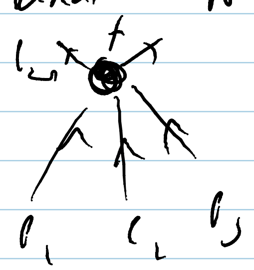
Dlogs-dilogs

31 May 2011
09:16

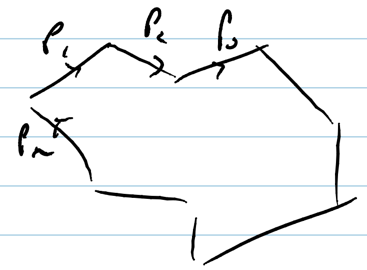
arxiv: 1212:6228
1307:1447

Credits: A Lipstein

Planar $N=4$ SYM



$\longleftrightarrow$



Amplitude $\subset$ dilog l-loop

Seen most concretely in MHV diagrams as planar duality diagram by diagram.
+ summ

These diagrams are Feynman diagrams from twistor action.

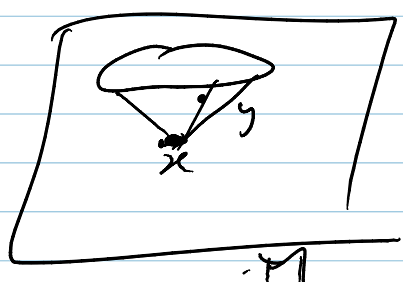
Agenda: 1. Explain how MHV diagrams arise from twistor action

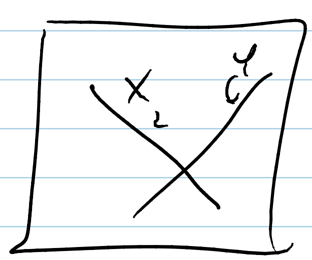
2. Show how Feynman diagrams immediately give loop integrand in dlog form
3. Sketch how to integrate such integrands directly without Feynman parameters --> dilogs etc.

Focus on Wilson-loop in the following:

Twistor space: $Z^\alpha = (\lambda_\alpha, \mu^{\dot\alpha}, \chi^a) \in \mathbb{CP}^{3|4}$

incidence: $\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha$, $\chi^a = \theta^{a\alpha}\lambda_\alpha$ $\quad (x,\theta) \in M^{4|8}$

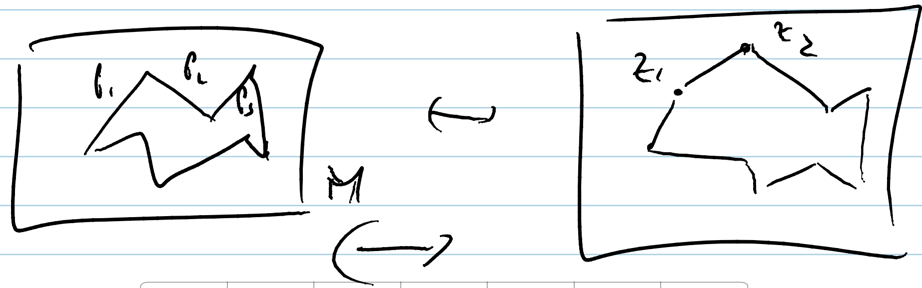




$x \iff \mathbb{CP}^1_x = X \subset \mathbb{PT}$

$$\text{"}x \leftrightarrow \mathbb{CP}^1 = X \subset \mathbb{PT}\text{"}$$
$$\lambda_\alpha$$

For null polygon $\leftrightarrow$ lines



Points						Lines

SYM on $\mathbb{PT}$: $\bar\partial$-operator on bundle

Data: $\bar\partial_A = d\bar Z^I \frac{\partial}{\partial \bar Z^I} + A$, $A \in \Omega^{0,1} \otimes \mathfrak{g}$

$$S = \int \left(A\bar\partial A + \tfrac{2}{3}A^3\right)\wedge\Omega + \int_M d^{4|8}x \int_X \log\det\bar\partial_A$$

Axial gauge: Choose reference twistor Z_*

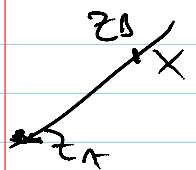
s.t. $Z_*\cdot\partial \,\lrcorner\, A = 0 \quad \rightsquigarrow A^3 = 0$

$\rightsquigarrow$ Propagator $A(Z,Z') = \bar\delta^{2|4}_{2\pi i}(Z,Z') := \frac{1}{2\pi i}\int \frac{ds}{s}\frac{dt}{t}\,\bar\delta^{4|4}(Z+sZ_*+tZ')$

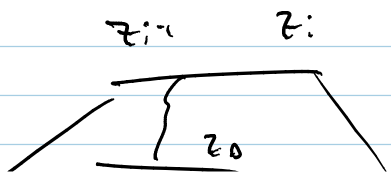
$$\left(\begin{array}{l} \bar\delta(z) = \bar\partial \frac{1}{z} \\ \delta(x^0) = x^0 \end{array}\right)$$

Vertices come from expanding log det $\rightsquigarrow$

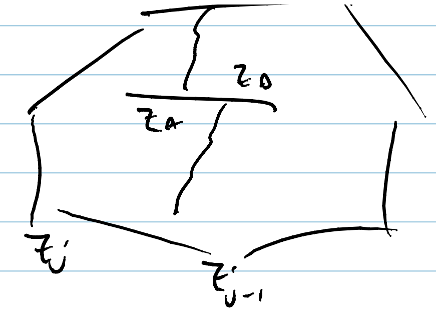
$$V(1,\dots,n) = \int \underbrace{\frac{d^{4|4}Z_A\, d^{4|4}Z_B}{\mathrm{Vol}\,GL_2}}_{d^{4|8}x} \prod_{i=1}^n \frac{d^2\sigma\;\bar\delta^{4|4}(Z_i - \sigma_{i0}Z_A - \sigma_{i1}Z_B)}{(\sigma_i\,\sigma_{i+1})}$$



Wilson loop on M $\leftrightarrow$ Hol Wilson loop in $\mathbb{PT}$

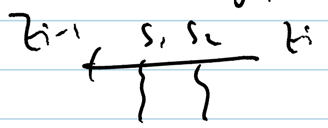


to form Feynman-diagrams draw propagators



To form [illegible] diagrams
draw propagators
between vertices = two
and edges or other
lines.

For edges:



$$\int \frac{ds_1}{s_1}\frac{ds_2}{s_2 - s_1}$$ and other propagators

and $Z_1 = Z_{i-1} + s_1 Z_i$
$Z_2 = Z_{i-1} + s_2 Z_i$
etc.

Amplitude :		Wilson Loop
Loop order	=	# Vertices
MHV-degree	=	# Props − 2 # Vertices

At MHV can identify 2 props per vertex

$<$ perform $d^{4|4}Z_A$ integrals against δ-fns

Ex: 1-Loop for $GL(2|n)$ $\quad \sigma_1 = \binom{1}{0}$, $\sigma_2 = \binom{0}{1}$

$$V_{ij} = \int d^{4|4}Z_A\, d^{4|4}Z_B \int \frac{ds}{s}\frac{dt}{t}\, \Delta(Z_A, Z_{i-1} + sZ_i)\, \Delta(Z_B, Z_{j-1} + tZ_j)$$

$$= \frac{-1}{4\pi^4}\int \frac{ds}{s}\frac{dt}{t}\frac{ds_0}{s_0}\frac{dt_0}{t_0}$$

δ-fns $\Rightarrow$ $Z_A = i s_0 Z_* + \frac{1}{1+s}(Z_{i-1} + sZ_i)$

$Z_B = i t_0 Z_* + \frac{1}{1+t}(Z_{j-1} + tZ_j)$

Normalize all Z_i s.t. $Z_i \cdot \bar{Z}_* = 1$
into $a_{ij} = Z_i \cdot \bar{Z}_j$

$\times$ Lorentz real $\Rightarrow$ $s_0, t_0 \in \mathbb{R}$ $\}$

[illegible]

$$s = \frac{\bar{t}(a_{ij-1} - v) + t(a_{ij} - v)}{\bar{t}(a_{i-1j-1} - v) \quad t(a_{i-1j} - v)}$$

$v = s_0 - t_0 \in \mathbb{R}$.

This is of course an integral over x_0

$$s_0 = \frac{-x_{0i}^2}{2x_{0i} \cdot n}, \quad t_0 = \frac{-x_{0j}^2}{2x_{0j} \cdot n}, \quad s = \frac{\langle i-1 | x_{0i} | \eta]}{\langle i | x_{0i} | \eta]}$$

η = reference spinor $\quad t = \frac{\langle j-1 | x_{0j} | \eta]}{\langle j | x_{0j} | \eta]}$

$n = \eta\tilde{\eta}$

[Note: $s = \frac{s_{i-1}}{s_i} \quad t = \frac{t_{j-1}}{t_j}$ ..

Notes: • Everything is DCI

• $s = s_{i-1}/s_i \quad t = t_{i-1}/t_i \rightsquigarrow$ decomposition into Grassmann boxes

• Same as result for BCFW of 1-loop MHV. In BCFW approach parameters are shifts.

• Typical even for non-planar & less susy

• Higher MHV - degree ~ d log & s-dep.

Performing integral:

Real poles $\frac{ds_0}{s_0} \frac{dt_0}{t_0}$ need $i\epsilon$ prescription

in fact $s_0 = 0 \Longleftrightarrow x_{0i}^2 = 0$

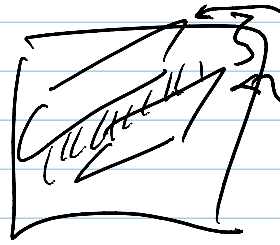
$\Rightarrow$ need Feynman $i\epsilon$ prescription.

$s_0 \to s_0 + i\epsilon f_i \quad , \quad f_i = \frac{1}{x_{0i} \cdot n}$

$t_0 \to t_0 + i\epsilon f_j$

$\rightsquigarrow$ Can do one of those integrals a result depends only on V. Using residues get

$$\sim \quad \Theta(-f_i d_j) \quad \mathrm{Sgn}\, f:$$

$\Rightarrow$ 

$x_{0:\cdots n} = 0$

$x_{0:\cdots n} = 0$

don't see ∞ !

$$\sim \int_{V_*}^{\infty} \frac{dV}{V} \underbrace{\int_\Gamma \frac{ds}{s}\frac{dt}{t}}, \quad V_* = \frac{x_{ij}^2}{x_{i,j+1}}$$

Lemma: $$\int_\Gamma \frac{ds}{s}\frac{dt}{t} = 4\pi i \log \frac{|b|}{|a|}$$

$\Gamma : s = c\left|\frac{t-a}{t-b}\right|$ Pf: use Stokes on $\int d(\ln s \, d\ln t)$

$$\sim \int_{V_*}^{\infty} \log \frac{\left(1 - \frac{a_{i,j-1}}{v}\right)\left(1 - \frac{a_{i-1,j}}{v}\right)}{\left(1 - \frac{a_{ij}}{v}\right)\left(1 - \frac{a_{i-1,j-1}}{v}\right)} \frac{dv}{v}$$

$$K_{ij} = \mathrm{Li}_2\left(\frac{a_{i-1,j}}{v_*}\right) + \mathrm{Li}_2\left(\frac{a_{i,j-1}}{v_*}\right) - \mathrm{Li}_2 \frac{a_{ij}}{v_*} - \mathrm{Li}_2 \frac{a_{i-1,j-1}}{v_*} + \text{c.c.}$$

- Finite < DCI for $|i-j| \geq 2$
- Check symbol = standard + telescopically cleaning
- Check = BST + telescopic ! (but non-trivial).
- Divergent for $|i-j| = 1$

Measures! Set $i\epsilon \to i\epsilon - M^2$

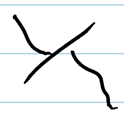
hard working

$$K_{i\,i+1} = -\frac{1}{4}\left|\ln\frac{\mu^2}{x_{i\,i+2}} - \log\frac{x_{i-1\,i+1}}{x_{i\,i+2}}\right|^2 + \frac{\pi^2}{4} + O(\epsilon)$$

- correct divergent behaviour
- Correct terms required to make

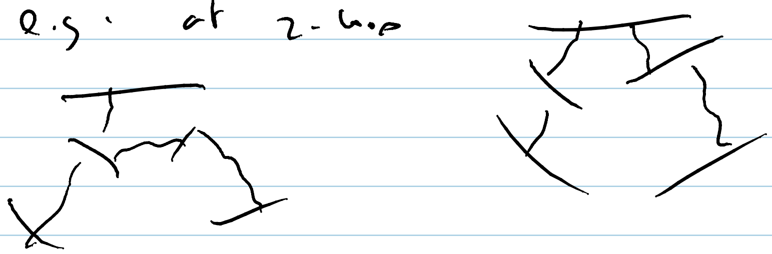
$$A = \frac{1}{2}\sum_{i,j} K_{ij} \qquad (\text{note } K_{ii} = 0)$$

Higher loops: Technique is directly to evaluate



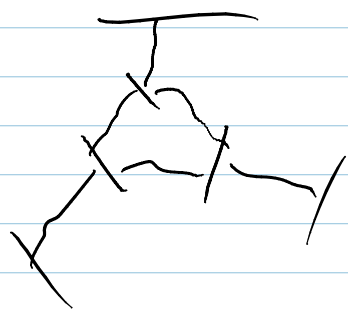
as subdiagram

e.g. at 2-loop



as at 2-loop integrals above concatenation.

3 loop



? Might need something new?

- Main ideas do not rely on planarity
- Dont even require much SUSY.
- Could use dim-reg.
- Can do general correlators More easily than amplitudes.