Quantum Information and the Interpretation of Quantum Mechanics

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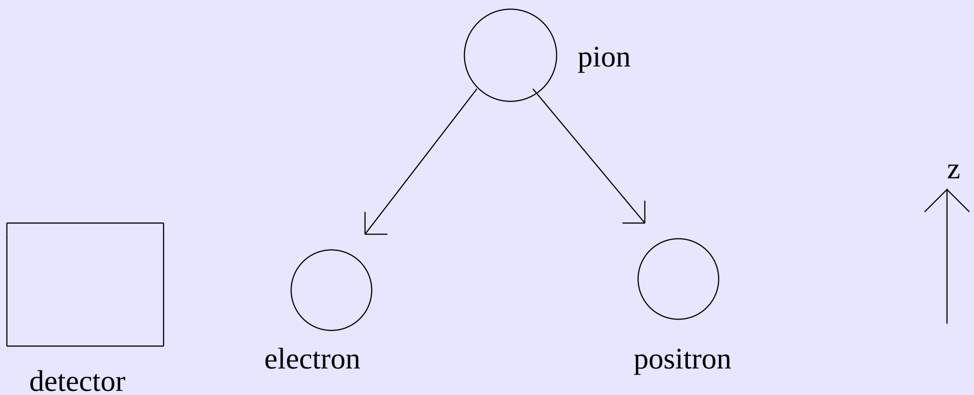
Contents

Entanglement

- Example: pion decays into a positron and electron with opposite spins.



- Bell State

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle)$$

  can't be decomposed.

- Measurement in one basis effects the state in the other, "spukhafte fernwirrkung".

Bell's Theorem and Inequality

- Imagine predetermined by some 'local hidden variables'

Population of state	Particle 1	Particle 2
N_1	$(\hat{a}_+, \hat{b}_+, \hat{c}_+)$	$(\hat{a}_-, \hat{b}_-, \hat{c}_-)$
N_2	$(\hat{a}_+, \hat{b}_+, \hat{c}_-)$	$(\hat{a}_-, \hat{b}_-, \hat{c}_+)$
N_3	$(\hat{a}_+, \hat{b}_-, \hat{c}_+)$	$(\hat{a}_-, \hat{b}_+, \hat{c}_-)$
N_4	$(\hat{a}_+, \hat{b}_-, \hat{c}_-)$	$(\hat{a}_-, \hat{b}_+, \hat{c}_+)$
N_5	$(\hat{a}_-, \hat{b}_+, \hat{c}_+)$	$(\hat{a}_+, \hat{b}_-, \hat{c}_-)$
N_6	$(\hat{a}_-, \hat{b}_+, \hat{c}_-)$	$(\hat{a}_+, \hat{b}_-, \hat{c}_+)$
N_7	$(\hat{a}_-, \hat{b}_-, \hat{c}_+)$	$(\hat{a}_+, \hat{b}_+, \hat{c}_-)$
N_8	$(\hat{a}_-, \hat{b}_-, \hat{c}_-)$	$(\hat{a}_+, \hat{b}_+, \hat{c}_+)$

- Bell's Inequality

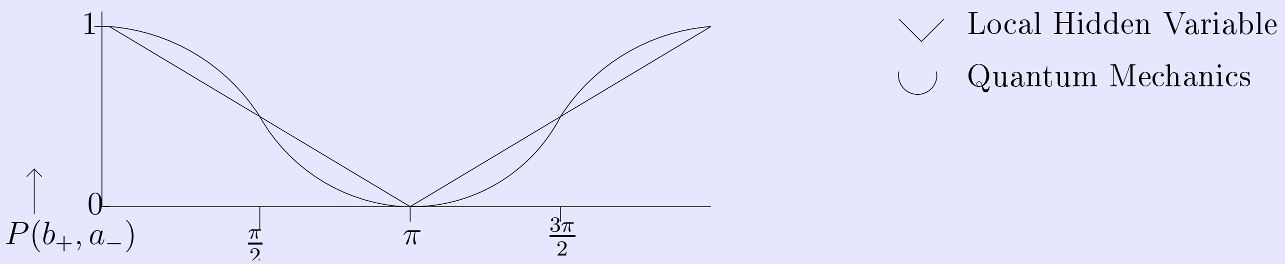
$$P(\hat{a}_+;\hat{b}_+) \leq P(\hat{a}_+;\hat{c}_+) + P(\hat{c}_+;\hat{b}_+)$$
$$N_3 + N_4 \leq (N_2 + N_4) + (N_3 + N_7)$$

- In Quantum Picture where θ_{ab} measures angle between directions $\hat{a}$ and $\hat{b}$

$$P(\hat{a}_+;\hat{b}_+) = \frac{1}{2} sin^2\left(\frac{\theta_{ab}}{2}\right)$$

- Inequality translates to

$$sin^2\left(\frac{\theta_{ab}}{2}\right) \leq sin^2\left(\frac{\theta_{ac}}{2}\right) + sin^2\left(\frac{\theta_{bc}}{2}\right)$$



Decoherence

- Mathematical mechanism for creating entangled states e.g. measurement apparatus

$$|\psi_i\rangle|E_r\rangle \longrightarrow |\psi_i\rangle|E_i\rangle$$

- Now consider superposition

$$\sum_i c_i|\psi_i\rangle|E_r\rangle \longrightarrow \sum_i c_i|\psi_i\rangle|E_i\rangle$$

- Superposition originally present at only the level of the system has been amplified to the level of the system-environment composite.

- Expressing our entangled state as a density matrix gives

$$\varrho_{SE} = \sum_{i,j} c_{ij}|\psi_i\rangle\langle\psi_j| \otimes |E_i\rangle\langle E_j|$$

- Trace over environment to obtain the reduced density matrix $\varrho_S = \sum_{i,j} c_{ij}|\psi_i\rangle\langle\psi_j|\langle E_j|E_i\rangle$

- Suppression of overlap $\langle E_i|E_j\rangle$ for $i \neq j$ is decoherence as diagonal density matrix emerges

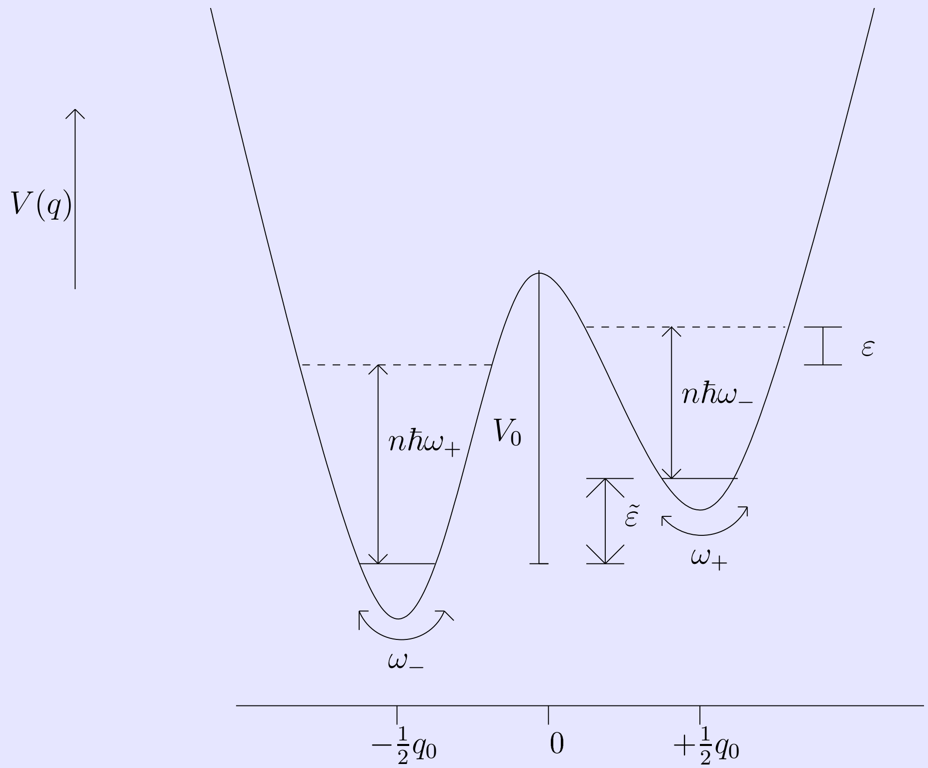
Canonical Models

System	Environment
Harmonic Oscillator	Harmonic Oscillators
Spin-$\frac{1}{2}$ particle	Spin-$\frac{1}{2}$ particles

- Feynman and Vernon 1963, can map any environment to harmonic oscillators provided "sufficiently weak interaction".

- Consider Spin-Boson model consisting of a Spin-$\frac{1}{2}$ particle linearly coupled to an environment of harmonic oscillators

Spin-Boson Model



- The asymmetric double well-potential truncated to a two-state system
- Quantum Computing
- $P(t)$ is expectation value for $t > 0$ for system to be in one well or other.
- Can solve exactly but equations are very unwieldy and prohibitively complicated!

Exact Solution

$$P(t) = \sum_{n=0}^{\infty} (-1)^n \Delta^{2n} K_n(t)$$

$$K_n(t) = 2^{-n} \sum_{\{\zeta_j\}} \int_0^t dt_{2n} \int_0^{t_{2n}} dt_{2n-1} \ldots \int_0^{t_2} dt_1 F_n(\{t_m\}, \{\zeta_i\}, \varepsilon)$$

$$F_n(\{t_m\}, \{\zeta_i\}, \varepsilon) = F_1 \{t_m\} F_2 \{t_m, \zeta_i\} F_3 \{t_m \zeta_i\} F_4 \{t_m \zeta_i \varepsilon\}$$

$$F_1 \equiv exp\left[-\frac{q_0^2}{\pi\hbar} \sum_{j=1}^{n} S_j\right] \qquad F_2 \equiv exp\left[-\frac{q_0^2}{\pi\hbar} \sum_{k=1}^{n} \sum_{j=k+1}^{n} \zeta_j \zeta_k \Lambda_{jk}\right]$$

$$F_3 \equiv \prod_{k=1}^{n-1} cos\left[\frac{q_0^2}{\pi\hbar} \sum_{j=k+1} n \zeta_j X_{jk}\right] \qquad F_4 \equiv cos\left[\sum_{j=1}^{n} \zeta_j \left[(t_{2j} - t_{2j-1})\frac{\varepsilon}{\hbar} - \frac{q_0^2}{\pi\hbar} X_{j0}\right]\right]$$

$$S_j = Q_2(t_{2j} - t_{2j-1})$$

$$\Lambda_{jk} = Q_2(t_{2j-1} - t_{2k}) + Q_2(t_{2j} - t_{2k-1}) - Q_2(t_{2j} - t_{2k}) - Q_2(t_{2j-1} - t_{2k-1})$$

$$X_{jk} = Q_1(t_{2j} - t_{2k+1}) - Q_1(t_{2j} - t_{2k}) - Q_1(t_{2j-1} - t_{2k+1}) + Q_1(t_{2j-1} - t_{2k})$$

$$Q_1(t) = \int_0^{\infty} \frac{J(\omega)}{\omega^2} sin(\omega t) d\omega$$

$$Q_2(t) = \int_0^{\infty} \frac{J(\omega)}{\omega^2} (1 - cos(\omega t)) coth\left(\frac{\beta\hbar\omega}{2}\right) d\omega$$

Ohmic Dissipation for Unbiased Case

- Applying an approximation known as the Non-Interacting Blip Approximation simplifies equations enormously allowing for evaluation of P(t)

- Following the method of Leggett et al. 1987, justify in extreme regions of (α, T)-phase space

- Ohmic Dissipation, spectral density function $J(\omega) \sim \omega$ for $\omega \lesssim \omega_c$ so we take $J(\omega) = \omega e^{-\frac{\omega}{\omega_c}}$

- Obtain result

$$P(t) = \frac{1}{2\pi i} \int_C e^{\lambda t} \left[\lambda + f(\lambda)\right]^{-1} d\lambda$$

$$f(\lambda) = \Delta^2 \int_0^\infty e^{-\lambda t} \frac{cos\left[2\alpha tan^{-1}(\omega_c t)\right]}{(1 + (\omega_c t)^2)^\alpha} \left[\frac{2\gamma t}{2sinh(\gamma t)}\right]^{2\alpha} dt$$

- Qualitatively different to the case of isolation which simply oscillates.

Possible further work

- Solve for non-ohmic spectral densitites
- Kondo model, i.e. Bosons $\longrightarrow$ Fermions
- Solve Quantum Brownian Motion Model

Summary

- Through entanglement superpositions become elevated from system to system-environment composite

- As environmental states become distinct coherent superpositions are suppressed and classical case emerges

- Can solve the dynamics of the spin-boson model exactly to compare the effect of the environment.

Any Questions

Thankyou for listening