The Euclidean Steiner Tree Problem

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Definition

- What is the Euclidean Steiner Tree Problem?

  Input: n terminals *in the plane.*
  Output: Minimum spanning tree using all n *as well as extra* Steiner points *from the plane.*

- Extension of the Euclidean Minimum Spanning Tree Problem

- Solution: **Steiner Minimal Tree**

Fermat Problem

- The $n = 3$ case *(Fermat, 1601-1665)*.

  "Find in the plane a point whose total distance from three given points is minimal".

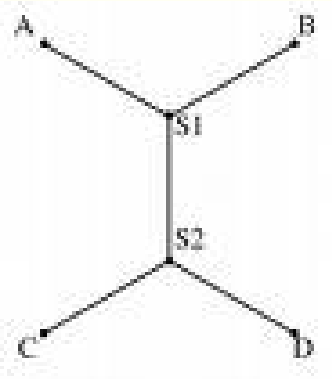
- Solution:

  1. **If all angles are less than 120°:** Point which makes 120° with each pair of the 3 given points.

  2. **If an angle is greater than 120°:** Vertex of the angle greater than 120°.

Properties of Steiner Minimal Tree

- No two edges meet at an angle less than 120°. (*angle condition*)
- Each Steiner point has **degree 3**.
- No crossing edges.
- At most $n-2$ Steiner points.



Steiner Topologies

- **Steiner topologies** show the connections between terminals and Steiner points.

- **Full Steiner topology** has $n-2$ Steiner points.

- Number of full Steiner topologies increases rapidly with n: $f(n) = (2n-4)!/[2^{n-2}(n-2)!]$

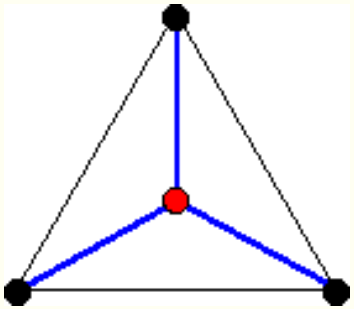
n	2	3	4	5	6	7
$f(n)$	1	1	3	15	105	945
$F(n)$	1	4	31	360	5625	110800

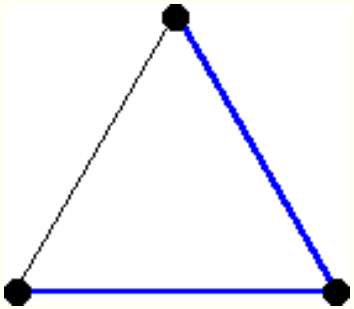
Exact Algorithms / Heuristics

- **Computational** problem - algorithmic, solved by a computer.
- Time complexity, $T_A(N)$.
- Best *exact algorithm* **GeoSteiner** solves up to $n = 2000$ in a day.
- *Heuristics* use the minimum spanning tree. **Prim's Algorithm** has $T_A(N) = O(n \log n)$

The Steiner Ratio

- For all n: $|SMT(n)| \leq |MST(n)|$





- **Steiner ratio** (largest ratio) is $\sqrt{3}/2$.

- Lengths never differ by more than 15.5%

Soap Film Model

- 3 physical devices: String model, **Soap film model**, Membrane model.

- Posts located at terminals.

- State of minimum energy of soap film forms Steiner minimal tree.

Conclusion

- Euclidean Steiner Tree Problem: find the mininum tree connecting n terminals with the addition of auxillary points.
- The Fermat problem is the $n = 3$ case.
- Steiner minimal trees have Steiner points which make 3 angles of 120°
- Problem is exponential due to number of possible topologies increasing raplidly with n.

- No exact polytime algorithm is known to solve the problem.
- Heuristic algorithms using the minimum spanning tree are used.
- The minimum spanning tree for set n is never more than 15.5% longer than the Steiner minimal tree.
- Soap films are a physical model used to study the problem.

The end