Quantum Information and the Interpretation of Quantum Mechanics

Author: Timothy GODDARD
Supervisors: Dr. Benjamin DOYON, Dr. Douglas SMITH

Abstract

SOME of the largest gaps in the operator formalism of quantum mechanics are labeled, slightly misleadingly, under the title of 'the measurement problem'. These problems can be better understood as regarding the quantum-to-classical transition. By introducing the phenomena of entanglement, as well as tools such as density matrices, it is possible to demonstrate, through the mechanism of 'decoherence', the emergence of classical physics from the underlying quantum laws. By considering a simple case such as the spin-boson model we can illustrate the influence and effects of the environment as compared to a system in isolation.

1. Entanglement

ENTANGLEMENT is a phenomenon whereby two particles interact in such a way that measurement of one allows the observer to immediately deduce information about the other. For example a pion decays into a positron and electron with opposite spins meaning measurement of either particle allows us to deduce the spin of both along the axis of measurement. Since an observer can measure in different directions, and such a choice affects the measurement statistics of the unobserved particle, Einstein called this "spukhafte fernwirrkung" [spooky action at a distance]. In 1964 John Stewart Bell [1] demonstrated that quantum mechanics would give experimentally distinct results to a case where 'local hidden-variables' had always encoded the result of a measurement.
We denote two entangled systems with $|a_i\rangle \varepsilon H_A$ and $|b_j\rangle \varepsilon H_B$ as

$$|\psi\rangle = \sum_{i,j} c_{i,j} |a_i\rangle \otimes |b_i\rangle$$

where $|\psi\rangle$ is entangled iff it cannot be written as the tensor product of two separate states i.e. $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ with $|\psi_A\rangle \varepsilon H_A$ and $|\psi_B\rangle \varepsilon H_B$. If it were possible to separate the state in this way then the states would have retained their independence.

References

[1] John Stewart Bell *On the Einstein Podolsky Rosen Paradox* Physics 1 no.3, 195 (1964)

[2] Leggett et al. *Dynamics of the dissipative two-state system* Rev. of Modern Phys., 1987

2. Decoherence

BEGINNING with a superposition $|\psi\rangle = \sum_i^2 c_i|\psi_i\rangle$ constituting orthonormal states $|\psi_i\rangle$, and consider an interaction, known as the von-Neumann ideal measurement scheme, whereby this state becomes entangled with the environment,

$$\sum_i^2 c_i|\psi_i\rangle \otimes |E\rangle \longrightarrow \sum_i^2 c_i|\psi_i\rangle \otimes |E_i\rangle$$

where we have here established entanglement between our system and the environment. The superposition initially present only on the level of the system is now present on the level of the system-environment composite. If we now define a density matrix as $\rho = |\psi\rangle\langle\psi|$ and trace over the environmental states to obtain the so-called 'reduced-density matrix', ρ_s we obtain

$$|c_1|^2|\psi_1\rangle\langle\psi_1| + |c_2|^2|\psi_2\rangle\langle\psi_2| + c_1\bar{c}_2|\psi_1\rangle\langle\psi_2|\langle E_1|E_2\rangle + c_2\bar{c}_1|\psi_2\rangle\langle\psi_1|\langle E_2|E_1\rangle$$

where all the interference terms of our superposition are now encapsulated in the overlap $\langle E_1|E_2\rangle$. It is the suppression of the overlap we instinctively expect due to the interaction with different states causing distinct environmental states that we call decoherence. As the overlap is suppressed we are left with only the diagonal elements of the reduced density matrix which represents an ensemble with probability $\approx |c_1|^2$ of being in state $|\psi_1\rangle$ etc.

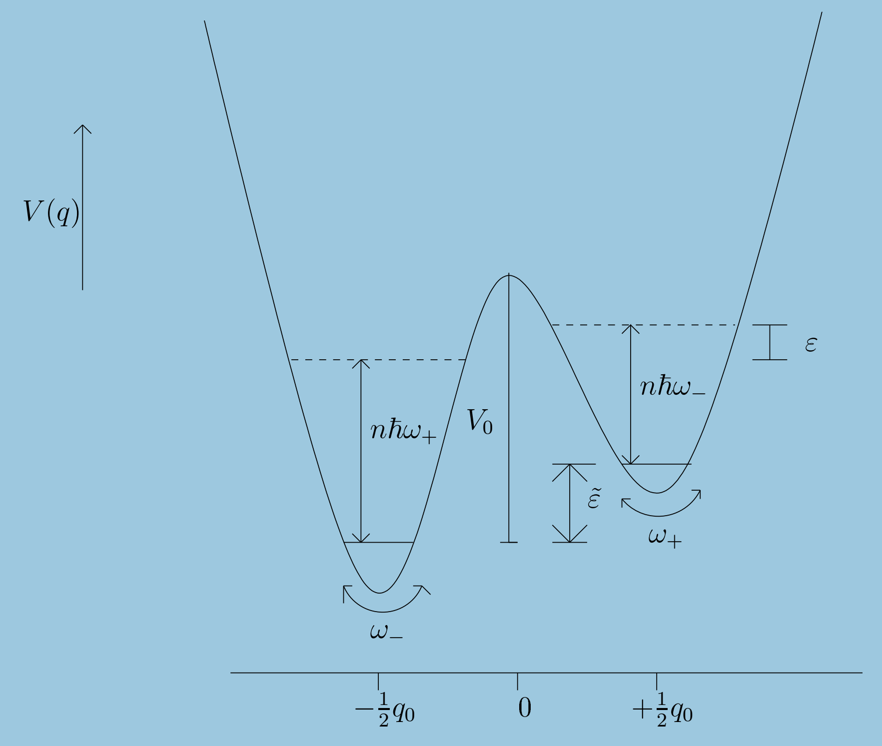

Figure 1: *A Double-Well Potential which, in the spin boson model, is truncated to two-states and interacts with an environment of harmonic oscillators.*

3. Spin-Boson Model

THE Spin-Boson model constitutes a central system restricted to two states (see Fig.1), interacting with an environment of harmonic oscillators. It is possible in the limit $\frac{\Delta}{\omega_c} \to 0$, where Δ is the tunneling amplitude and ω_c is the high-frequency cutoff, to obtain exact expressions for variables of interest, in terms of the spectral density function $J(\omega)$. We define $P(t)$ for $t > 0$ to be the expectation value (where +1 represents the right well in Fig.1) given certain initial conditions. Following the method of [2] using the Non-Interacting Blip Approximation in the case of a symmetric potential ($\varepsilon = 0$), and ohmic dissipation ($J(\omega) \sim \omega$ for $\omega \lesssim \omega_c$) to obtain the results

$$P(t) = \frac{1}{2\pi\hbar}\int_C e^{\lambda t}\left[\lambda + f(\lambda)\right]^{-1} d\lambda$$

$$f(\lambda) \equiv \Delta_{\text{eff}}\left(\frac{2\gamma}{\Delta_{\text{eff}}}\right)^{2\alpha-1} \frac{\Gamma(\alpha+\lambda/2\gamma)}{\Gamma(1-\alpha+\lambda/2\gamma)}$$

$$\gamma \equiv \frac{\pi k T}{\hbar} \qquad \Delta_{\text{eff}} \equiv \Delta\left(\frac{\Delta}{\omega_c}\right)^{\frac{\alpha}{1-\alpha}} \qquad \alpha \equiv \frac{\eta q_0^2}{2\pi\hbar}$$

where C is the standard Bromwich contour. Comparing this to an isolated system we see that the decoherence induced by interaction with the environment gives qualitatively different results.

4. Applications and Relevance of Decoherence

THE intense interest in decoherence results primarily from its role in quantum computing. The qubits need to be shielded from the environment to prevent decoherence from corrupting them and reducing them to classical bits. However, there is also the need to be able to manipulate your qubits to do computations, which prevents you from completely shielding them from outside influence. The establishing of entanglement between the qubits and apparatus required to manipulate the data necessitates quantum error correction codes, which are based on entanglement and which along with decoherence free subspaces are used to combat errors. Qubits are a two-state system interacting with an environment which (provided the interaction is weak enough) can be modeled as harmonic oscillators, meaning it can be equated with the spin-boson model.