

Holographic Graphene in a Cavity

Durham YTF - 17th December 2014

Peter Jones

STAG Research Centre
University of Southampton

Based on arXiv:1407.3097 (Phys. Rev. D 90, 086008) in
collaboration with Nick Evans

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Holographic Graphene

- **AdS/CFT** or **holography** is now a well-established tool for studying **strongly-coupled** systems.

Holographic Graphene

- **AdS/CFT** or **holography** is now a well-established tool for studying **strongly-coupled** systems.
- **Graphene** has been conjectured to be such a system. System consists of 1-atom thick carbon ("2-dimensional") - low energy description is a QFT of massless 2+1-dim. fermions interacting via 3+1-dim. electromagnetism.

Holographic Graphene

- **AdS/CFT** or **holography** is now a well-established tool for studying **strongly-coupled** systems.
- **Graphene** has been conjectured to be such a system. System consists of 1-atom thick carbon ("2-dimensional") - low energy description is a QFT of massless 2+1-dim. fermions interacting via 3+1-dim. electromagnetism.
- The effective speed of light of the fermionic theory is considerably less than the vacuum value, pushing up the fine structure constant \rightarrow strongly coupled?

Holographic Graphene

- **AdS/CFT** or **holography** is now a well-established tool for studying **strongly-coupled** systems.
- **Graphene** has been conjectured to be such a system. System consists of 1-atom thick carbon ("2-dimensional") - low energy description is a QFT of massless 2+1-dim. fermions interacting via 3+1-dim. electromagnetism.
- The effective speed of light of the fermionic theory is considerably less than the vacuum value, pushing up the fine structure constant \rightarrow strongly coupled?
- One way to drive graphene to stronger coupling is to place it in a **cavity** e.g. placing a sheet between two mirrors. The separation **length scale** enters into the effective coupling.

Holographic Graphene

- **AdS/CFT** or **holography** is now a well-established tool for studying **strongly-coupled** systems.
- **Graphene** has been conjectured to be such a system. System consists of 1-atom thick carbon ("2-dimensional") - low energy description is a QFT of massless 2+1-dim. fermions interacting via 3+1-dim. electromagnetism.
- The effective speed of light of the fermionic theory is considerably less than the vacuum value, pushing up the fine structure constant \rightarrow strongly coupled?
- One way to drive graphene to stronger coupling is to place it in a **cavity** e.g. placing a sheet between two mirrors. The separation **length scale** enters into the effective coupling.
- Graphene-like systems can be realised holographically using particular **probe-brane** constructions that give rise to **defect QFTs**. We would like to put such systems in a cavity.

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Holographic Principle and AdS/CFT

- **Holographic Principle:** Quantum gravity in a given region of spacetime can be described by degrees of freedom living on the **boundary** of that region e.g. black hole entropy.

Holographic Principle and AdS/CFT

- **Holographic Principle:** Quantum gravity in a given region of spacetime can be described by degrees of freedom living on the **boundary** of that region e.g. black hole entropy.
- First concrete realization comes from string theory via the **AdS/CFT correspondence**, which relates a bulk theory with gravity in 5-dimensions to a boundary theory without gravity in 4-dimensions. In particular:

Holographic Principle and AdS/CFT

- **Holographic Principle:** Quantum gravity in a given region of spacetime can be described by degrees of freedom living on the **boundary** of that region e.g. black hole entropy.
- First concrete realization comes from string theory via the **AdS/CFT correspondence**, which relates a bulk theory with gravity in 5-dimensions to a boundary theory without gravity in 4-dimensions. In particular:

$$\mathcal{N} = 4 \text{ SYM in } \mathbb{R}^{1,3} \Leftrightarrow \text{Type IIB String Theory in } AdS_5 (\times S^5)$$

where note that $\mathcal{N} = 4$ SYM is a **conformal field theory**, hence the name AdS/CFT.

Holographic Principle and AdS/CFT

- **Holographic Principle:** Quantum gravity in a given region of spacetime can be described by degrees of freedom living on the **boundary** of that region e.g. black hole entropy.
- First concrete realization comes from string theory via the **AdS/CFT correspondence**, which relates a bulk theory with gravity in 5-dimensions to a boundary theory without gravity in 4-dimensions. In particular:

$$\mathcal{N} = 4 \text{ SYM in } \mathbb{R}^{1,3} \Leftrightarrow \text{Type IIB String Theory in } AdS_5 (\times S^5)$$

where note that $\mathcal{N} = 4$ SYM is a **conformal field theory**, hence the name AdS/CFT.

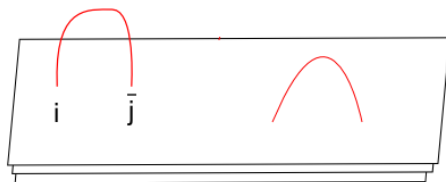
- Terminology: “QFT side” vs. “gravity side”.

Dp-Branes

- Type IIB contains various **Dp-branes**. Dp-Branes are $p+1$ -dim. extended objects which open strings end on. The open string endpoints give rise to **gauge theories on the brane worldvolume**.

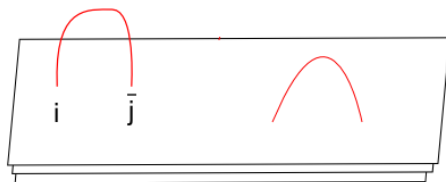
Dp-Branes

- Type IIB contains various **Dp-branes**. Dp-Branes are $p+1$ -dim. extended objects which open strings end on. The open string endpoints give rise to **gauge theories on the brane worldvolume**.



Dp-Branes

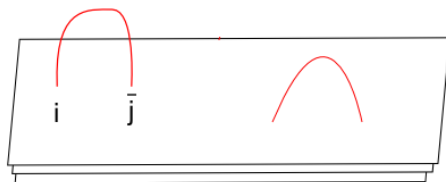
- Type IIB contains various **Dp-branes**. Dp-Branes are $p+1$ -dim. extended objects which open strings end on. The open string endpoints give rise to **gauge theories on the brane worldvolume**.



- E.g. worldvolume theory on stack of N_c D3-branes in IIB is $\mathcal{N} = 4$ SYM with gauge group $SU(N_c)$.

Dp-Branes

- Type IIB contains various **Dp-branes**. Dp-Branes are $p+1$ -dim. extended objects which open strings end on. The open string endpoints give rise to **gauge theories on the brane worldvolume**.



- E.g. worldvolume theory on stack of N_c D3-branes in IIB is $\mathcal{N} = 4$ SYM with gauge group $SU(N_c)$.
- Near-brane geometry of a stack of N_c D3-branes is $AdS_5 \times S^5$!

General Gauge/Gravity Duality or Holography

- The correspondence has been **generalised** in many ways e.g. break conformal symmetry or supersymmetry.

General Gauge/Gravity Duality or Holography

- The correspondence has been **generalised** in many ways e.g. break conformal symmetry or supersymmetry.
- A precise **dictionary** exists for mapping observables between the theories e.g. bulk fields \Leftrightarrow QFT operators, QFT global symmetries \Leftrightarrow isometries etc.

General Gauge/Gravity Duality or Holography

- The correspondence has been **generalised** in many ways e.g. break conformal symmetry or supersymmetry.
- A precise **dictionary** exists for mapping observables between the theories e.g. bulk fields \Leftrightarrow QFT operators, QFT global symmetries \Leftrightarrow isometries etc.
- The correspondence is a **weak/strong duality** - makes holography very useful!

General Gauge/Gravity Duality or Holography

- The correspondence has been **generalised** in many ways e.g. break conformal symmetry or supersymmetry.
- A precise **dictionary** exists for mapping observables between the theories e.g. bulk fields \Leftrightarrow QFT operators, QFT global symmetries \Leftrightarrow isometries etc.
- The correspondence is a **weak/strong duality** - makes holography very useful!
- In fact, the form in which holography is most commonly used and in which we will use it is:

QFT at Strong Coupling \Leftrightarrow <u>Classical</u> Gravity in (A)AdS

Anti de-Sitter Space (AdS)

- AdS is a solution of the vacuum Einstein equations with a **negative cosmological constant**: $G_{MN} = |\Lambda|g_{MN}$.

Anti de-Sitter Space (AdS)

- AdS is a solution of the vacuum Einstein equations with a **negative cosmological constant**: $G_{MN} = |\Lambda|g_{MN}$.
- AdS_{d+1} isometries $SO(2,d) \Leftrightarrow$ conformal group in $\mathbb{R}^{1,d-1}$.

Anti de-Sitter Space (AdS)

- AdS is a solution of the vacuum Einstein equations with a **negative cosmological constant**: $G_{MN} = |\Lambda|g_{MN}$.
- AdS_{d+1} isometries $SO(2,d) \Leftrightarrow$ conformal group in $\mathbb{R}^{1,d-1}$.
- AdS space has a **boundary**. AdS_{d+1} can be written:

$$ds^2 = \frac{R^2}{u^2}(du^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

where $\mu = 0, 1, \dots, d-1$, and R is the **AdS radius**. The boundary is at $u = 0$, and is a copy of Minkowski space $\mathbb{R}^{1,d-1}$. This is where the dual QFT lives!

Overview

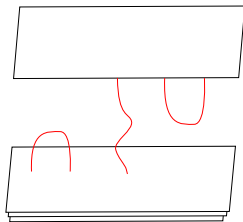
- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Adding Flavour to the Correspondence

- Original correspondence only contains **adjoint DoF** in the gauge theory i.e. vector multiplet $(A_\mu^a, \lambda^a, \phi^a)$. We want 'matter' or quarks ψ^i i.e. **fundamental DoF**.

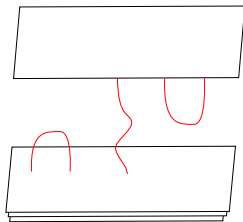
Adding Flavour to the Correspondence

- Original correspondence only contains **adjoint DoF** in the gauge theory i.e. vector multiplet $(A_\mu^a, \lambda^a, \phi^a)$. We want 'matter' or quarks ψ^i i.e. **fundamental DoF**.
- These can be incorporated by adding N_f **probe branes**:



Adding Flavour to the Correspondence

- Original correspondence only contains **adjoint DoF** in the gauge theory i.e. vector multiplet $(A_\mu^a, \lambda^a, \phi^a)$. We want 'matter' or quarks ψ^i i.e. **fundamental DoF**.
- These can be incorporated by adding N_f **probe branes**:



- The **probe limit** is $N_f \ll N_c$ and means there is **no backreaction** on the geometry. Very useful approximation!

Defect QFTs and Holographic Graphene

- Consider a D5 probe brane. Supersymmetry requires that it is embedded in the following manner:

Defect QFTs and Holographic Graphene

- Consider a D5 probe brane. Supersymmetry requires that it is embedded in the following manner:

	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	•	•	•	•	•	•
D5	-	-	-	•	-	-	-	•	•	•

so 0-3 are the QFT directions, and the AdS radial direction is, say, 4.

Defect QFTs and Holographic Graphene

- Consider a D5 probe brane. Supersymmetry requires that it is embedded in the following manner:

	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	•	•	•	•	•	•
D5	-	-	-	•	-	-	-	•	•	•

so 0-3 are the QFT directions, and the AdS radial direction is, say, 4.

- This gives rise to a **defect QFT** i.e. the matter is confined to a 2+1-dim. defect within the spacetime of the 3+1-dim. QFT.

Defect QFTs and Holographic Graphene

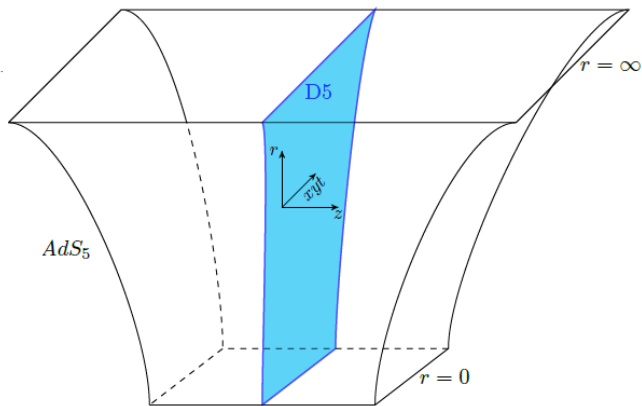
- Consider a D5 probe brane. Supersymmetry requires that it is embedded in the following manner:

	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	•	•	•	•	•	•
D5	-	-	-	•	-	-	-	•	•	•

so 0-3 are the QFT directions, and the AdS radial direction is, say, 4.

- This gives rise to a **defect QFT** i.e. the matter is confined to a 2+1-dim. defect within the spacetime of the 3+1-dim. QFT.
- The picture is thus that of 2+1-dim. matter interacting via gauge DoF that propagate in 3+1-dim. c.f. the low-energy effective field theory of **graphene**.

Illustration of Probe Brane and Defect QFT



[Semenoff et. al. 2014]

Outlook

- Holography allows us to use classical gravity to study strongly-coupled systems, and graphene might be strongly-coupled.

Outlook

- Holography allows us to use classical gravity to study strongly-coupled systems, and graphene might be strongly-coupled.
- Probe D5 systems resemble graphene-like systems.

Outlook

- Holography allows us to use classical gravity to study strongly-coupled systems, and graphene might be strongly-coupled.
- Probe D5 systems resemble graphene-like systems.
- We can push graphene further to strong coupling by putting it in a cavity.

Outlook

- Holography allows us to use classical gravity to study strongly-coupled systems, and graphene might be strongly-coupled.
- Probe D5 systems resemble graphene-like systems.
- We can push graphene further to strong coupling by putting it in a cavity.
- Would now like to study this from the gravity side!

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

AdS Soliton Geometry

- Begin by putting a graphene sheet in a **compact space**.
Superficially correct, since it introduces a conformal symmetry breaking **length scale**, namely the length of the compact dimension.

AdS Soliton Geometry

- Begin by putting a graphene sheet in a **compact space**.
Superficially correct, since it introduces a conformal symmetry breaking **length scale**, namely the length of the compact dimension.
- The gravity dual for $\mathcal{N} = 4$ SYM on a compact space is given by the **AdS Soliton**:

$$ds^2 = \frac{1}{u^2} (h(u)^{-1} du^2 + \eta_{ij} dx^i dx^j + h(u) dz^2) + d\Omega_5^2$$

where $i = 0, 1, 2$, the compact direction is z , and:

$$h(u) = 1 - \left(\frac{u}{u_0} \right)^4$$

AdS Soliton Geometry (cont.)

$$ds^2 = \frac{1}{u^2} (h(u)^{-1} du^2 + \eta_{ij} dx^i dx^j + h(u) dz^2) + d\Omega_5^2$$

$$h(u) = 1 - \left(\frac{u}{u_0} \right)^4$$

AdS Soliton Geometry (cont.)

$$ds^2 = \frac{1}{u^2} (h(u)^{-1} du^2 + \eta_{ij} dx^i dx^j + h(u) dz^2) + d\Omega_5^2$$

$$h(u) = 1 - \left(\frac{u}{u_0} \right)^4$$

- This space is **asymptotically AdS**, and has a new parameter u_0 which is related to the length, Δz , of the compact direction z - regularity fixes $\Delta z = \pi u_0$.

AdS Soliton Geometry (cont.)

$$ds^2 = \frac{1}{u^2} (h(u)^{-1} du^2 + \eta_{ij} dx^i dx^j + h(u) dz^2) + d\Omega_5^2$$

$$h(u) = 1 - \left(\frac{u}{u_0} \right)^4$$

- This space is **asymptotically AdS**, and has a new parameter u_0 which is related to the length, Δz , of the compact direction z - regularity fixes $\Delta z = \pi u_0$.
- Note that the space closes off smoothly at the horizon $u = u_0$ where $h(u)$ vanishes.

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space**
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet**
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 Summary

Putting in a Single Graphene Sheet

- Use a different coordinate system when putting the probe brane in:

$$ds^2 = (\rho^2 + L^2)(g_x dx_{2+1}^2 + g_z dz^2) + \frac{1}{(\rho^2 + L^2)}(d\rho^2 + \rho^2 d\Omega_2^2 + dL^2 + L^2 d\tilde{\Omega}_2^2)$$

So the 10 coordinates are $(x_i, z, \rho, \Omega_2, L, \tilde{\Omega}_2)$.

Putting in a Single Graphene Sheet

- Use a different coordinate system when putting the probe brane in:

$$ds^2 = (\rho^2 + L^2)(g_x dx_{2+1}^2 + g_z dz^2) + \frac{1}{(\rho^2 + L^2)}(d\rho^2 + \rho^2 d\Omega_2^2 + dL^2 + L^2 d\tilde{\Omega}_2^2)$$

So the 10 coordinates are $(x_i, z, \rho, \Omega_2, L, \tilde{\Omega}_2)$.

- Recall:

	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	•	•	•	•	•	•
D5	-	-	-	•	-	-	-	•	•	•

The transverse directions are $(z, L, \tilde{\Omega}_2)$ - note the transverse $SO(3)$ symmetry (important later!).

Putting in a Single Graphene Sheet (cont.)

- Look for **ground-state** embeddings of the D5-probe. By symmetry we look for embeddings $L(\rho)$ (look for $z(\rho)$ later).

Putting in a Single Graphene Sheet (cont.)

- Look for **ground-state** embeddings of the D5-probe. By symmetry we look for embeddings $L(\rho)$ (look for $z(\rho)$ later).
- We need the brane action - a generalisation of the relativistic point-particle action known as the **DBI action**:

$$S_{DBI} \sim -T \int d\xi^6 \sqrt{-\det \left[g_{MN} \frac{dX^M}{d\xi^a} \frac{dX^N}{d\xi^b} \right]}$$

where ξ^a are the coordinates on the 6-dimensional worldvolume of the D5-brane.

Putting in a Single Graphene Sheet (cont.)

- Look for **ground-state** embeddings of the D5-probe. By symmetry we look for embeddings $L(\rho)$ (look for $z(\rho)$ later).
- We need the brane action - a generalisation of the relativistic point-particle action known as the **DBI action**:

$$S_{DBI} \sim -T \int d\xi^6 \sqrt{-\det \left[g_{MN} \frac{dX^M}{d\xi^a} \frac{dX^N}{d\xi^b} \right]}$$

where ξ^a are the coordinates on the 6-dimensional worldvolume of the D5-brane.

- Fixing a gauge, plugging in the ansatz $L(\rho)$ and taking g_{MN} as above, the action becomes:

$$S \sim - \int d\rho \left(1 + \frac{1}{(\rho^2 + L^2)^2} \right)^{3/2} \rho^2 \sqrt{1 + L'^2}$$

Single Probe Embedding and Asymptotics

$$S \sim - \int d\rho \left(1 + \frac{1}{(\rho^2 + L^2)^2} \right)^{3/2} \rho^2 \sqrt{1 + L'^2}$$

- Asymptotically to the boundary (i.e. $\rho \rightarrow \infty$) one can see that the solution goes as:

$$L(\rho) \sim m + c/\rho + \dots$$

Single Probe Embedding and Asymptotics

$$S \sim - \int d\rho \left(1 + \frac{1}{(\rho^2 + L^2)^2} \right)^{3/2} \rho^2 \sqrt{1 + L'^2}$$

- Asymptotically to the boundary (i.e. $\rho \rightarrow \infty$) one can see that the solution goes as:

$$L(\rho) \sim m + c/\rho + \dots$$

By the standard **AdS/CFT dictionary**, m is the quark mass (**source**) and c is the chiral condensate $c \equiv \langle \bar{\psi}\psi \rangle$ (**VEV**).

Single Probe Embedding and Asymptotics

$$S \sim - \int d\rho \left(1 + \frac{1}{(\rho^2 + L^2)^2} \right)^{3/2} \rho^2 \sqrt{1 + L'^2}$$

- Asymptotically to the boundary (i.e. $\rho \rightarrow \infty$) one can see that the solution goes as:

$$L(\rho) \sim m + c/\rho + \dots$$

By the standard **AdS/CFT dictionary**, m is the quark mass (**source**) and c is the chiral condensate $c \equiv \langle \bar{\psi}\psi \rangle$ (**VEV**).

- Chiral condensate in graphene is a condensate of **excitons** - bound state of an electron and a hole.

Single Probe Embedding and Asymptotics

$$S \sim - \int d\rho \left(1 + \frac{1}{(\rho^2 + L^2)^2} \right)^{3/2} \rho^2 \sqrt{1 + L'^2}$$

- Asymptotically to the boundary (i.e. $\rho \rightarrow \infty$) one can see that the solution goes as:

$$L(\rho) \sim m + c/\rho + \dots$$

By the standard **AdS/CFT dictionary**, m is the quark mass (**source**) and c is the chiral condensate $c \equiv \langle \bar{\psi}\psi \rangle$ (**VEV**).

- Chiral condensate in graphene is a condensate of **excitons** - bound state of an electron and a hole.
- Non-zero c for zero m signals **chiral symmetry breaking** (c.f. QCD - confinement, mass gap etc).

Single Probe Embedding and Asymptotics (cont.)

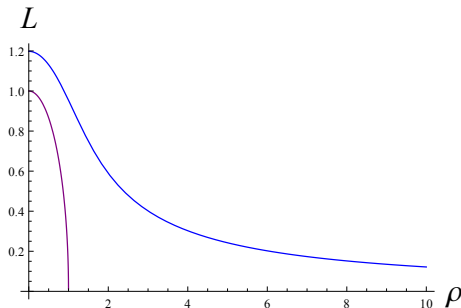
$$L(\rho) \sim m + c/\rho + \dots$$

- We find the **massless** embedding numerically by finding the solution with $L(\infty) \rightarrow 0$. Solution is:

Single Probe Embedding and Asymptotics (cont.)

$$L(\rho) \sim m + c/\rho + \dots$$

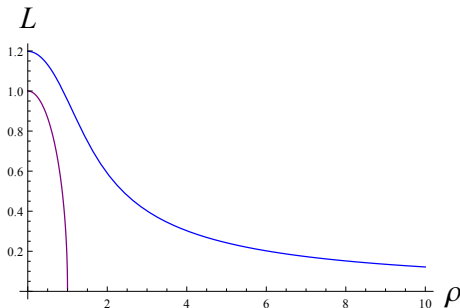
- We find the **massless** embedding numerically by finding the solution with $L(\infty) \rightarrow 0$. Solution is:



Single Probe Embedding and Asymptotics (cont.)

$$L(\rho) \sim m + c/\rho + \dots$$

- We find the **massless** embedding numerically by finding the solution with $L(\infty) \rightarrow 0$. Solution is:



- Clearly have $c \neq 0 \rightarrow$ chiral symmetry breaking!

Single Probe Embedding and χ SB.

- The non-zero L -embedding breaks the $SO(3)$ to $SO(2)$ - recall that QFT global symmetries are mapped to spacetime symmetries in gravity.

Single Probe Embedding and χ SB.

- The non-zero L -embedding breaks the $SO(3)$ to $SO(2)$ - recall that QFT global symmetries are mapped to spacetime symmetries in gravity.
- The non-zero value of $L(0)$ can be interpreted as a dynamically generated **mass gap** - for example, gives rise to massive mesons.

Single Probe Embedding and χ SB.

- The non-zero L -embedding breaks the $SO(3)$ to $SO(2)$ - recall that QFT global symmetries are mapped to spacetime symmetries in gravity.
- The non-zero value of $L(0)$ can be interpreted as a dynamically generated **mass gap** - for example, gives rise to massive mesons.
- So putting in a single graphene sheet leads to χ SB - doesn't happen in pure AdS case.

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space**
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets**
- 4 Graphene in a Cavity
- 5 Summary

Joined Configurations

- Another way to have χ SB is to have joined D5/anti-D5 brane (true even in pure AdS). This represents condensation occuring between fermions on the two defects: $\langle \bar{\psi}_1 \psi_2 \rangle \neq 0$.

Joined Configurations

- Another way to have χ_{SB} is to have joined D5/anti-D5 brane (true even in pure AdS). This represents condensation occurring between fermions on the two defects: $\langle \bar{\psi}_1 \psi_2 \rangle \neq 0$.
- Now consider an embedding $z(\rho)$ i.e. we allow the brane embedding to vary in the compact direction. This leads to:

Joined Configurations

- Another way to have χ SB is to have joined D5/anti-D5 brane (true even in pure AdS). This represents condensation occurring between fermions on the two defects: $\langle \bar{\psi}_1 \psi_2 \rangle \neq 0$.
- Now consider an embedding $z(\rho)$ i.e. we allow the brane embedding to vary in the compact direction. This leads to:

$$S \sim - \int d\rho \left(1 + \frac{1}{\rho^4}\right)^{3/2} \rho^2 \sqrt{1 + \frac{(\rho^4 - 1)^2}{2(\rho^4 + 1)}} z'^2$$

Joined Configurations

- Another way to have χ SB is to have joined D5/anti-D5 brane (true even in pure AdS). This represents condensation occurring between fermions on the two defects: $\langle \bar{\psi}_1 \psi_2 \rangle \neq 0$.
- Now consider an embedding $z(\rho)$ i.e. we allow the brane embedding to vary in the compact direction. This leads to:

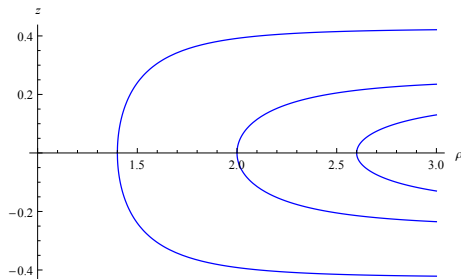
$$S \sim - \int d\rho \left(1 + \frac{1}{\rho^4}\right)^{3/2} \rho^2 \sqrt{1 + \frac{(\rho^4 - 1)^2}{2(\rho^4 + 1)}} z'^2$$

- Proceed as before. Action is independent of z explicitly so there is a constant of motion $\Pi_z \equiv \partial \mathcal{L} / \partial z'$. Thus, problem reduces to a first order ODE:

$$z' = \pm \frac{\sqrt{2} \Pi_z \rho^2}{\sqrt{(1 + 1/\rho^4)(\rho^4 - 1)^4 - \Pi_z^2 \rho^4 (\rho^4 + 4/(1 + \rho^4) - 3)}}$$

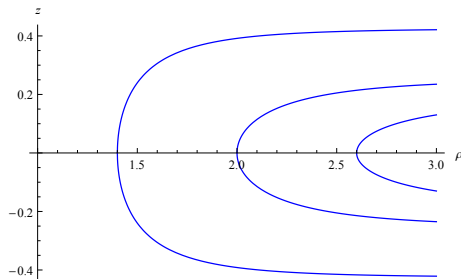
Example Solutions and Properties

- Plot the joined configurations for various values of the turning point ρ_0 :



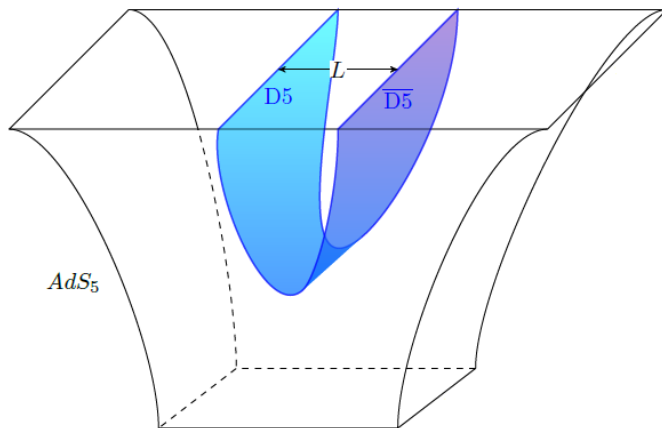
Example Solutions and Properties

- Plot the joined configurations for various values of the turning point ρ_0 :



- Maximum separation** is given by $\pi/2$ (i.e. half the width of the space) - corresponds to the branes extending right down to the horizon.

Illustration of Joined Configurations



[Semenoff et. al. 2014]

Vacuum Alignment for 2 sheets

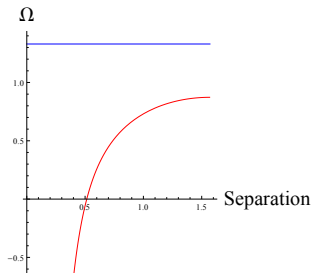
- Suppose we put in two graphene sheets. Does condensation occur across the graphene sheets or on the sheets individually?

Vacuum Alignment for 2 sheets

- Suppose we put in two graphene sheets. Does condensation occur across the graphene sheets or on the sheets individually?
- Need to calculate the energy of the configurations, given by $\Omega = -S|_{on-shell}$. Naively, the on-shell action diverges - one needs to perform **holographic renormalization**.

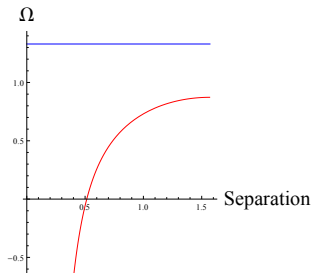
Vacuum Alignment for 2 sheets

- Suppose we put in two graphene sheets. Does condensation occur across the graphene sheets or on the sheets individually?
- Need to calculate the energy of the configurations, given by $\Omega = -S|_{on-shell}$. Naively, the on-shell action diverges - one needs to perform **holographic renormalization**.



Vacuum Alignment for 2 sheets

- Suppose we put in two graphene sheets. Does condensation occur across the graphene sheets or on the sheets individually?
- Need to calculate the energy of the configurations, given by $\Omega = -S|_{on-shell}$. Naively, the on-shell action diverges - one needs to perform **holographic renormalization**.



- The joined configuration is always favourable - condensation across the sheets is the vacuum configuration for all separations!

Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity**
- 5 Summary

Gravity Dual of QFTs with Boundary - AdS/BCFT

- Before we considered graphene in a compact space. Would really like to put it in a cavity e.g. between two fixed mirrors, walls etc.

Gravity Dual of QFTs with Boundary - AdS/BCFT

- Before we considered graphene in a compact space. Would really like to put it in a cavity e.g. between two fixed mirrors, walls etc.
- How to find the gravity dual for a QFT with a boundary?

Gravity Dual of QFTs with Boundary - AdS/BCFT

- Before we considered graphene in a compact space. Would really like to put it in a cavity e.g. between two fixed mirrors, walls etc.
- How to find the gravity dual for a QFT with a boundary?
- There exists an **AdS/BCFT** prescription [Takayanagi et. al. 11']. Add the Gibbons-Hawking boundary term for a **new interior boundary** to the usual Einstein-Hilbert action:

$$I = \frac{1}{16\pi G_N} \int_{\mathcal{M}} \sqrt{-g}(R - 2\Lambda) + \frac{1}{8\pi G_N} \int_{\partial\mathcal{M}} \sqrt{-h}(K - \mathcal{T})$$

for extrinsic curvature K .

Gravity Dual of QFTs with Boundary - AdS/BCFT

- Before we considered graphene in a compact space. Would really like to put it in a cavity e.g. between two fixed mirrors, walls etc.
- How to find the gravity dual for a QFT with a boundary?
- There exists an **AdS/BCFT** prescription [Takayanagi et. al. 11']. Add the Gibbons-Hawking boundary term for a **new interior boundary** to the usual Einstein-Hilbert action:

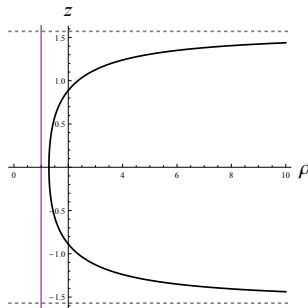
$$I = \frac{1}{16\pi G_N} \int_{\mathcal{M}} \sqrt{-g}(R - 2\Lambda) + \frac{1}{8\pi G_N} \int_{\partial\mathcal{M}} \sqrt{-h}(K - \mathcal{T})$$

for extrinsic curvature K .

- Have also added a boundary matter Lagrangian \mathcal{T} which is taken to be constant and interpreted as the **tension** of the boundary.

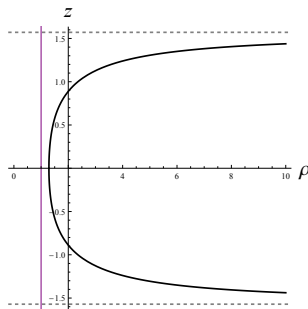
Gravity Dual of Strip

- Following their prescription, the gravity dual for a strip would again be the AdS soliton but now with a boundary in the interior:



Gravity Dual of Strip

- Following their prescription, the gravity dual for a strip would again be the AdS soliton but now with a boundary in the interior:



- However, we find that the brane embedding does not close off before hitting this new boundary!

How to proceed?

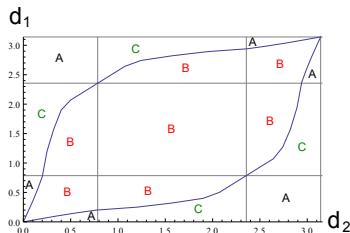
- Maybe we need additional BC when the probes meet the interior boundary? Or perhaps the whole AdS/BCFT construction does not make sense?

How to proceed?

- Maybe we need additional BC when the probes meet the interior boundary? Or perhaps the whole AdS/BCFT construction does not make sense?
- We take the most naive prescription - use the AdS soliton background to represent the vacuum of the theory between the mirrors, and put in the probes and their **mirror images by hand**. Now the additional possibility of **condensation with the mirror image**.

How to proceed?

- Maybe we need additional BC when the probes meet the interior boundary? Or perhaps the whole AdS/BCFT construction does not make sense?
- We take the most naive prescription - use the AdS soliton background to represent the vacuum of the theory between the mirrors, and put in the probes and their **mirror images by hand**. Now the additional possibility of **condensation with the mirror image**.
- Result is a complicated phase diagram:



Overview

- 1 Introduction and Motivation
- 2 Review of Holography
 - Basic Holography
 - Adding Flavour
- 3 Graphene in a Compact Space
 - The Background Geometry - AdS Soliton
 - Single Graphene Sheet
 - Putting in 2 Sheets
- 4 Graphene in a Cavity
- 5 **Summary**

Summary

- Have studied D5-probes in AdS soliton geometry. This geometry is dual to vacuum of $\mathcal{N} = 4$ SYM on a space with one compact direction.

Summary

- Have studied D5-probes in AdS soliton geometry. This geometry is dual to vacuum of $\mathcal{N} = 4$ SYM on a space with one compact direction.
- The scale Δz generates fermion condensation and mass gap formation - saw this for a single graphene sheet, and also for 2 sheets (where the joined configuration was always favourable).

Summary

- Have studied D5-probes in AdS soliton geometry. This geometry is dual to vacuum of $\mathcal{N} = 4$ SYM on a space with one compact direction.
- The scale Δz generates fermion condensation and mass gap formation - saw this for a single graphene sheet, and also for 2 sheets (where the joined configuration was always favourable).
- Including mirror images, there are additional possible phases where one or more sheets condenses with its mirror image. The resulting phase diagram is then somewhat more interesting.

Summary

- Have studied D5-probes in AdS soliton geometry. This geometry is dual to vacuum of $\mathcal{N} = 4$ SYM on a space with one compact direction.
- The scale Δz generates fermion condensation and mass gap formation - saw this for a single graphene sheet, and also for 2 sheets (where the joined configuration was always favourable).
- Including mirror images, there are additional possible phases where one or more sheets condenses with its mirror image. The resulting phase diagram is then somewhat more interesting.
- Hope is that these qualitative predictions could be looked for experimentally.