II, Algebra and Mathematical Physics II.</td>
<td>Electromagnetism III, Quantum Mechanics III and a Level 2 or 3 mathematics module to the value of 20 credits.</td>
<td>Level 4 mathematics modules to the value of 40 credits from List B, and Project IV (optional).</td>
</tr>
</tbody>
</table>