AdS/CMT and Consistent Kaluza-Klein Truncations

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AdS/CMT

- The AdS/CFT correspondence is a powerful tool to study strongly coupled quantum field theories.

Can holography be applied to Condensed Matter Theory?

- One focus: systems with strongly coupled "quantum critical points" - phase transitions at zero temperature.

- Another focus: superconductors (superfluids) [Gubser].

  In fact some superconductors ("heavy fermions", high Tc cuprates) are associated with quantum critical points.

- Some critical points have full relativistic conformal invariance in the far IR. If the corresponding CFT is strongly coupled one can aim to find AdS solutions of string or M-theory that describe the system.

- The dual description of the CM system at finite temperature is given by asymptotically AdS black hole solutions.

- If the CM system has a global symmetry, then there is a gauge field in the bulk that is dual to the conserved current. One can study the CM system at finite chemical potential by constructing electrically charged AdS black holes.

- Leads to novel black hole solutions with interesting thermodynamic instabilities

- The CM critical points may exhibit a scaling that is not isotropic

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i, \qquad z \neq 1$$

where z is the "dynamical exponent"

- Lifshitz(z) geometries [Kachru, Liu, Mulligan]

$$ds^2 = -r^{2z} dt^2 + r^2 (dx^i dx^i) + \frac{dr^2}{r^2}$$

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i, \qquad r \to \lambda^{-1} r$$

Also want to construct black holes that asymptote to these geometries.
Note: the holographic dictionary for these geometries is much less well established than in the case of AdS geometries.

Most work has been carried out in “Bottom Up” models.
Find solutions in a simple theory of gravity with a few other degrees of freedom e.g. a vector, plus one or two scalar fields.

Advantages:
- Simple
- Models should exist somewhere in string landscape?
- Might capture some universal behaviour

Disadvantages:
- Does the model arise in string theory? Is there any well defined dual (conformal) field theory?
- If phenomenological model is viewed just as an approximation to a model that can be embedded in string theory, it might not capture e.g. interesting low temperature behaviour.

Alternative "Top Down" approach - construct explicit solutions of D=10 or D=11 supergravity.

Advantages:

- One is studying bone-fide dual field theories - not all bottom up models might be realised.
- One can study small parts of landscape of solutions.

Disadvantages:

- Hard!
- Solutions may not be of direct physical relevance.

We have been pursuing Top Down constructions.
Main Tool: Consistent Kaluza-Klein truncations

Lifshitz(z) Solutions

Bottom Up examples are well studied:
D=4 theory of gravity consisting of metric, cosmological constant and massive vector field. Provided that the mass is appropriately tuned to the cosmological constant, then obtain solutions with Lifshitz(z) in d=1+2.

Top Down: Can they be realised in String/M-theory?

Li, Nishioka, Takayangi: No-go theorem for SUGRA solutions
Hartnoll, Polchinski, Silverstein, Tong: Three construction but all implicit/ schematic.
Balasubramanian, Narayan : Time direction becomes null.

Donos, JPG, Kim, Varela: New result:

d=1+2 Lifshitz(z) solutions of D=11 SUGRA with z~39!

Consistent KK Truncations

- Consider KK reduction of some high D dimensional theory on some internal manifold M. Obtain a low d dimensional theory with an infinite number of fields.

- A consistent KK truncation is one where we can keep a finite set of fields such that any solution of the low d theory involving these fields uplifts to an exact solution of the high D theory.

- E.g. KK reduction on S^1 - keep ALL $U(1)$ invariant modes, the usual g, A_μ, ϕ .The modes which are truncated are charged and can't source these neutral modes

- Note that in this case it so happens that the modes that are kept are massless and the ones that are discarded are massive, so one can also view the truncation in an effective field theory sense

- Usually consistent KK truncations don't exist.

Some general results known in context of AdS/CFT

Conjecture-Theorem (in many cases):
Consider most general supersymmetric $AdS \times M$ solutions of D=10/11 SUGRA.
Can always consistently KK reduce on M keeping the supermultiplet containing the graviton.
In the dual CFT language keep the fields dual to the superconformal current multiplet $(T_{ab}, J_a, \ldots)$
[JPG, O.Varela]

For $AdS_4 \times SE_7$ solutions of D=11 SUGRA and
$AdS_5 \times SE_5$ solutions of type IIB SUGRA we can do more...

(here SE is a Sasaki-Einstein space)

$AdS_4 \times SE_7$ -dual to N=2 SCFTs in d=1+2 (U(1) R-symmetry)
There is a consistent KK reduction to [JPG,Kim,Varela,Waldram]

D=4 N=2 gauged supergravity (metric, vector)

+1 Vector multiplet (1 vector + 2 scalars)
+1 Hypermultiplet (4 scalars)

Comments: Reduced theory has a parameter $\epsilon = \pm 1$
$\epsilon = +1:$ AdS_4 vac uplifts to N=2 supersymmetric $AdS_4 \times SE_7$
$\epsilon = -1:$ AdS_4 vac uplifts to N=0 skew-whiffed $AdS_4 \times SE_7$
Also has two additional AdS vacua with N=0

HOW? KK truncation exploits the geometric structures of the SE_7 space that are implied by supersymmetry. Most general?

$AdS_5 \times SE_5$ -dual to N=1 SCFTs in d=1+3 (U(1) R-symmetry)

There is a consistent KK reduction not only to D=5 N=2 gauged supergravity but also to

[JPG,Varela;Cassani,Dalla'gatta,Faedo;Liu,Szepietowski,Zhao]

D=5 N=4 gauged supergravity (metric + 6 vectors + 1 scalar)
+2 N=4 vector multiplets (2 vectors + 10 scalars)

Comments:

AdS_5 susy vac uplifts to N=2 supersymmetric $AdS_5 \times SE_5$

There is also one additional AdS_5 vac with N=0 susy

Any solution of these D=4/5 gauged SUGRA theories gives an infinite number of solutions of D=11/type IIB SUGRA by uplifting on an arbitrary SE_7/SE_5 space.

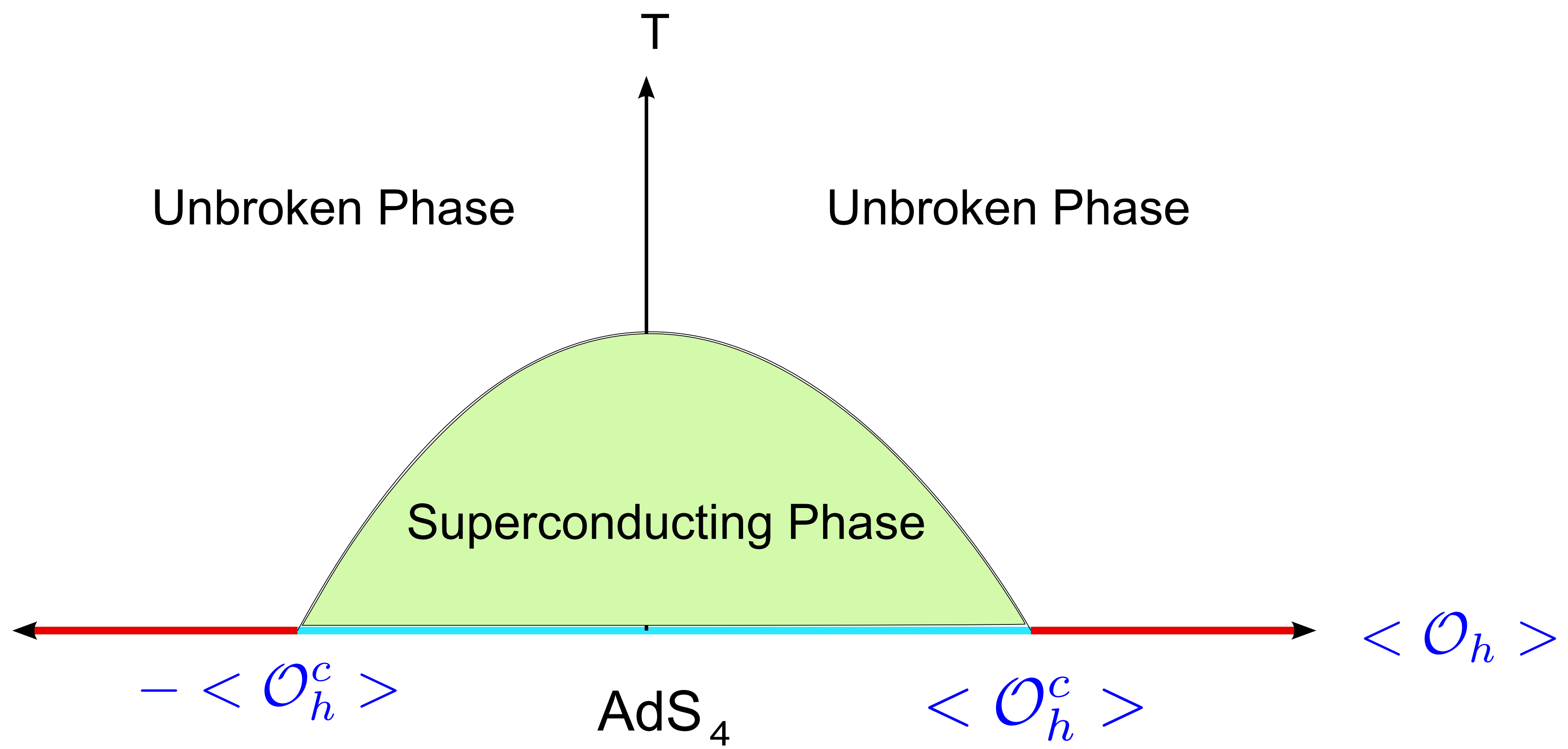
Application: Holographic Superconductors

These theories have been used to study holographic superconductivity in string/M-theory. Conveniently, there are further consistent truncations that one can use.
JPG,Sonner,Wiseman;Gubser,Herzog,Pufu,Tesileanu

eg For D=4 case (dual to d=1+2 systems) we can truncate to
metric, vector, charged scalar, neutral scalar.
Electrically charged black holes describe the skew-whiffed SCFT at finite temperature and non-zero chemical potential.

At high temp there are black holes with no charged scalar hair that describe the non-superconducting phase where U(1) not broken.
At low temp there are black holes with charged hair, spontaneously breaking the U(1) symmetry, that describe the superconducting phase.

The neutral scalar is dual to a relevant operator, breaking parity invariance, which gives a dome of superconducting solutions.



At T=0 superconducting black holes become charged domain walls and approach N=0 AdS_4 solution in far IR: universal emergent conformal symmetry.

In particular, the finite entropy density of the unbroken phase black holes at T=0 is completely cloaked by the dome.

Consistent Truncation via M5-branes Wrapping SLag3 and Lifshit(z) Solutions of D=11 SUGRA

- Calabi-Yau (M_6, J, Ω) with a SLag 3-cycle Σ_3

$$Vol(\Sigma_3) = Re(\Omega)|_{\Sigma_3}$$

- An M5-brane can wrap Σ_3 and preserve supersymmetry.

- Worldvolume of M5 is $\mathbb{R}^{1,2} \times \Sigma_3$ and in the IR we obtain a d=1+2 QFT with N=2 susy.

- Via AdS/CFT we know that if $\Sigma_3 = H_3/\Gamma$ then this QFT is actually an N=2 SCFT.

Described by $AdS_4 \times \Sigma_3 \times S^4$ solutions of D=11 SUGRA
JPG,Kim,Waldram

There exists a consistent KK truncation of D=11 SUGRA on
$\Sigma_3 \times S^4$ where $\Sigma_3 = H^3/\Gamma, S^3, T^3$ Donos,JPG,Kim,Varela

Obtain:

D=4 N=2 gauged supergravity (metric, vector)
+1 Vector multiplet (1 vector + 2 scalars)
+2 Hypermultiplets (8 scalars)

When $\Sigma_3 = H_3/\Gamma$ this theory has a susy AdS_4 solution which uplifts to the $AdS_4 \times H_3/\Gamma \times S^4$ solution.

In addition we find that there is a Lifshitz(z) solution with $z \sim 39$

No-go theorem of Li, Nishioka, Takayanagi:

No D=4 Lifshitz(z) solutions of D=11 SUGRA of the form given by $Lif(z) \times M_7$ allowing for warped product metrics, but not allowing for the M_7 to be fibred over the $Lif(z)$

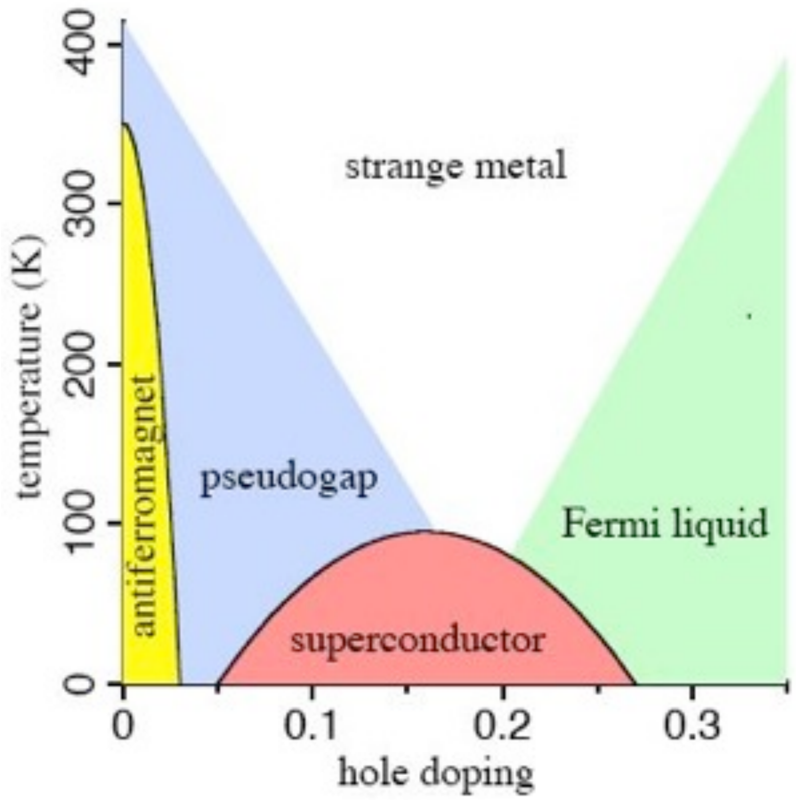
Many things to be further explored....

Final Comments

Consistent KK truncations are powerful tools to construct solutions of D=10/11 SUGRA with many applications including Top Down solutions for AdS/CMT.

- Superconductors: additional black holes, magnetic fields, fermions...
- Lifshitz: black holes, superconductors,.... Also, need to develop holographic dictionary.
- For SE cases, do we have the most general truncations?
- For SLAG 3-case, does the truncation also work for the more general class of N=2 susy $AdS_4 \times M_7$ solutions?
- Why do consistent truncations work?

Phase diagram: of High Tc cuprates e.g. $La_{2-x}Sr_xCuO_4$



Quantum critical point under the superconducting dome?