DURHAM UNIVERSITY
Department of Mathematical Sciences

Level 1 Mathematics modules
Course Booklet
2010 - 2011
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1 General Information

Welcome to the Department of Mathematical Sciences! 1,200 undergraduates take modules provided by the Department. This booklet provides information on first-year modules offered by the department.

It also contains summary information on key policies related to assessment and academic progress.

Full details of the department’s policies and procedures are available in the departmental degree programme handbooks at http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/, which also contains an on-line version of the course descriptions contained in this booklet.

Information concerning general University regulations, examination procedures etc., are contained in the Faculty Handbooks (www.dur.ac.uk/faculty.handbook/) and the University Calendar, which provide the definitive versions of University policy.

The Teaching and Learning Handbook (www.dur.ac.uk/teachingandlearning.handbook/) contains information about assessment procedures, amongst other things.

You should keep this booklet for future reference. For instance, prospective employers might find it of interest.

You can look forward to an enjoyable year.

1.1 Useful Contacts

The first point of contact for issues referring to a particular course or module should be the relevant lecturer. For more general questions or difficulties you are welcome to consult the Course Director, your Adviser (if you have one) or Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk).

For issues involving University registration for mathematics modules, please see the Registration Co-ordinator.

Head of Department:
Prof P. Mansfield (CM207, p.r.w.mansfield@durham.ac.uk)
Director of Undergraduate Studies:
Prof S. Ross (s.f.ross@durham.ac.uk)
Director of Support Teaching:
Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Director of Registrations:
Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Department Disability Representative (DDR):
Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Chair of Staff Student Consultation Committee A (SSCCA):
Prof. S. F. Ross (CM218, s.f.ross@durham.ac.uk)
Chair of Staff Student Consultation Committee B (SSCCB):
(tba)
Chair of Board of Examiners: Prof J. Parker (CM220, j.r.parker@durham.ac.uk)
Deputy Chair of Board of Examiners: Dr. S. Borgan (CM208, sharry.borgan@durham.ac.uk)
Secretary of Board of Examiners: Dr H. Gangl (CM108, herbert.gangl@durham.ac.uk)
The Course Directors for students are determined by their programme and level of study as follows:

Students on Mathematics programmes at level one:
Dr P. Bowcock (CM307, peter.bowcock@durham.ac.uk)

Students on Mathematics programmes at level two:
Dr N. Peyerimhoff (CM320, norbert.peyerimhoff@durham.ac.uk)

Students on Mathematics programmes at levels three and four:
Prof. C. S. Chu (CM305, chong-sun.chu@durham.ac.uk)

Students on Natural Sciences and Combined Honours programmes at all levels:
Dr V. Kurlin (CM324, vitaliy.kurlin@durham.ac.uk)

Students on programmes other than Mathematics and Natural Sciences and Combined Honours at all levels:
Prof R. Ward (CM202, richard.ward@durham.ac.uk)

For each Joint Honours degree there is a designated member of staff from each participating department whom you may contact if you wish to discuss any aspect of your joint degree course. The relevant contacts in the Department are as follows:

Joint degrees with Physics:
Prof R. Ward (CM202, richard.ward@durham.ac.uk)

Joint degree with Chemistry:
Dr D. J. Smith (CM231a, douglas.smith@durham.ac.uk)

Joint degree with Education:
Dr V. E. Hubeny (CM306, veronika.hubeny@durham.ac.uk)

We may also wish to contact you! Please keep the Mathematics Office informed of your current term-time residential address and e-mail address.

1.2 Course Information

Term time in Durham is Michaelmas (10 weeks), Epiphany (9 weeks) and Easter (9 weeks). There are 22 teaching weeks, and the last seven weeks are dedicated to private revision, examinations and registration for the subsequent academic year.

Timetables giving details of places and times of your commitments are available on Departmental web pages and noticeboards in the first floor corridor of the Department. It is assumed that you read them!

You may access your own Maths timetable at [www.maths.dur.ac.uk/teaching/](http://www.maths.dur.ac.uk/teaching/) and then clicking on the ‘My Maths timetable’ link.

Also, teaching staff often send you important information by e-mail to your local [@dur.ac.uk](mailto:@dur.ac.uk) address, and so you should scan your mailbox regularly.

Note that in October it takes time to sort out groups for tutorials and practicals, and so these classes start in week 2.
1.3 Assessment

Full details of the University procedures for Examinations and Assessment may be found in Section 6 of the Learning and Teaching Handbook, http://www.dur.ac.uk/learningandteaching.handbook/. The Department’s policies and procedures are described in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/. The Department follows the marking guidelines set out by the University Senate:

<table>
<thead>
<tr>
<th>Degree Class</th>
<th>Marking Range(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>70 – 80</td>
</tr>
<tr>
<td>II(i)</td>
<td>60 – 69</td>
</tr>
<tr>
<td>II(ii)</td>
<td>50 - 59</td>
</tr>
<tr>
<td>III</td>
<td>40 – 49</td>
</tr>
<tr>
<td>Fail</td>
<td>0 – 39</td>
</tr>
</tbody>
</table>

Core A (MATH 1012) and Core B1 (MATH 1051) are assessed by written examination. For Core B2 (MATH 1041), 50% of the assessment is based on summative coursework submitted in the problem-solving part of the module and 50% is based on a written examination on the Dynamics part of the module.

For all other first year modules offered by the department, 10% of the assessment is based on summative coursework and 90% is on a written examination. All courses include either summative or formative assessed work, with assignments being set on a regular basis in lecture-based courses. The purpose of formative and summative assessment of coursework is to provide feedback to you on your progress and to encourage effort all year long.

Regular assignments are marked A-E to the following conventions:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Equivalent Mark</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>≥ 80%</td>
<td>Essentially complete and correct work</td>
</tr>
<tr>
<td>B</td>
<td>60%—79%</td>
<td>Shows understanding, but contains a small number of errors or gaps</td>
</tr>
<tr>
<td>C</td>
<td>40%—59%</td>
<td>Clear evidence of a serious attempt at the work, showing some understanding, but with important gaps</td>
</tr>
<tr>
<td>D</td>
<td>20%—39%</td>
<td>Scrappy work, bare evidence of understanding or significant work omitted</td>
</tr>
<tr>
<td>E</td>
<td>&lt;20%</td>
<td>No understanding or little real attempt made</td>
</tr>
</tbody>
</table>

1.4 Academic progress

The Department is responsible for ensuring that students are coping with the courses and meeting their academic commitments.

For 1st year modules you are required:
- to attend tutorials/problems classes/computer practical classes
- to sit collections exams
- to submit summative or formative assessed work on time to a satisfactory standard.

Assessed work which is graded D or E is counted as being of an unsatisfactory standard. Attendance and submission of work is monitored through a database. It is your responsibility to
ensure that your attendance is recorded by signing the relevant attendance sheets. Students who are not keeping up with their commitments will be contacted by course directors to help get them back on track. Persistent default will result in a formal written warning, which may be followed by the initiation of Faculty procedures.

Full details of academic progress requirements for the department are available in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/.

1.5 Durham University Mathematical Society

MathSoc: Necessary and Sufficient

Durham University Mathematical Society, affectionately known as MathSoc, gives people with an interest in maths an opportunity to meet away from lectures, and also provides a number of valuable services to its members.

We arrange a variety of events throughout the year. A given year might involve a freshers’ social, a trip paintballing, a Christmas meal, a couple of themed bar crawls, along with what never fails to be the highlight of the MathSoc year - a trip to see Countdown being filmed!

MathSoc helps to arrange the Undergraduate Colloquia, where members of the department and external speakers explain their research, or an area of maths that interests them, at a level accessible to undergraduates. There are two lectures per term which have covered a range of topics, including how viruses exploit symmetry and Fermat’s Last Theorem. We also organise other non-academic talks last year, we had a careers talk, along with a visit from a stand-up mathematician!

In addition to all this, we buy and sell books, meaning that students can get rid of their old ones, and new students can buy them for substantially reduced prices. We also publish a MathSoc newsletter, containing the term’s MathSoc news along with some interesting features (interviews, department news and more).

Our society aims to bring people together, whether its answering questions on what modules to take next year, a casual drink at the pub or actually talking about maths! Its a great way to meet students from other years and get to know other mathematicians.

Want to join?

If so, either come and see our stand at the freshers’ fair, or email us at any time: it costs just 7.50 for a life membership (6 if you pay at the freshers’ fair), or 4 for a year.

This year’s Exec is:

<table>
<thead>
<tr>
<th>Role</th>
<th>Name</th>
<th>Email</th>
</tr>
</thead>
<tbody>
<tr>
<td>President</td>
<td>Jennifer Avery</td>
<td><a href="mailto:jennifer.avery@durham.ac.uk">jennifer.avery@durham.ac.uk</a></td>
</tr>
<tr>
<td>Secretary</td>
<td>Matthew Palmer</td>
<td><a href="mailto:m.i.palmer@durham.ac.uk">m.i.palmer@durham.ac.uk</a></td>
</tr>
<tr>
<td>Treasurer</td>
<td>Chloe Green</td>
<td><a href="mailto:chloe.green@durham.ac.uk">chloe.green@durham.ac.uk</a></td>
</tr>
<tr>
<td>Social Secretary</td>
<td>Eloise Cook</td>
<td><a href="mailto:e.m.e.cook@durham.ac.uk">e.m.e.cook@durham.ac.uk</a></td>
</tr>
<tr>
<td>Publicity Officer</td>
<td>Tash Morrison</td>
<td><a href="mailto:m.m.morrison@durham.ac.uk">m.m.morrison@durham.ac.uk</a></td>
</tr>
</tbody>
</table>
1.6 Disclaimer

The information in this booklet is correct at the time of going to press in May 2010. The University, however, reserves the right to make changes without notice to regulations, programmes and syllabuses. The most up-to-date details of all undergraduate modules can be found in the Faculty Handbook on-line at [www.dur.ac.uk/faculty.handbook/](http://www.dur.ac.uk/faculty.handbook/).

1.7 Booklists and Descriptions of Courses

The following pages contain brief descriptions of the courses in Mathematics Core Mathematics A, B1 and B2, and of the three optional modules Data Analysis & Simulation, Discrete Mathematics and Statistics.

- **Core Mathematics A** (MATH1012) is a double module starting with courses in CALCULUS (36 lectures) and LINEAR ALGEBRA (30 lectures). Alongside at one hour a week in the first term is the optional ‘Brush up your skills’ (BUYS 1) course. In the second term there are courses in PROBABILITY (26 lectures) and LINEAR ALGEBRA (26 lectures).

- **Core Mathematics B1** (MATH1051) is a course in ANALYSIS (37 lectures) occupying terms 1 and 2.

- **Core Mathematics B2** (MATH1041) comprises a first-term course PROBLEM SOLVING (20 lectures and seminars) plus a second-term course DYNAMICS (17 lectures). Alongside at one hour a week in the second term is the optional ‘Brush Up Your Skills’ course (BUYS 2).

- **Data Analysis & Simulation** (MATH1711) is a single module (40 lectures).

- **Discrete Mathematics** (MATH1031) is a single module (40 lectures).

- **Mathematics for Engineers and Scientists** (MATH1551) is a single module (62 lectures). Note: this module is not available to Mathematics students.

- **Single Mathematics A** (MATH1561) is a single module (62 lectures). Note: this module is not available to Mathematics students. **Single Mathematics B** (MATH1571) is a single module (62 lectures). Note: this module is not available to Mathematics students.

- **Statistics** (MATH1541) is a single module (40 lectures).

Each course description is followed by a list of recommended books and a syllabus. For some courses you are advised to buy a particular book, indicated by an asterisk; for others a choice of titles is offered or no specific recommendation is given.

Syllabuses, timetables, handbooks, exam information, and much more may be found at [www.maths.dur.ac.uk/teaching/](http://www.maths.dur.ac.uk/teaching/) or by following the link ‘teaching’ from the Department’s home page ([www.maths.dur.ac.uk](http://www.maths.dur.ac.uk)).

The syllabuses are intended as guides to the modules. No guarantee is given that additional material will not be included and examined nor that all topics mentioned will be treated.
Calculus is elementary mathematics (algebra, geometry, trigonometry) enhanced by the limit process. Its invention is credited to Isaac Newton and Gottfried Leibnitz in the late seventeenth century. Leibnitz started his work in 1673, eight years after Newton, but initiated the basic modern notation for derivative and integral, \( dx \) and \( \int \). From 1690 onward, calculus grew rapidly and reached its present state in roughly a hundred years. This course will seek to consolidate and expand the knowledge you already have of this extremely important area of mathematics. It is designed to be completely accessible to the beginning calculus student without sacrificing appropriate mathematical rigour. The underlying emphasis is on the three basic concepts of calculus: limit, derivative and integral. Applications from the sciences, engineering, business and economics are often used to motivate or illustrate mathematical ideas. This course will be concerned with the nuts and bolts of calculus, while the Core B1 module will revisit the above concepts and provide a deeper knowledge. Differential equations are introduced in connection with applications to exponential growth and decay. Many standard ordinary differential equations (ODEs) that appear frequently in applications are first and second order linear differential equations and are solved by methods that take advantage of their natural association with the technique of integration. The course will provide numerous exercises, some of them involving the use of the computer algebra package MAPLE.

**Recommended Books**


The above books are strongly recommended. Both are useful in several modules at level 1 and 2 (Core A Geometry, Core B1, Core B2, Mathematical Physics II, Analysis in Many Variables). All mathematicians have to understand calculus, so there are many large books aimed at this vast market. A typical example of the genre is listed below. The more concise book by Haggarty might appeal to some students. All books are stocked in libraries in the University.


**Preliminary Reading:** Revise A-level Core Mathematics material in your favourite books.

**Calculators:** The use of electronic calculators is forbidden in the examinations.
Outline of course

**Core A Calculus**

**Aim:** To master a variety of methods for solving problems and acquire some skill in writing and explaining mathematical arguments.

**Term 1** (30 lectures)

**Elementary Functions of a Real Variable:** Domain and range. Graphs of elementary functions. Even and odd functions. Trigonometric and hyperbolic functions. Algebraic combinations and composition. One-to-one functions and inverses.

**Limits and Continuity:** Informal treatment of limits. Statement of main properties (uniqueness, calculus of limits theorem). Continuity at a point and on intervals. The pinching theorem and trigonometric limits. Statement of Intermediate Value Theorem and applications.

**Differentiation:** Derivative as slope of tangent line (the latter being a limit of chords). Differentiability and continuity. Differentiation formulas. Chain rule. Tangent as a linear approximation. Derivative as rate of change. Mean value and Rolle’s theorems. Increasing and decreasing functions. Max-min problems. L’Hopital’s rule. Partial differentiation.

**Taylor’s Theorem:** Statement of Taylor’s theorem with integral and Lagrange remainders. Statement of Taylor’s theorem in more than one variable. Examples of how to estimate remainder terms. Taylor series expansions of $e^x$, $\sin x$, $\sinh x$, $\log (1 + x)$.

**Integration:** Very basic treatment of definite integral in terms of Riemann sums. Indefinite integrals. Relation between the two types of integral (Fundamental theorem of Calculus). Use of partial fractions to integrate rational functions.

**Ordinary Differential Equations:** First order: separable, exact, homogeneous, linear. Second and higher order: linear with constant coefficients, importance of boundary conditions, reduction to a set of first order equations, treatment of homogeneous and inhomogeneous equations, particular integral and complementary function. Applications to particle dynamics (constant force, harmonic oscillator with damping).

**Term 3** (6 lectures)

**Fourier Series:** Orthogonal functions and Fourier series. Convergence, periodic extension, sine and cosine series, half-range expansion.
Techniques from linear algebra are used in all of mathematics. This course gives an introduction to all the major ideas in the topic. The things you learn in this course will be very useful for most modules you take later on.

The first term is concerned with the solution of linear equations and the various ways in which the ideas involved can be interpreted including those given by matrix algebra, vector algebra and geometry. This enables us to determine when a system of equations has a unique solution and gives us a systematic way of finding it. These ideas are then developed further in terms of the theory of vector spaces and linear transformations. We will discuss examples of linear transformations that are familiar from geometry and calculus.

Any linear map can be put into a particularly easy form by changing the basis of the space on which it acts. The second term begins with the solution of the eigenvalue problem which tells you how to find this basis. We then go on to generalise the notions of length, distance and angle to any vector space. These ideas may be used in a surprisingly large range of contexts. We conclude the course by showing how all these ideas come together in the applications to geometry and calculus introduced in the first term.

Recommended Books

Outline of course (continued on next page)

Core A Linear Algebra

Term 1 (30 lectures)

Vectors in $\mathbb{R}^n$ (8 lectures)

- vectors, addition and scalar multiplication in $\mathbb{R}^n$ with concrete examples in $\mathbb{R}^2$ and $\mathbb{R}^3$
- scalar product, vector product, triple product
- equations of lines and planes
- examples: scalar and vector equations of lines and planes in $\mathbb{R}^3$
- solutions of linear equations as generalisations of lines and planes in $\mathbb{R}^3$

Matrices and determinants (8 lectures)

- matrices as mappings in $\mathbb{R}^n$
- examples: dilation, projection, reflection and rotation in $\mathbb{R}^2$
- multiplication and inversion of matrices
- determinants and explicit methods for their calculation (row and column expansion)
- examples: areas of parallelograms, volumes of parallelepipeds
- Gauss–Jordan elimination using matrix notation

Vector spaces over $\mathbb{R}$ (7 lectures)

- vector spaces and subspaces
- examples: lines and planes in $\mathbb{R}^3$
- linear independence, spanning sets, bases and coordinates, dimension
- vector spaces of polynomials
- affine subspaces

Linear mappings (7 lectures)

- definition of linear mapping (examples: projections, reflections, rotations in $\mathbb{R}^3$)
- differentiation as a linear mapping (example: polynomials)
- representation of linear mappings by matrices
- change of basis and of coordinates
- composition of linear mappings and matrix multiplication
- kernel, (row and column) rank and image of a linear mapping
**Term 2** (26 lectures plus collection)

**Complex numbers and \( \mathbb{C}^n \) as a vector space** (4 lectures)
- complex numbers: addition, multiplication, complex conjugate
- geometric illustration: Argand diagram, de Moivre formula
- complex numbers and roots of polynomials
- \( \mathbb{C}^n \) as a vector space

**Diagonalisation and Jordan normal form** (7 lectures)
- eigenvalues and eigenvectors
- explicit calculation with characteristic polynomial
- diagonalisation by change of basis
- Jordan normal forms: invariant subspaces, normal blocks

**Inner product spaces** (8 lectures)
- Definition and examples: \( \mathbb{R}^n, \mathbb{C}^n \), polynomials
- Cauchy–Schwarz inequality
- orthonormal bases and Gram–Schmidt procedure
- orthogonal and unitary matrices
- examples: projection, reflections and distances in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)
- orthogonal complement of a subspace
- diagonalisation of symmetric matrices by orthogonal matrices

**Special polynomials** (4 lectures)
- axioms of groups
- examples: \( \text{GL}(n), \text{SL}(n), \text{O}(n) \)
- matrix realisation of symmetry groups of polygons

**Special polynomials** (3 lectures)
- linear differential operators
- special polynomials as eigenfunctions
Multiple sums and multiple integrals appear throughout mathematics and this part of Core A starts with a brief introduction to the standard methods for evaluating and re-expressing sums and double and triple integrals.

Probability is a concept with applications in all numerate disciplines e.g. in mathematics, science and technology, medicine, engineering, agriculture, economics and many other fields. In this course, the theory of probability is developed with the calculus and analysis available and with applications in mind. Among the topics covered are: probability axioms, conditional probability, special distributions, random variables, expectations, generating functions, applications of probability, laws of large numbers, central limit theorems.

**Recommended Books**

The following book is very good:


The publishers will produce a customised version of DeGroot & Schervish containing only the material relevant to Core A. It will be priced at about £30 but no ISBN was available in time for this booklet.

The DUO site will provide information about some other textbooks.

A lot of information is available from the website [en.wikipedia.org/wiki/Probability](en.wikipedia.org/wiki/Probability)

**Calculators**

Approved electronic calculators are needed in the examinations (Paper II, Core A Mathematics).
Outline of course Core A Probability

Aim: to develop probabilistic insight and computational skills.

Term 2 (26 lectures)

Multiple Integration: iterated sums, double and triple integrals by repeated integration, volume enclosed by surface, Jacobians and change of variables.

Introduction to probability: chance experiments, sample spaces, events, assigning probabilities. Probability axioms and interpretations.

Conditional probability: theorem of total probability, Bayes theorem, independent events. Applications of probability.

Random variables: discrete probability distributions and distribution functions, binomial, Poisson, Poisson approximation to binomial, transformations of random variables. Continuous random variables: probability density functions, normal distribution, normal approximation to binomial.

Joint, marginal and conditional distributions.

Expectations: expectation of transformations, variance, covariance, expectations of expectations, Chebyshev’s inequality, weak law of large numbers. Moment-generating functions.

Central-limit theorems.
This course deals mainly with ‘limits of infinite processes’. It provides a firm foundation for the operations of differentiation and integration that you already know something about. In addition, you will learn how to answer questions such as the following:

(a) What is the limit of the sequence $(2/1)^1, (3/2)^2, (4/3)^3, (5/4)^4, \ldots$ of rational numbers? [Answer: the transcendental number $e$.]

(b) It is not hard to believe that the geometric series $1 + 1/2 + 1/4 + 1/8 + \ldots$ converges to the value 2, but what does the series $1 + 1/2 + 1/3 + 1/4 + \ldots$ converge to? [Answer: it does not converge.]

(c) What is the value of the integral $\int_0^\infty \frac{x^{5/2}}{1+x^2} \, dx$? [Answer: it does not exist.]

We shall discuss techniques for answering questions of this sort. But analysis consists of more than simply problem-solving. Ultimately, it is about constructing logical arguments (proofs), using the correct language and style, and what mathematicians call rigour. Acquiring this skill is more important than learning problem-solving tricks, but also more difficult, especially at first. We hope that by the end of the year, you will be able to invent and write out simple proofs.

**Recommended Books**

The course material is covered in many books on calculus or analysis that you will find in the various libraries. The book by Salas et al, recommended for several other modules, also covers most of the material in this course.

The following are standard American blockbusters, which also cover material in several other first-year courses:


The following are smaller and more specialised English-style books:


R. Maude, *Mathematical Analysis*, Edward Arnold, 1986

C. Clark, *Elementary Mathematical Analysis*, Wadsworth, 1982

**Calculators**: The use of electronic calculators is forbidden in the examinations.
Outline of course

Core B1 Analysis

Term 1 (20 lectures)

**Aim:** An understanding of the real and complex number systems, an introduction to methods of analysis.

**Introduction**

**Numbers:** Introduction: the need for a better understanding of real (and complex) numbers. The number systems \( \mathbb{Z}, \mathbb{Q}, \mathbb{R} \) (not axiomatics). \(|x| < c \iff -c < x < c, |a| + |b| \geq |a+b| \geq ||a| - |b|| \) for real (and complex) numbers.

**Sup and inf:** \( \mathbb{Q}, \mathbb{R} \) and the completeness axiom. Sup and inf of subsets of \( \mathbb{R} \) and of real valued functions. Relation to maxima/minima. \( \sup f + \sup g \geq \sup (f + g) \geq \sup f + \inf g \).

**Limits of Sequences:** \( e, \mathbb{N} \) definition. Basic theorems (uniqueness of limits, COLT, pinching theorem). \( \sup \) \( f \) \( + \) \( \sup \) \( g \) \( \geq \) \( \sup \) \( f \) \( + \) \( \inf \) \( g \).

**Convergence of Series:** Infinite series; convergence, examples including \( \sum n^{-a} \). Comparison test, absolute convergence theorem, ratio test, alternating sign test, conditional convergence. Convergence and absolute convergence of complex sequences and series.

Term 2 (17 lectures)

**Aim:** To construct calculus rigorously, to further develop methods of analysis.

**Limits and Continuity:** Functions of real and complex variables. Epsilon-delta definition of limit of a function. Proof of one or more of basic theorems on limits (sums, pinching theorem etc). Limit of a function as \( x \) tends to infinity, \( \lim_{x \to \infty} x^a/e^x \), \( \lim_{x \to \infty} \log x/x^a \). Continuity and equivalence with \( f(\lim x_n) = \lim (f(x_n)) \). Sum, composite of continuous functions is continuous. Intermediate Value theorem and applications. Bisection proof of max-min theorem.

**Differentiability:** Definition. Differentiability implies continuity. Proof of product rule of differentiation. Proof of Rolle’s theorem, Mean Value theorem and applications (NB. Some of these applications will already have been covered in the Core A Calculus module).

**Integration:** Brief discussion of Riemann sums if necessary (already mentioned in the Core A Calculus module). Fundamental theorem of calculus. \( | \int f | \leq \int |f| \) for real and complex valued \( f \). Convergence of \( \int_0^\infty f(x) \, dx \), comparison test, absolute convergence theorem, examples. Convergence of integrals with bounded range but unbounded integrand, comparison test, absolute convergence, examples. State formula for differentiation under the integral sign. (Integral test for convergence of series).

**Real and Complex Power Series:** Radius of convergence, term-by-term differentiation and integration with examples to show these results are not necessarily true for general (pointwise convergent) series of functions. Taylor series (NB Taylor’s theorem has already been covered in the Core A Calculus module).
Dynamics concerns evolution with time. In this course we study a model of time-development called ‘classical mechanics’. This applies to the world around us and describes the motion of everyday objects via ‘forces’. It was invented by Isaac Newton in the 17th century, when it stimulated revolutions in astronomy, physics and mathematics. Today it is a cornerstone of applied science.

This introductory course treats firstly the motion of point particles, and then the motion of a certain extended body - a flexible stretched string. Highlights include conservation laws and use of Fourier series.

We use what you have covered in Core A Calculus (ordinary and partial differential equations, Fourier analysis) and from Core A Geometry (vectors). It is vital to be familiar with this material!

The Dynamics course leads on naturally to the second-year courses ‘Mathematical Physics’ and ‘Analysis in Many Variables’.

Recommended Books


There are many other textbooks on Mechanics in the Library at shelfmarks 531, 531.1, 531.2, 531.3. eg. French & Ebison, *Introduction to Classical Mechanics*.

For vibrating strings and Fourier series, use the books recommended for partial differential equations in Core A Calculus (especially Boas) or else consult the relevant chapter in almost any book on ‘Mathematics for Physical Scientists’ (or Engineers). These are at Library shelfmarks 51:53, 51:54, 51:62.

Calculators

The use of electronic calculators is forbidden in the examinations.
Outline of course
Core B2 Dynamics

Aim: to provide an introduction to Newton Mechanics applied to simple physical systems.

Term 2 (17 lectures)

Frames of reference, reminder of Newton’s laws in vector form: forces, mass, momentum, gravitational force, Lorentz force.

Examples of work, energy, angular momentum.


Two-body system: central orbits, energy, angular momentum, planetary motion.

Waves and strings: derivation of wave equation for small amplitude oscillations, solution by separation of variables.
This module gives you the opportunity to engage in mathematical problem solving and to develop problem solving skills through reflecting on a set of heuristics. You will work both individually and in groups on mathematical problems, drawing out the strategies you use and comparing them with other approaches.

**General aims**

This module will enable you to develop your problem solving skills; use explicit strategies for beginning, working on and reflecting on mathematical problems; draw together mathematical and reasoning techniques to explore open ended problems; use and develop schema of heuristics for problem solving.

**Topics/scope**

This module provides an underpinning for subsequent mathematical modules. It should provide you with the confidence to tackle unfamiliar problems, think through solutions and present rigorous and convincing arguments for your conjectures. While only small amounts of mathematical content will be used in this course which will extend directly into other courses, the skills developed should have wide ranging applicability.

**Recommended Books**

Much of what you will do is based on the following book. You will need access to a copy, but please don’t read any of it until after the relevant lecture!


Some “deep content” may be included in the module following one of the books

- Robert Burn, Numbers and functions: Steps into Analysis, CUP 1993;
- Robert Burn, Pathway into Number Theory, CUP 1996;
Outline of course

Core B2 Problem Solving

**Aim**: To enable students to develop their problem solving skills; use explicit strategies for beginning, working on and reflecting on mathematical problems; draw together mathematical and reasoning techniques to explore open ended problems; use and develop schema of heuristics for problem solving.

**Term 1** (lectures & seminars)

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Term 1 - Data Analysis: Lectures for the first term of the module coincide with the Statistics module (MATH1541) and provide an introduction to data analysis. The topics to be covered are: sources of data, descriptive statistics, exploration of relationships between two or more variables, and a selection of more advanced techniques.

Term 2 - Modelling and Simulation: The second term deals with problems arising in deterministic modelling, allowing us to predict the behaviour of physical systems (or to learn that the behaviour is unpredictable). For instance biological systems modelling populations with diseases which also experience birth and death.

Computers will be used for some demonstrations and for practical classes. The software will be R for Windows and Maple for Windows.

There are two lectures and an average of 1.5 hours of computing practicals and problems classes per week. Weekly problems may be taken from the exercise sheets and for practicals. There will be a Collection examination in January. All these form an integral part of the module.

Recommended Books

Purchase of a book is not necessary. However, background reading is strongly recommended. Much of the material covered in first term lectures may be found in [1]. Many other introductory statistics texts cover most of the basic techniques addressed. Note that various formulae and methods may differ slightly from book to book, and from lecture material to books. The latter two references cover material for the second term. Other books may be recommended when appropriate.


Calculators

Approved electronic calculators are allowed in the examinations.
Outline of course

Aim: The module is a first course in practical data analysis and computer modelling. The emphasis of the module is upon the understanding of real-life statistical and mathematical problems, and develops the basic concepts and methods by example.

Term 1 (20 lectures)

Sources of Data: Controlled experiments. Randomisation. Observational studies. Ethical practice.

Descriptive Statistics: Displaying distributions: stem and leaf plots, histograms. Notation; summation formulae. Describing and summarising distributions: location (mode, mean, median, percentiles); spread (variance, inter-quartile range); boxplots. Standardisation. Measurements and errors: outliers (link from boxplots), bias, randomness, chance errors, (informally) central tendency. Normal curve; areas under Normal curve; assessing Normality. Misleading graphs.


Methods for More than Two Variables: Least squares and multiple regression; two way tables, mean polish and median polish.

Data Analysis Topics: Chosen from the following: non-linear least squares, smoothing, transformations, design of experiments.

Terms 2 & 3 (21 lectures) Smoothing Data: (2): Least-squares, solving normal equations from first principles with data errors.


Continuous Models (6): Chemical reactions, continuous population problems, first-order ordinary differential equations, Euler’s method, mechanical models (2nd-order systems), phase portraits, equilibria, stability, phase paths and isoclines.

Stochastic Models (5): Random walks and Monte-Carlo quadratures, problems like Buffon’s needle.
1.7.8 DISCRETE MATHEMATICS – MATH1031 (41 lectures)

Dr S. Borgan

This module introduces a wide variety of topics, all of them about things which are discrete (like the integers) rather than continuous (like the real numbers). We will often ask ‘how many?’; these counting problems can be simple to state, using ordinary language, but surprisingly difficult to solve, needing both careful common sense and some specific techniques. The second term of the course is mostly about graphs. These are not the familiar graphs of functions, but networks - like for example railway lines and stations.

Many of the problems you will tackle cannot be done by any standard method, so you must learn to explain your thinking clearly, in some suitable combination of words, symbols and diagrams. Of course this skill will be very useful for other modules, and the rest of your life.

Discrete Maths has some of its origins in mathematical puzzles and games, but now finds many and varied applications, usually in setting up structure or organising something. It is fundamental to computer science.

There are two lectures and one problems class per week. Problems are set weekly to be handed in and there is a compulsory examination (Collections) in January to see how you are going on. In May/June there is a 3-hour written examination.

Recommended Books

There is no required text, but any of these might be helpful or interesting.

Grimaldi is perhaps the most comprehensive.

Tucker also covers most of the material.

Graham, Knuth and Patashnik is a mine of interesting information and examples, written in a very chatty style.

Wilson’s book is excellent for the graph theory part of the course and goes well beyond.

Marcus is very good on the non-graph theory parts of the course.


Calculators

Approved electronic calculators are allowed in the examinations.
Outline of course

Discrete Mathematics

Aim: To provide students with a range of tools for counting discrete mathematical objects. To provide experience of a range of techniques and algorithms in the context of Graph Theory, many with every day applications.

Term 1 (20 lectures)


Recurrence Relations and Generating Functions: Recurrence relations, generating functions, partitions.

Terms 2 & 3 (21 lectures)

Graphs: Basic concepts (paths circuits, connectedness etc.) Euler paths, maze algorithms. Planar graphs, Euler’s theorem, the Platonic graphs. A brief introduction to graph colouring, the Six Colour Theorem. Greedy algorithm.

Note: This module is not available to Mathematics students.

This module is intended to supply the basic mathematical needs for students in Engineering and other sciences.

There will be a short online diagnostic test to be completed during the first week. This test is based on a wide range of maths A-level material. The purpose is to help you brush-up on any material you have forgotten or did not cover in great detail at A-level (not everyone has the same mathematical background.) It does not count in any way towards your final mark for this module. Note that there are also revision classes during the first two weeks of term where you can practise problems and ask questions.

There are 3 lectures each week and fortnightly tutorials. The tutorials start in Week 3. Problems will be set to be handed in each week and there is a Collection examination in December to test your understanding of the first term material. All these form an integral part of the module.

**Recommended Books**

Students should buy either the two books by Stroud or the book by Stephenson.


If you are not too confident about the mathematics module then the books by Stroud will provide you with much support throughout the module. Students have found these books very helpful in previous years. You will probably already know some of the material in the first book. Stephenson is a more concise text but should also prove useful for parts of the second year mathematics module for Engineering students.

You may also like to refer to: (all paperbacks)


**Calculators**

The use of electronic calculators is forbidden in examinations.
Outline of course

Mathematics for Engineers and Scientists

**Term 1** (28 lectures)

**Elementary Functions** (Practical): Their graphs, trigonometric identities and 2D Cartesian geometry: To include polynomials, trigonometric functions, inverse trigonometric functions, $e^x$, $\ln x$, $x$, $\sin(x+y)$, sine and cosine formulae. Line, circle, ellipse, parabola, hyperbola.

**Differentiation** (Practical): Definition of the derivative of a function as slope of tangent line to graph. Local maxima, minima and stationary points. Differentiation of elementary functions. Rules for differentiation of sums, products, quotients and function of a function.

**Integration** (Practical): Definition of integration as reverse of differentiation and as area under a graph. Integration by partial fractions, substitution and parts. Reduction formula for $\int \sin^n x dx$.

**Complex Numbers**: Addition, subtraction, multiplication, division, complex conjugate. Argand diagram, modulus, argument. Complex exponential, trigonometric and hyperbolic functions. Polar coordinates. de Moivre’s theorem. Positive integer powers of $\sin u; \cos u$ in terms of multiple angles.

**Differentiation**: Limits and Continuity. L’Hopital’s rule. Leibniz rule. Tangents, normals. Newton-Raphson method for roots of $f(x) = 0$. Power series, Taylor’s and MacLaurin’s theorem, and applications.


**Terms 2 & 3** (34 lectures)

**Partial Differentiation**: Functions of several variables. Chain rule. Level curves and surfaces. Gradient of a scalar function. Normal lines and tangent planes to surfaces. Local maxima, minima, and saddle points.


**Ordinary Differential Equations**: First order differential equations: separable, homogeneous, exact, linear. Second order linear equations: superposition principle, complementary function and particular integral for equations with constant coefficients, fitting initial conditions, application to circuit theory and mechanical vibrations.
Note: This module is not available to Mathematics students.

The module follows on from A-level, although certain A-level topics may be covered afresh. There are three lectures and one tutorial per week. Problems are set to be handed in each week and there is a compulsory practice examination (Collections) in January to see how you are getting on. It is important to do the written work conscientiously throughout the year both to prepare yourself for the examination and because there is continuous assessment for written work. The material consists of important basic ideas and techniques in calculus and linear algebra which have applications in a huge variety of areas of science and mathematics.

**Recommended Books**

Course notes will be available for purchase (price £2) though examples considered in the lectures may not be included in these notes.

The recommended books for the course are:


The first covers the material in the first term, the second covers terms 2 & 3 of Single A and virtually everything in Single B. These books are strongly recommended.

Many other books contain useful material covering parts of the course. They can be useful sources for further worked examples and problems. A few suggestions are:


**Calculators**

Approved electronic calculators are allowed in the examinations.
Outline of course

**Single Mathematics A**

**Term 1** (30 lectures)

**Diagnostic Test** (1)

**Elementary Algebra and Basic Functions** (4):
Elementary algebra: Simple algebraic manipulations. Roots of \( ax^2 + bx + c = 0 \) by completion of the square. Binomial theorem. Trigonometric functions and their inverses, addition formulae and derivatives.

**Integration** (10):
Fundamental theorem of calculus. Natural log and exponential; hyperbolic functions. Basic methods of integration including substitution, integration by parts, partial fractions, reduction formulae. Line integrals and arc length.

**Complex Numbers** (7):
Addition, subtraction, multiplication, division, complex conjugate, modulus, argument, polar form. Argand diagram, de Moivre’s theorem, \( e^{i\theta} \), positive integral powers of sin and cos in terms of multiple angles, complex exponential, trigonometric and hyperbolic functions. Roots of unity, solutions of simple equations in terms of complex numbers, the fundamental theorem of algebra.

**Limits and Real Analysis** (8):

**Terms 2 & 3** (32 lectures)

**Taylor’s theorem** (7):
Taylor polynomials, Taylor’s theorem with Lagrange form of the remainder, applications to simple numerical approximations, application to L’Hopital’s rule. Convergence of Taylor series. Taylor series of \( 1/(1+x) \); \( e^x \); \( \cos x \); \( \sin x \); \( \ln(1+x) \).

**Linear equations and matrices** (18):

**Numerical methods** (7):
Note: This module is not available to Mathematics students.

This module follows on from A-level mathematics, although many topics will be covered afresh. There are three lectures and one tutorial per week. Problems are set to be handed in each week and there is a compulsory examination (Collections) in January. These are all integral parts of the module.

In the first term we will discuss vector algebra and some applications to mechanics and geometry, ordinary differential equations – their classification and solutions, and Fourier analysis – the representation of functions as linear superpositions of sines and cosines.

In the second and third terms we cover functions of several variables, partial differential equations, and probability. The ideas of differentiation and integration extended to functions of two or more variables give rise to partial derivatives and multiple integrals. A partial differential equation expresses a relationship involving a function of two or more variables and some of its partial derivatives. Wave motion is one of the many phenomena described by partial differential equations; an example is vibration of a stretched string, such as a guitar string. The Fourier analysis discussed in the first term will be used in this context. The final part of the module provides an introduction to probability and statistics.

**Recommended Books**

First Term: Ordinary differential equations, vector methods and Fourier analysis can be found in most books on mathematical methods, for example:


**Calculators**

The use of electronic calculators is forbidden in the examinations.
Outline of course: Single Mathematics B

Term 1 (30 lectures)

Diagnostic Test (1)

Vectors (9):

Ordinary Differential Equations (12):
Properties of the exponential function. First order: \(y' = \lambda y\), separable, homogeneous, Bernoulli’s, linear equations. Second order: linear equations with constant coefficients, superposition, complementary functions and particular integrals. Applications to particle dynamics, using Newton’s Laws of Motion (constant force, harmonic oscillator with damping).

Fourier Analysis (8):
Periodic functions, orthogonality of trigonometric functions, Fourier representation and coefficients. Odd and even functions. Mention of complex form.

Terms 2 & 3 (32 lectures)

Functions of several variables (10):
(The aim is to develop techniques of multivariable calculus with special emphasis on two and three dimensions) Basic description, graphs. Partial derivatives, chain rule, coordinate transformations. Taylor expansions, critical points. Gradients and examples.

Multiple integration (8):
Double integrals, in Cartesian and polar coordinates. Triple integrals and integration in Cylindrical and spherical polars. Line integrals, Green’s theorem.

Partial differential equations (8):
Simple examples of partial differential equations (Laplacian in cylindrical and spherical polars, wave eqn, heat kernel eqn, Schrodinger eqn). D’Alembert’s solution. Separation of variables, harmonics.

Probability (6):
Sample space, random variables, independence, probability distributions (binomial and normal distributions), expectation and variance.
Statistics attempts to make evaluations concerned with uncertainty and numerical conjectures about perplexing questions. The focus of the course is upon the understanding of real-life statistical problems. The first term’s lectures coincide with those for the Data Analysis, Modelling and Simulation module (MATH1711), and develop the basic concepts, with an emphasis on using computer packages for exploratory data analysis. In term 2 we address mainly inferential techniques.

No prior statistical knowledge is assumed. Students are required to have an A-level (with grade ‘C’ at least) in a mathematics subject which may or may not be statistics, or an equivalent qualification.

There are two lectures per week and three other hours (a mixture of tutorials, problems classes, and computer practicals) per fortnight. Problems are set weekly to be handed in for assessment. There will be a Collection examination in January.

**Recommended Books**

Purchase of a book is not necessary. However, background reading is strongly recommended.

Some of the material covered in first term lectures may be found in [1], and this also provides good background for second term lectures. Many other introductory statistics texts cover most of the basic techniques addressed. Note that various formulae and methods may differ slightly from book to book, and from lecture material to books.


**Calculators**

Approved electronic calculators are allowed in the examinations.
**Outline of course**

**Aim**: The module is designed to be a first statistics course. The emphasis is upon the understanding of real-life statistical problems, and develops the basic concepts and statistical methods by example.

**Term 1** (20 lectures)

**Sources of data**: Controlled experiments. Randomisation. Observational studies. Ethical practice.

**Descriptive statistics**: Displaying distributions: stem and leaf plots, histograms. Notation; summation formulae. Describing and summarising distributions: location (mode, mean, median, percentiles); spread (variance, inter-quartile range); boxplots. Standardisation. Measurements and errors: outliers (link from boxplots), bias, randomness, chance errors, (informally) central tendency. Normal curve; areas under Normal curve; assessing Normality. Misleading graphs.


**Methods for more than two variables**: Least squares and multiple regression; two way tables, mean polish and median polish.

**Data analysis topics**: Chosen from the following. Non-linear least squares, smoothing, transformations, design of experiments.

**Terms 2 & 3** (21 lectures)


**Introducing inference**: Binomial distribution. Random sampling; the sample mean. Distribution of the sample mean. Central limit theorem. Normal approximation to binomial.

**Introduction to confidence intervals and hypothesis testing**: Generating confidence intervals. Basic ideas about hypothesis testing, type I and type II errors. Significance tests. P values. Sensible statistical reporting.

**Inferences for means of Normally distributed populations**: Procedures where the variance is known. Procedures where the sample size is large. t tests. Matched pairs problems. Comparing two population means. Comparing population variances. Comparing several population means (Analysis of variance).

**Methods for categorical data**: Fitting hypothesized frequencies to data. Fitting hypothesized probability distributions to data. Chi-square tests of homogeneity. Chi-square tests of independence.

**Distribution-free methods**: Spearman’s rank correlation coefficient. Mann-Whitney-Wilcoxon test, exact and approximate. Wilcoxon signed rank test, exact and approximate.
1.7.13  BRUSH UP YOUR SKILLS (Foundation)

Dr P. Heslop / Prof. P. M. Sutcliffe

Term 1: Because of widening access, a broadening A-level syllabus and differences in the syllabuses of different boards, we facilitate revision and consolidation of the key skills required to embark on a mathematics degree through the “Brush Up Your Skills course”. The course covers material that well over 90% of the students will have seen at A-level, but as well as revision, the course is intended to cover any gaps there may be in any particular combination of A-level modules.

The course consists of 9 problems classes in the first term that complement the Core A module. Attendance is not compulsory but is initially advised on the basis of a diagnostic test administered to all students at the beginning of the first term. The course is voluntary and does not form part of the degree, so students may attend only those sessions that deal with subjects where they feel weak. This facility is intended to help students take control of their own learning, recognize areas of weakness and use the resources available to improve them. It is the first step on the road to becoming an independent learner.

Recommended Books

There is a wide variety of books that cover the topics here. I have picked two. Stroud has a modern structured approach that you may or may not find helpful. Jordan and Smith is more compact, traditional and slightly more advanced.

D.W. Jordan and P. Smith, Mathematical Techniques, 3rd ed., OUP

Term 2: Core B2 Dynamics uses ODEs from the start, and uses PDEs and Fourier Series later on. Understanding Dynamics needs practice and confidence with these topics. The voluntary second-term BUYS classes are designed to help as required.

Recommended Books

As for Term 1.
Outline of course

Brush up your skills

**Term 1** (9 sessions)

**Basics:** rational and irrational numbers, indices and surds, quadratic equations, manipulation of polynomials, (algebraic division, factor and remainder theorems), partial fractions, linear and non-linear inequalities, \((a + b)^n\) when \(n\) is an integer, law of logarithms.

**Functions:** definition, domain and range, linear and quadratic functions, composition, inverse, modulus function, functions \((\sinh, \cosh, \exp, \log)\) and their graphs including \(kf(x)\), \(f(x) + k\) and \(f(kx)\).

**Coordinate Geometry:** equations and properties of straight lines, general equation of circle, centre and radius, Cartesian and parametric equations of curves.

**Trigonometry:** sine and cosine rules, radian measure, arc length, area of sector of circle, trigonometric functions \((\sin, \cos, \tan, \sec, \cosec \text{ and } \cot)\) inverses of \((\sin, \cos, \tan)\), simple identities, double angle formulae, compound angle formulae, solution of simple trigonometric equations.

**Differentiation:** interpretation as slope, derivative of \(x^n\), \(e^x\), \(\ln x\), \(\sin x\), \(\cos x\) and \(\tan x\), chain rule, sum, product and quotient rules, simple functions defined implicitly or parametrically, maxima and minima, equations of tangents and normals.

**Integration:** basic definition, as inverse of differentiation, as area under curve, integration of \(x^n\), \(e^x\), \(\ln x\), \(\sin x\), \(\cos x\) and \(\tan x\), simple examples using substitution, by parts integration using simple trigonometric identities, definite integrals, applications to volumes and surfaces of revolution.

**Vectors:** vectors in 2 and 3 dimensions, vector addition and subtraction, multiplication by scalar, magnitude, orthogonal unit vectors, distance between two points, vector line, scalar product.

**Term 2** (9 sessions)

Aspects of calculus required for Core B2 Dynamics, including ODEs and PDEs.
1.7.14 MAPLE

Dr K. Peeters

Background:

As part of the redesign of the undergraduate programme, the department has decided to incorporate
the use of Maple in its teaching.

"Maple is a general purpose computer algebra system, designed to solve mathematical problems
and produce high-quality technical graphics. It is easy to learn, but powerful enough to calculate
difficult integrals in seconds. Maple incorporates a high-level programming language which allows
the user to define his own procedures; it also has packages of specialized functions which may be
loaded to do work in group theory, linear algebra, and statistics, as well as in other fields. It can be
used interactively or in batch mode, for teaching or research.” (Centre for Statistical and
Mathematical Computing, Indiana University)

Maple makes many mathematical calculations and derivations straightforward. It may be used to
reduce the tedium of extended calculations, to verify correctness of hand calculation and also for
exploration of a topic. Because Maple is designed by mathematicians for use by mathematicians,
it usually gets the right answer!

Content:

There will be an initial supervised/guided session where students will go through a worksheet
showing how Maple may be used in the context of some A-level topics.

During the remainder of the year, from time to time lecturers will demonstrate ways in which
Maple may be used to check calculations, carry out more difficult computations and gain insight
into the material being studied. Lecturers may also set problems to be solved by students using
Maple. Some computer classroom sessions will be provided for students who feel the need to work
on Maple problems with support.

Software availability:

Maple will be available to use on all IT service networked computers. Once registered and in
possession of a campus card, students who wish to purchase Maple for their own computers will
be able to do so at the substantially discounted price of £15.

Reference materials:

It’s not easy to get much insight into software from purely written materials. There are many books
about Maple or which use Maple but none is particularly suitable for this level.

The best start with Maple is to take the "New Users’ Tour" (available from the Help menu once
Maple is running). A good starting point for on-line materials is the Google Web Directory’s Maple
section:

directory.google.com/Top/Science/Math/Software/Maple/.
A Details of Modules and Programmes

All mathematics modules are open, except for

- **MATH3121** (Mathematics Teaching III) — tied to G100, G103, G104, CFG0, FGC0, QRV0, QRVA and X1G1.
- **MATH3131** (Communicating Mathematics III) — tied to G100, G104, CFG0 and QRV0.
- **MATH3161** (Independent Study III) — tied to G100, G103 and G104

G100: B.Sc. in Mathematics
G103: Master of Mathematics
G104: B.Sc. in Mathematics (European Studies)

- If you do **MATH3131** (Communicating Mathematics III), you cannot take another project module in another department.

- If you do **MATH3121** (Mathematics Teaching III), you cannot take another into schools module in another department.

- The double module Project IV is open to students doing an M.Sci. in Natural Sciences, provided the minimal requirements for taking a Level 4 mathematics module are met, provided the corequisite of one other Level 4 mathematics module and provided no Level 4 Project is taken in another department.
Level 1

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<tr>
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Level 2

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<td>MATH2131</td>
<td>Codes &amp; Actuarial Mathematics II</td>
<td>[P: 1012 ; C:-; EC: 2141, 2161, 2171 &amp; 2571]</td>
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<tr>
<td>MATH2141</td>
<td>Codes &amp; Geometric Topology II</td>
<td>[P: 1012 ; C:-; EC: 2131,2151 &amp; 2571]</td>
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<td>MATH2011</td>
<td>Complex Analysis II</td>
<td>[P: 1012; P/C: 1051; EC: SM*]</td>
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<tr>
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<td>Elementary Number Theory &amp; Cryptography II</td>
<td>[P: 1012; C:-; EC: SM*]</td>
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<td>MATH2041</td>
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</tbody>
</table>

AA*: A level Mathematics at grade A.
AC*: A level Mathematics at grade C or above.
SM*: 1551, 1561 & 1571

Prob‡: 2151 or 2161 or 2561 or 2571
Act‡: 2131 or 2161

P: Pre-requisite; C: Co-requisite; P/C: Pre- or Co-requisite; EC: Excluded combination.

CHEM1012: Core 1A Chemistry;
PHYS1*: PHYS1111 (Fundamental Physics A) or PHYS1122 (Foundations of Physics I).
Level 3

**MATH3131** Communicating Mathematics III \((A_3)\) \([P:\) At least 3 Maths modules taken in 2nd year, at least two of which are at Level 2; C: At least two other Level 3 maths module; EC: 3121 or Level 3 project modules other Depts.\]  
**MATH3121** Mathematics Teaching III \((A_3)\) \([P:\) At least 3 Maths modules taken in 2nd year, at least two of which are at Level 2; C: At least two other Level 3 maths modules; EC: 3131, GEOL3251, PSYC3191 & PHYS3611\]

**List A**

**MATH3321** Algebraic Geometry III \((A_1)\) \([P:\) 2011 & 2581 ; C:--; EC: 4011\]  
**MATH3011** Analysis III \((A_1, A_2)\) \([P:\) 2011 & 2031 ; C:--; EC: 4201\]  
**MATH3081** Approx. Th. & Solns to ODEs III \((A_2)\) \([P:\) 2051 and (1051* or 1*); C:--; EC: 4221\]  
**MATH3341** Bayesian Statistics III \((A_1)\) \([P:\) 2041; C:--; EC: 4031\]  
**MATH3101** Continuum Mechanics III \((A_1)\) \([P:\) 2031 & (1* & (1041 or PHYS1*); or (1041 & 1051*)); C:--; EC: 4081\]  

**MATH3071** Decision Theory III \((A_3)\) \([P:\) 1012 C--; EC: -\]  
**MATH3021** Differential Geometry III \((A_3)\) \([P:\) 2031; C:--; EC: -\]  
**MATH3091** Dynamical Systems III \((A_3)\) \([P:\) 2031 & 2011; C:--; EC: -\]  
**MATH3181** Electromagnetism III \((A_3)\) \([P:\) 2031 & (2071 or PHYS2*); C:--; EC:--;\]  
**MATH3221** Elliptic Functions III \((A_2)\) \([P:\) 2011 & (1051* or 1*); C:--; EC: 4151\]  
**MATH3041** Galois Theory III \((A_3)\) \([P:\) 2581 C--; EC: -\]  
**MATH3331** General Relativity III \((A_1)\) \([P:\) 2031 & (2071 or PHYS2*); C:--; EC: 4051\]  
**MATH3201** Geometry III \((A_2)\) \([P:\) 2581, 2031 & 2011; C:--; EC: 4141\]  
**MATH3161** Independent Study III \((A_3)\) \([P:\) 2H Honours Mathematics; C:--; EC: -\]  
**MATH3171** Mathematical Biology III \((A_3)\) \([P:\) 2031; C:--; EC: -\]  
**MATH3301** Mathematical Finance III \((A_3)\) \([P:\) (1051* or 1*) or 2*; C:--; EC: 4181\]  
**MATH3031** Number Theory III \((A_2)\) \([P:\) 2581; C:--; EC: 4211\]  
**MATH3141** Operations Research III \((A_3)\) \([P:\) 1012; C:--; EC: -\]  
**MATH3291** Partial Differential Equations III \((A_3)\) \([P:\) 2031 & (1051* or 1*); C:--; EC: 4041\]  
**MATH3211** Probability III \((A_2)\) \([P:\) 2031, 2011 & Prob*; C:--; EC: 4131\]  
**MATH3111** Quantum Mechanics III \((A_3)\) \([P:\) 2031 & 2071; C:--; EC: -\]  
**MATH3191** Rep. Theory & Modules III \((A_1)\) \([P:\) 2581; C:--; EC: 4101\]  
**MATH3231** Solitons III \((A_2)\) \([P:\) (2031 & 2011; C:--; EC: 4121\]  
**MATH3351** Statistical Mechanics III \((A_2)\) \([P:\) 2031 & (1051* or 1*); C:--; EC: 4231\]  
**MATH3051** Statistical Methods III \((A_3)\) \([P:\) & 2041; C:--; EC: -\]  
**MATH3251** Stochastic Processes III \((A_1)\) \([P:\) 2031 & Prob*; C:--; EC: 4091\]  
**MATH3361** Topics in Statistics III \((A_2)\) \([P/C:3051; C:--; EC: 4071\]  
**MATH3281** Topology III \((A_3)\) \([P:\) 2581, 2031 & 2011; C:--; EC: 4021\]

* If taken in Year 2.  
1* One Level 2 mathematics module.  
2* Two Level 2 mathematics modules.  
† One Level 3 mathematics module.  
Prob1 2151 or 2161 or 2561 or 2571  
PHYS1**: PHYS1111 (Fundamental Physics A) or PHYS1122 (Foundations of Physics I).  
PHYS2**: PHYS2511 (Foundations of Physics II).
**Level 4**

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Course Title</th>
<th>List</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH4072</td>
<td>Project IV</td>
<td>(B3)</td>
<td>[P: 5‡; C: one level 4 maths module; EC:†]</td>
</tr>
<tr>
<td>MATH4061</td>
<td>Advanced Qntm. Th. IV</td>
<td>(B3)</td>
<td>[P: 5†, and including 3111 or PHYS3522; C:--; EC: -]</td>
</tr>
<tr>
<td>MATH4011</td>
<td>Algebraic Geometry IV</td>
<td>(B1)</td>
<td>[P: 3* and including 2581; C:--; EC: 3321]</td>
</tr>
<tr>
<td>MATH4161</td>
<td>Algebraic Topology IV</td>
<td>(B3)</td>
<td>[P: 5†, and including 3281; C:--; EC: -]</td>
</tr>
<tr>
<td>MATH4201</td>
<td>Analysis IV</td>
<td>(B1)</td>
<td>[P: 3*; C:--; EC: 3011]</td>
</tr>
<tr>
<td>MATH4221</td>
<td>Approx. Th. &amp; Solns to ODEs IV</td>
<td>(B2)</td>
<td>[P: 5†, and including 2051; C:--; EC: 3081]</td>
</tr>
<tr>
<td>MATH4031</td>
<td>Bayesian Statistics IV</td>
<td>(B2)</td>
<td>[P: 5†, and including 2041; C:--; EC: 3341]</td>
</tr>
<tr>
<td>MATH4081</td>
<td>Continuum Mechanics IV</td>
<td>(B1)</td>
<td>[P: 5†, and including 2031 &amp; (1041 or PHYS1111 or PHYS1122); C:--; EC: 3101]</td>
</tr>
<tr>
<td>MATH4151</td>
<td>Elliptic Functions IV</td>
<td>(B3)</td>
<td>[P: 5†, and including 3111; C:--; EC: -]</td>
</tr>
<tr>
<td>MATH4051</td>
<td>General Relativity IV</td>
<td>(B1)</td>
<td>[P: 3* and 2†; C:--; EC: 3331]</td>
</tr>
<tr>
<td>MATH4141</td>
<td>Geometry IV</td>
<td>(B2)</td>
<td>[P: 3* And 2†; C:--; EC: 3201]</td>
</tr>
<tr>
<td>MATH4181</td>
<td>Mathematical Finance IV</td>
<td>(B3)</td>
<td>[P: 5† or (4† &amp; COMP3361); C:--; EC: 3301]</td>
</tr>
<tr>
<td>MATH4211</td>
<td>Number Theory IV</td>
<td>(B2)</td>
<td>[P: 5†, and including 2581; C:--; EC: 3031]</td>
</tr>
<tr>
<td>MATH4041</td>
<td>Partial Diff. Eqns IV</td>
<td>(B3)</td>
<td>[P: 2031 &amp; (4‡ or (3†, &amp; COMP3361)); C:--; EC: 3291]</td>
</tr>
<tr>
<td>MATH4131</td>
<td>Probability IV</td>
<td>(B2)</td>
<td>[P: 3*.And, 2†; C:--; EC: 3211]</td>
</tr>
<tr>
<td>MATH4101</td>
<td>Rep. Th. &amp; Moduless IV</td>
<td>(B1)</td>
<td>[P: 5†, and including 2581; C:--; EC: 3191]</td>
</tr>
<tr>
<td>MATH4171</td>
<td>Riemannian Geometry IV</td>
<td>(B1)</td>
<td>[P: 5†, and including 3021; C:--; EC: -]</td>
</tr>
<tr>
<td>MATH4121</td>
<td>Solitons IV</td>
<td>(B2)</td>
<td>[P: 3* And, 2†; C:--; EC: 3231]</td>
</tr>
<tr>
<td>MATH4231</td>
<td>Statistical Mechanics IV</td>
<td>(B2)</td>
<td>[P: 5†, and including 2031; C:--; EC: 3351]</td>
</tr>
<tr>
<td>MATH4091</td>
<td>Stochastic Processes IV</td>
<td>(B1)</td>
<td>[P: 3* And Prob‡; C:--; EC: 3251]</td>
</tr>
<tr>
<td>MATH4071</td>
<td>Topics in Statistics IV</td>
<td>(B2)</td>
<td>[P: 3*; C:--; EC: 3361]</td>
</tr>
</tbody>
</table>

3*: See Level 3 module.
† See appendix A.
2† in addition, a minimum of two maths modules at Level 3.
3† Three maths modules in Years 2 and 3, with at least one module at Level 3
4‡ Four maths modules in Years 2 and 3, with at least one module at Level 3
4‡ Four maths modules in Years 2 and 3, with at least two modules at Level 3
5‡ Five maths modules in Years 2 and 3, with at least two modules at Level 3
5‡ Five or more maths modules in Years 2 and 3, with at least two modules at Level 3

**P**: Pre-requisite; **C**: Co-requisite; **P/C**: Pre- or Co-requisite; **EC**: Excluded combination.

A2, B2: Modules taught in alternate years, starting in 2010 - 2011; A1, B1: Modules taught in alternate years, starting in 2009 - 2010; A3, B3: Modules taught every year.

Table 1: Honours degrees in Mathematics*

**B.Sc. Mathematics (G100)**
Year 1: Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies.

Year 2: Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.

Year 3: Communicating Mathematics III or Mathematics Teaching III, and *either* modules to the value of 100 credits chosen from List A *or* modules to the value of 80 credits chosen from List A AND one open 20 credit module chosen from those offered by any other Board of Studies.

**Master of Mathematics (G103)**
Year 1: Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies.

Year 2: Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.

Year 3: Modules to the value of 120 credits chosen from Mathematics Teaching III and List A.

Year 4: Project IV and *either* modules to the value of 80 credits chosen from List B *or* modules to the value of 60 credits chosen List B AND an open 20 credit module (at level 4) chosen from those offered by any other Board of Studies.

**BSc Mathematics (European Studies) (G104)**
Year 1: Core Mathematics A, Core Mathematics B1, Core Mathematics B2 and Level 1 open modules to the value of 40 credits chosen from those offered by any Board of Studies, of which at least 20 credits must be an appropriate language module. The language requirement does not apply to students spending the year abroad at Trinity College, Dublin.

Year 2: Analysis in Many Variables II, Complex Analysis II and mathematics modules to the value of 80 credits.

Year 3: Students must study and be assessed in a mathematics programme (together, possibly, with other topics) in a European university under the ERASMUS/SOCRATES Programme.

Year 4: Communicating Mathematics III or Mathematics Teaching III, and *either* modules to the value of 100 credits chosen from List A *or* modules to the value of 80 credits chosen from List A AND an open 20 credit module chosen from those offered by any other Board of Studies.

*The Ordinary degree regulations are the same as the Honours degree regulations.*
Table 2: Mathematics module content of the BSc/MSci Mathematics and Physics, and MSci Chemistry and Mathematics Programmes. Joint Honours students also take modules prescribed by the partner department. Of course, you must have the prerequisites for any module you choose.

<table>
<thead>
<tr>
<th>Programme</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
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<tbody>
<tr>
<td><strong>B.Sc. JH Natural Sciences (Mathematics and Physics) (CFG0)</strong></td>
<td>Analysis in Many Variables II, Complex Analysis II and one other level 2 mathematics module.</td>
<td>Level 3 mathematics modules to the value of 40 credits, and possibly an extra 20 credit Level 3 mathematics module.</td>
</tr>
<tr>
<td><strong>MSci JH Natural Sciences (Mathematics and Physics) (NatSci3)</strong></td>
<td>Analysis in Many Variables II, Complex Analysis II and one other level 2 mathematics module.</td>
<td>Level 3 mathematics modules to the value of 60.</td>
</tr>
<tr>
<td></td>
<td>Year 4: Level 4 mathematics modules to the value of 40 credits from list B and Project IV (optional).</td>
<td></td>
</tr>
<tr>
<td><strong>MSci JH Natural Sciences (Chemistry and Mathematics) (NatSci1)</strong></td>
<td>Analysis in Many Variables II, Algebra and Mathematical Physics II.</td>
<td>Electromagnetism III, Quantum Mechanics III and a Level 2 or 3 mathematics module to the value of 20 credits.</td>
</tr>
<tr>
<td></td>
<td>Year 4: Level 4 mathematics modules to the value of 40 credits from List B, and Project IV (optional).</td>
<td></td>
</tr>
</tbody>
</table>