Data Assimilation and Modelling the Carbon Cycle

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Outline

- The Data Assimilation-Linked ECosystem model (DALEC)
- A dynamical systems approach
- Sensitivity analysis
- Data assimilation
- Model and data resolution matrices

Constraining DALEC v2 using multiple data streams and ecological constraints: analysis and application – Delahaies, Nichols and R, Geophys. Model Dev. 2017

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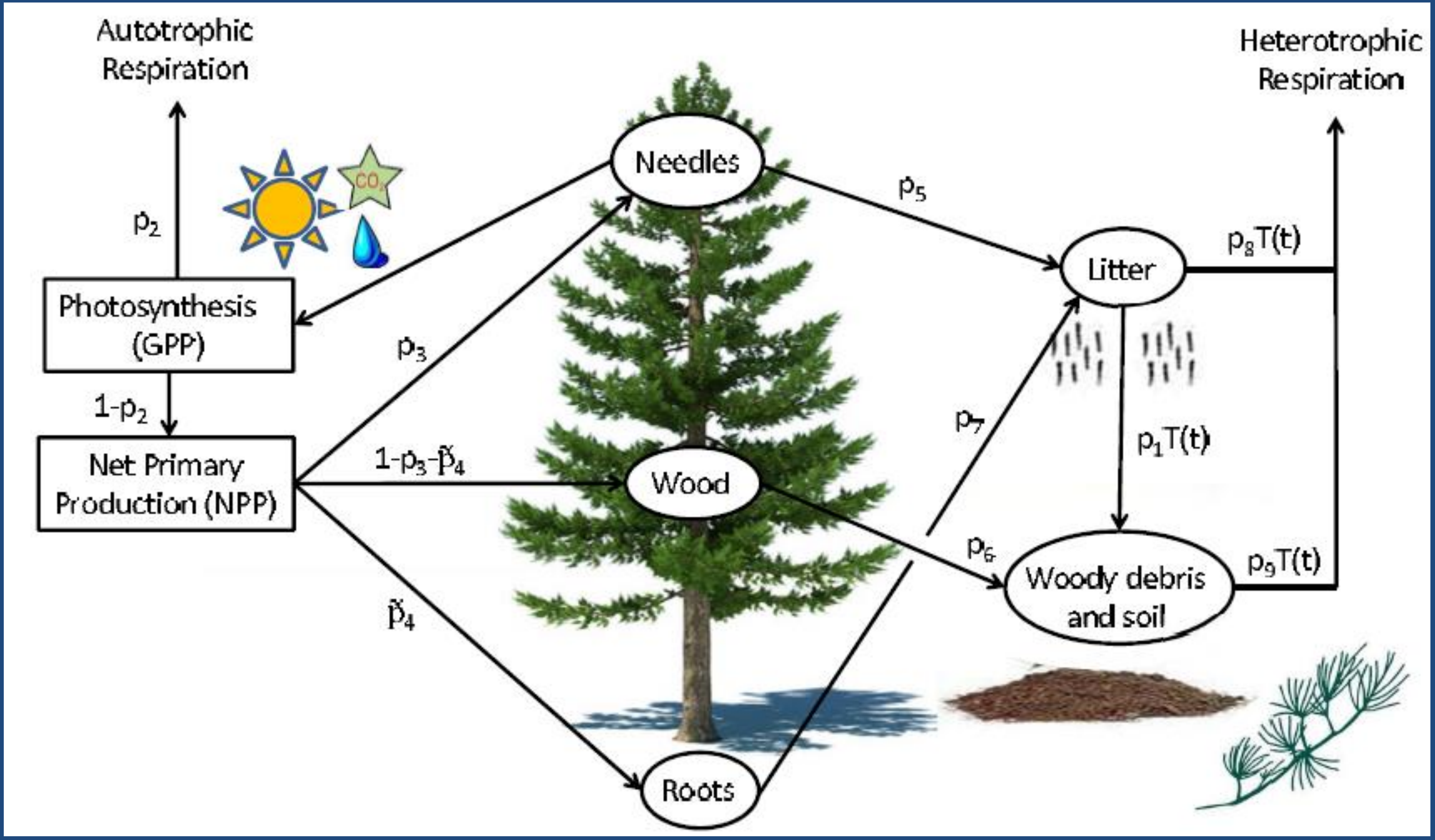
Carbon Cycle



DALEC EV

DALEC Evergreen Model

- The Gross Primary Production (GPP) function $(G(C_\mathrm{f}(t), t))$ represents a daily accumulation of photosynthate which is based on the Aggregated Canopy Model (ACM) (Williams *et al.*, 1997)
- GPP is a complicated function that depends on C_f, a variety of parameters, and on the daily drivers:
  - maximum temperature
  - minimum temperature
  - irradiance
- DALEC is a ‘simple’ model, but is essentially at the heart of all the more complex models
- Our aim is to understand the dynamical behaviour of this model

Pools and Parameters

- 5 carbon pools $(C_f, C_r, C_w, C_l, C_s)$
- 11 parameters $(p_1, \ldots, p_{11})$
- 3 meteorological drivers $(Temp, rad, CO2)$

Parameters used in DALEC EV

	Description	**Value**
p_1	Daily decomposition rate	0.0000044100
p_2	Fraction of GPP respired	0.52
p_3	Fraction of NPP allocated to foliage	0.29
$\tilde{p}_4$	Fraction of NPP allocated to roots	0.2911
p_5	Daily turnover rate of foliage	0.0028
p_6	Daily turnover rate of wood	0.00000206
p_7	Daily turnover rate of roots	0.003
p_8	Daily mineralisation rate of litter	0.02
p_9	Daily mineralisation rate of soil and organic matter	0.00000265
p_{10}	Parameter in exponential term of temperature dependent parameter	0.0693
p_{11}	Nitrogen use efficiency parameter in ACM	7.4

Canopy Model and Leaf Area Index

The aggregated canopy model (ACM) predicts the gross primary production (GPP) at a daily time step

$$GPP = \mathrm{ACM}(Lai, p_{11}, Temp, rad, CO2),$$

where

$$Lai = C_f / LMA.$$

ACM is based on the more complex soil-plant-atmosphere (SPA) model calibrated across a wide range of driving variables to produce a simple model that maintains the essential behaviour of the fine scale model.

Towards Differential Equations

The daily map for the carbon pools are given by

$$\begin{aligned}
C_f(n+1) &= (1-p_5)C_f(n) + p_3(1-p_2)GPP(C_f(n), p_{11}, n) \\
C_r(n+1) &= (1-p_7)C_r(n) + p_4(1-p_3)(1-p_2)GPP(C_f(n), p_{11}, n) \\
C_w(n+1) &= (1-p_6)C_w(n) + (1-p_4)(1-p_3)(1-p_2)GPP(C_f(n), p_{11}, n) \\
C_l(n+1) &= (1-(p_8+p_1)T(p_{10}, n))C_l(n) + p_5 C_f(n) + p_7 C_r(n) \\
C_s(n+1) &= [1-p_9 T(p_{10}, n)]C_s(n) + p_6 C_w(n) + p_1 T(p_{10}, n)C_l(n)
\end{aligned}$$

The net ecosystem exchange (NEE) defined as the difference between Gross primary production and respiration

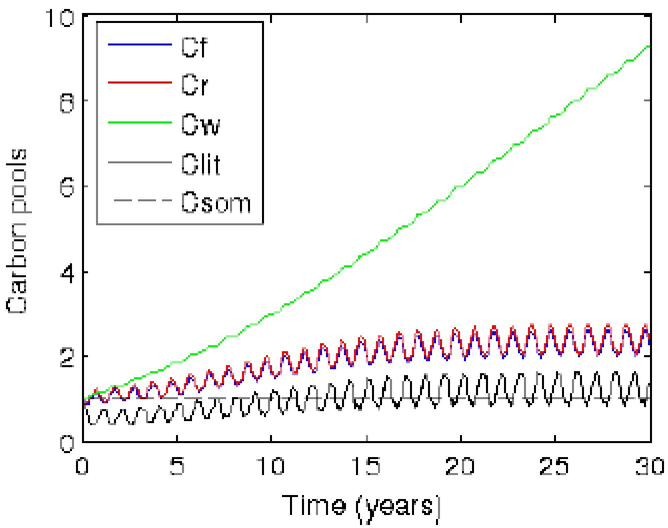
$$NEE = Ra + Rh1 + Rh2 - GPP,$$

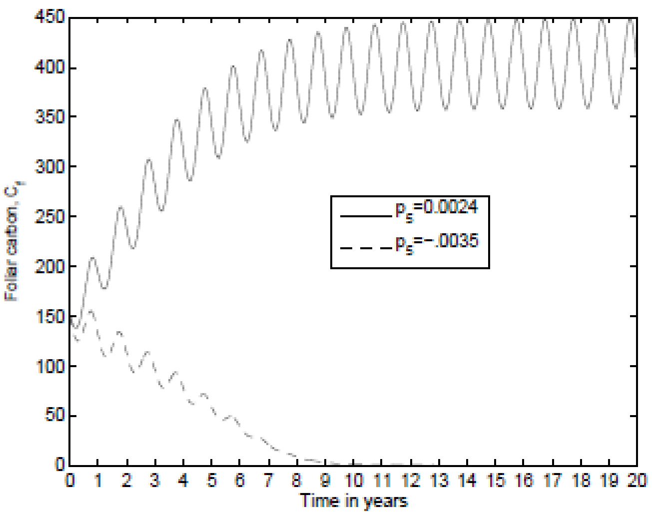
can be expressed as

$$NEE(n) = (1-p_2)GPP - p_8 T(p_{10}, n)C_l(n) - p_9 T(p_{10}, n)C_s(n).$$

DALEC Dynamics

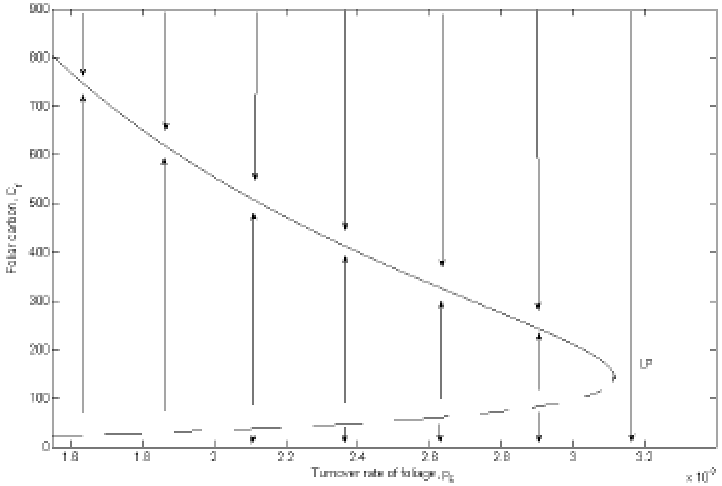
- If the drivers are made periodic with period 1 year, then the carbon pools evolve to a periodic solution
- Note that the pools evolve on different timescales
- The qualitative behaviour is parameter dependent





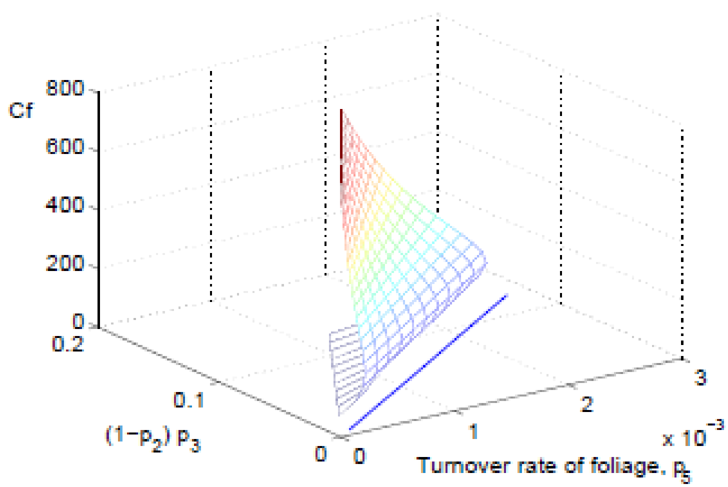
Periodic Solutions

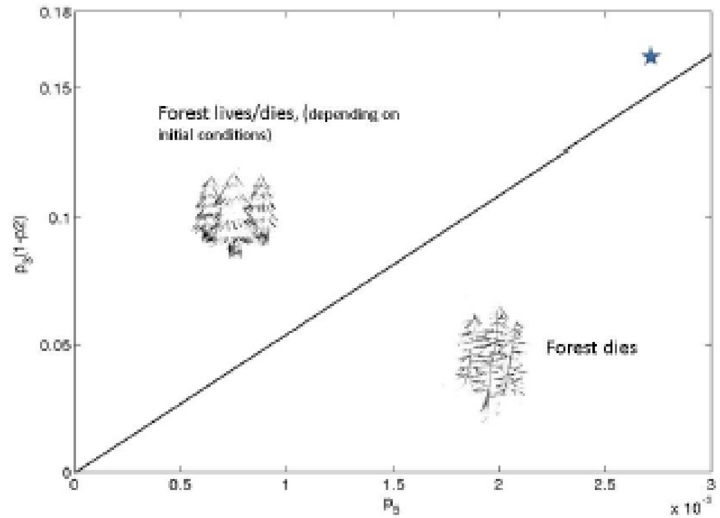
- We find periodic solutions as fixed points of an annual map, which satisfy $C_{\mathrm{f}}(0) = C_{\mathrm{f}}(365)$
- $C_{\mathrm{f}} = 0$ is always a stable fixed point
- If $C_{\mathrm{f}} = 0$, then all the other pools converge to zero as well – one dead forest!
- Non-zero fixed points of C_{f} can be found numerically
- p_5 is the rate at which foliar carbon goes into the litter
- If the needles drop at a high enough rate, then there is not sufficient carbon to sustain the tree and it will die



Limit Points

- The C_f equation depends only on p_5 (the rate at which foliar carbon goes into the litter) and $p_2(1-p_3)$ (the fraction of GPP allocated to the foliar carbon)
- We can find fixed points of C_f as a function of these two parameters
- This gives a line of limit points





- Maximum growth would be achieved by allocating as much carbon as possible to the wood and roots, keeping the foliar carbon to a minimum

Sensitivity

Parameter	Value	$\frac{\delta C_f}{\delta p_i} \frac{p_i}{100}$	$\frac{\delta NEE}{\delta p_i} \frac{p_i}{100}$
p_1	0.0000044	0	-0.00065
p_2	0.52	-1.8	8.2
p_3	0.29	1.7	-2.5
p_4	0.41	0	0.48
p_5	0.0028	-1.6	3.4
p_6	0.0000021	0	0.000038
p_7	0.0030	0	0.95
p_8	0.020	0	0.43
p_9	0.0000027	0	0.11
p_{10}	0.069	0	0.34
p_{11}	7.4	0.83	-3.8

Sensitivity

- From LAI measurements one can only estimate
  - (i) the turnover rate of foliar carbon (p_5),
  - (ii) the fraction of GPP allocated to foliar carbon ($p_3(1-p_2)$).
  - (iii) p_{11} (parameter in GPP).
- NEE is only sensitive to some of the parameters and hardly at all to the decomposition rate of litterfall, p_1, and turnover rate of wood, p_6.

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A dynamical systems analysis of the data assimilation linked ecosystem carbon models

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REgional Flux Estimation EXperiment (REFLEX)

A. Fox et al. (2009) Agricultural and Forest Meteorology 149.

Aims

- To compare the strengths and weaknesses of various data assimilation techniques for estimating carbon model parameters and predicting carbon fluxes.
- To quantify errors and biases introduced when extrapolating fluxes.

Experiments

- Nine participants using Monte Carlo methods and EnKF,
- Assimilation of both real and synthetic NEE and LAI observations over a two year period.

Results

- parameters directly linked to GPP and respiration were best constrained and characterised,
- parameters related to the allocation to and turnover of fine root/wood pools.

4D VAR

Variational Data Assimilation

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$$J(x)=\frac{1}{2}(x_0-x_b)^T B^{-1}(x_0-x_b)+\frac{1}{2}\sum_{i=1}^{N}(y_i-H(x_i))^T R_n^{-1}(y_i-H(x_i))$$

Find

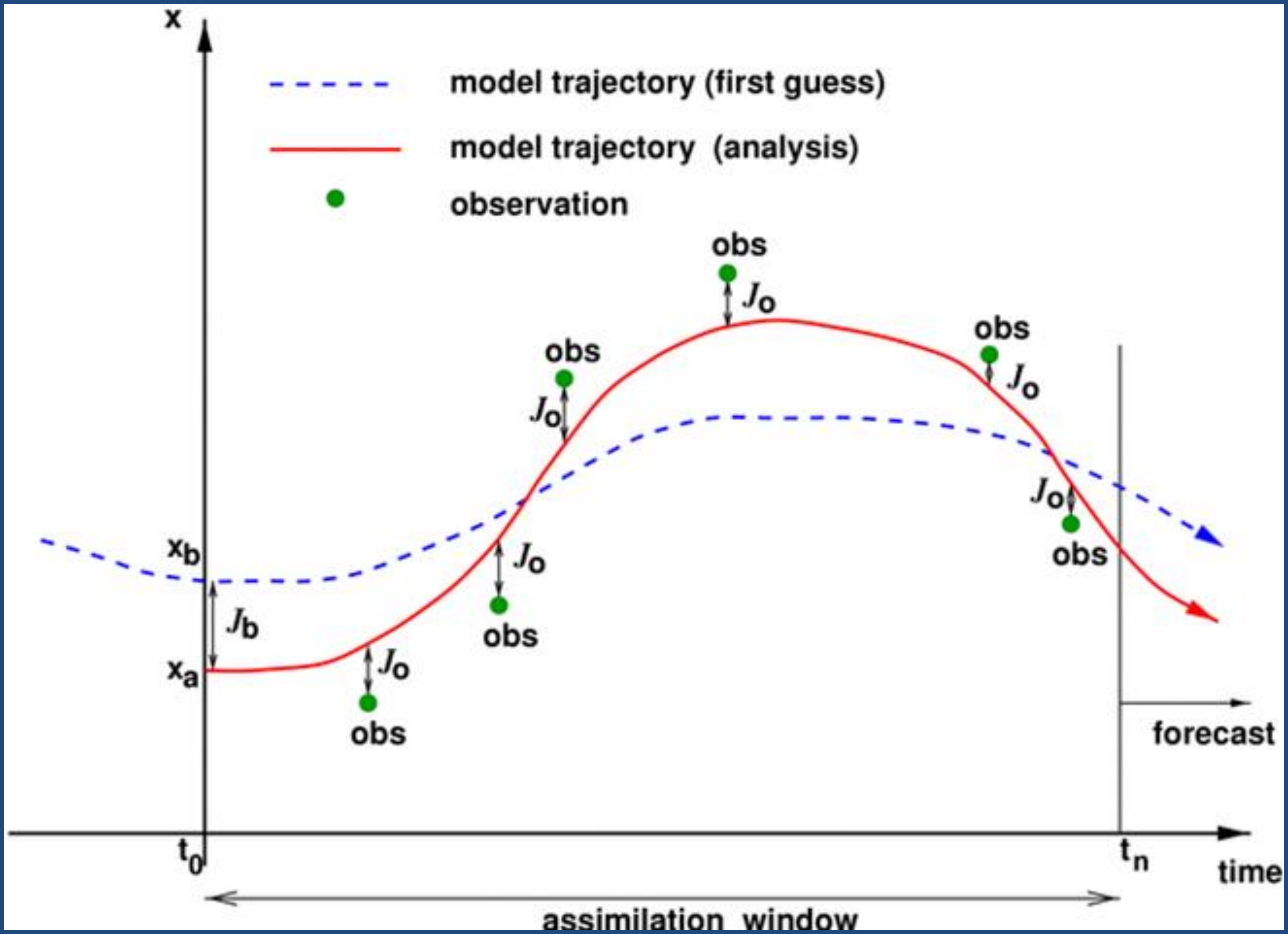
$$\min_{x_0} J(x)$$

Subject to the strong constraint that the model states are a solution to the numerical model and that the tangent linear hypothesis holds.

Adjoint variable λ:

$$\frac{\partial J}{\partial x_0} \Leftarrow -\lambda_0$$

4D VAR



x
model trajectory (first guess)
model trajectory (analysis)
observation
obs
J_o
obs
J_o
obs
J_o
J_o
x_b
J_o
J_b
obs
J_o
obs
x_a
obs
forecast
t_0
t_n
time
assimilation window

Inverse Problem and Data Assimilation

At the heart of incremental-4DVAR lies a linear inverse problem $Ax = b$, the least square solution is given by

$$x^* = \mathrm{argmin}\|Ax - b\|^2 = \sum_{i=1}^{n} \frac{u_i^T b}{\sigma_i} v_i,$$

using a singular value decomposition $A = U\Sigma V^T$ with $\Sigma = \mathrm{diag}(\sigma_1, \dots, \sigma_n)$.

When the data b is contaminated with noise the least square solution can be unreliable. Given $\bar{x}$ and $\bar{b}$ such that $A\bar{x} = \bar{b}$ we have

$$\frac{\|\delta x\|}{\|\bar{x}\|} \leq \kappa(A) \frac{\|\delta b\|}{\|\bar{b}\|},$$

where

- $\delta x = x^* - \bar{x}$ and $\delta b = b - \bar{b}$,
- $\kappa(A) = \sigma_1 / \sigma_n$ is the condition number of A.

Well-posedness

A generic inverse problem consists in finding a n-dimensional state vector ***x*** such that

$$\mathbf{h}(\boldsymbol{x}) = \boldsymbol{y}$$

for a given N-dimensional observation vector ***y***, including random noise, and a given model **h**. The problem is well posed in the sense of Hadamard (1923) if the three following conditions hold:

1) a solution exists,
2) the solution is unique, and
3) the solution depends continuously on the input data.

An ill-posed problem

	$\mathbf{x}_0$	$\delta\mathbf{x}_0$	RE_0	RE_1	RE_2	RE_3
C_f	298.4	25.0	1.1×10^{-12}	2.3×10^{-9}	1.2×10^{-4}	0.3
C_r	280.1	19.0	3.5×10^{-7}	5.7×10^{-5}	6.3×10^{1}	3.9×10^{4}
C_w	10410.6	920.0	1.8×10^{-6}	7.1×10^{-4}	1.11×10^{3}	3.1×10^{5}
C_l	38.9	16.0	2.8×10^{-8}	4.5×10^{-6}	5.0	3.1×10^{3}
C_s	10631.0	1100.0	8.7×10^{-7}	1.1×10^{-5}	1.0×10^{-5}	1.7×10^{4}
p_1	4.41×10^{-6}	4.0×10^{-7}	5.9×10^{-4}	9.4×10^{-2}	1.0×10^{5}	6.4×10^{7}
p_2	0.51	5.2×10^{-2}	1.3×10^{-10}	1.5×10^{-8}	3.2×10^{-3}	2.7
p_3	0.29	2.9×10^{-2}	8.4×10^{-11}	1.0×10^{-8}	3.5×10^{-3}	1.7
p_4	0.41	4.1×10^{-2}	2.3×10^{-7}	3.7×10^{-5}	4.1×10^{-5}	2.5×10^{4}
p_5	2.8×10^{-3}	2.8×10^{-4}	2.8×10^{-12}	4.6×10^{-9}	9.2×10^{-4}	1.0
p_6	2.06×10^{-6}	2.06×10^{-7}	5.8×10^{-6}	6.0×10^{-4}	4.3×10^{-2}	4.1×10^{5}
p_7	3.0×10^{-3}	3.0×10^{-4}	2.2×10^{-9}	1.1×10^{-6}	1.4	8.2×10^{2}
p_8	2.0×10^{-2}	2.0×10^{-3}	1.1×10^{-7}	1.8×10^{-5}	2.0×10^{-5}	1.2×10^{4}
p_9	2.65×10^{-5}	2.65×10^{-7}	9.0×10^{-6}	1.1×10^{-4}	1.0×10^{2}	1.8×10^{5}
p_{10}	6.93×10^{-2}	6.93×10^{-3}	1.9×10^{-10}	2.1×10^{-8}	2.2×10^{-3}	6.4
p_{11}	7.4	0.74	1.4×10^{-10}	2.4×10^{-8}	1.9×10^{-3}	4.9
$\|RE\|$	–	–	2.4×10^{-24}	1.8×10^{-8}	1.0×10^{2}	2.2×10^{7}

Table: The condition number is the same for each simulation $\kappa(A)\approx 1.1\times10^{23}$. $\delta\mathbf{x}_0$ is the perturbation used to generate the observations. The row RE_0 gives the relative error for observations without noise, the column RE_1 (resp. RE_2, RE_3) gives the relative error for the analysis for observations with a gaussian noise with variance $\sigma = 1.1\times10^{-16}$ (resp. $\sigma = 1.0\times10^{-5}$, $\sigma = 5.0\times10^{-1}$).

Model Reduction?

	RE
C_f	1.2×10^1
C_r	3.9×10^4
C_w	3.1×10^5
C_l	3.1×10^3
C_s	1.7×10^4
p_1	6.4×10^7
p_2	7.4×10^{-3}
p_3	1.1×10^1
p_4	2.5×10^4
p_5	3.6
p_6	2.4×10^5
p_7	8.0×10^2
p_8	1.2×10^4
p_9	1.2×10^5
p_{10}	2.3
p_{11}	11.6

	RE
C_f	6.5
C_l	0.5
C_s	7.4
p_2	1.2
p_3	1.3
p_4	0.2
p_5	3.1
p_8	1.8
p_{10}	3.5×10^{-2}
p_{11}	4.3

Linear Analysis

Considerable theoretical insights into the nature of the inverse problem, and the ill-posedness, can be obtained by studying a linearisation of the operator **h**. A first approximation to the inverse problem consists in finding a perturbation ***z*** which best satisfies the linear equation

$$\mathbf{H}\boldsymbol{z} = \boldsymbol{d}$$

where **H** is the tangent linear operator for **h** and ***d*** is a perturbation of the observations.

- Finding a solution $\boldsymbol{z}$ amounts to constructing a generalized inverse $\mathbf{H}^g$ such that formally

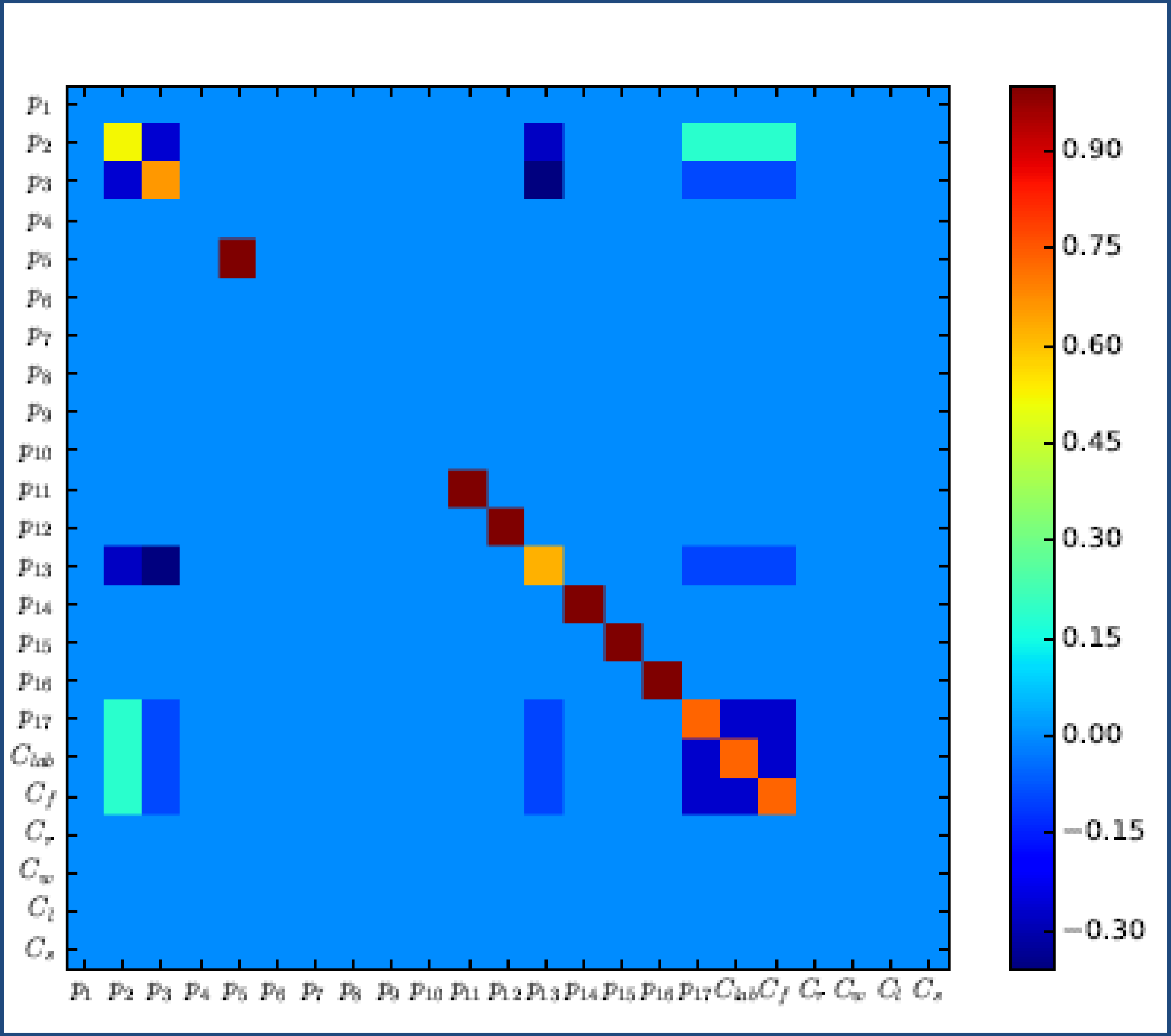
$$z = \mathbf{H}^g \boldsymbol{d}$$

- Assuming a true state $\boldsymbol{z^*}$ exists, possibly unknown, then we can define an operator **N** called the <u>model resolution matrix</u> which relates the solution $\boldsymbol{z}$ to the true state

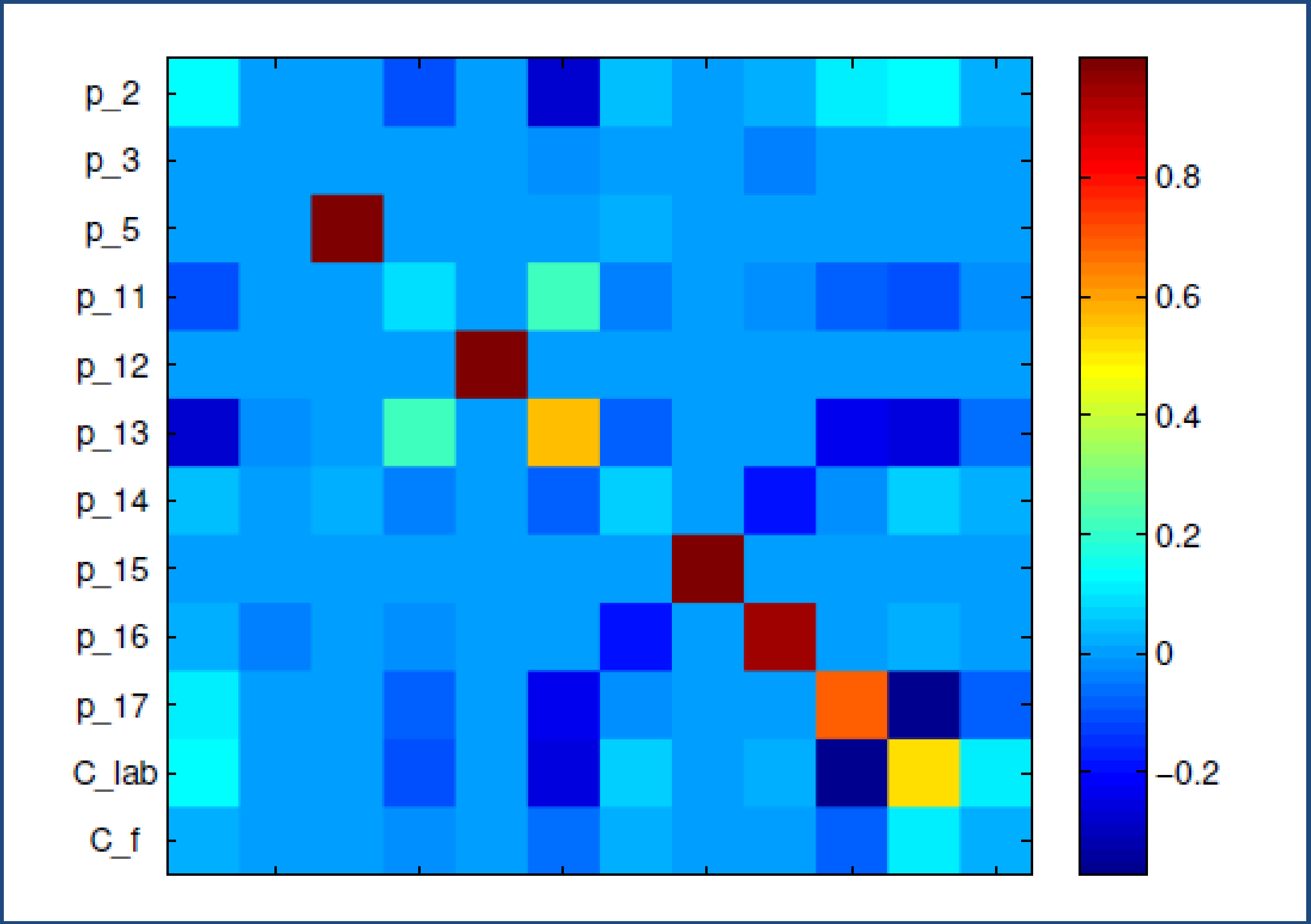
$$\boldsymbol{z} = \mathbf{H}^g \mathbf{H}\boldsymbol{z^*} = \mathbf{N}\boldsymbol{z^*}$$

- This matrix **N** gives a practical tool to analyse the resolution power of an inverse method, that is its ability to retrieve the true state, including or not any regularization method
- The closer **N** is to the identity the better the resolution.
- The trace of the matrix defines a natural notion of information content (IC).
- Similarly a <u>data resolution matrix</u> can be defined to study how well data can be reconstructed and its diagonal elements naturally define a notion of data importance.

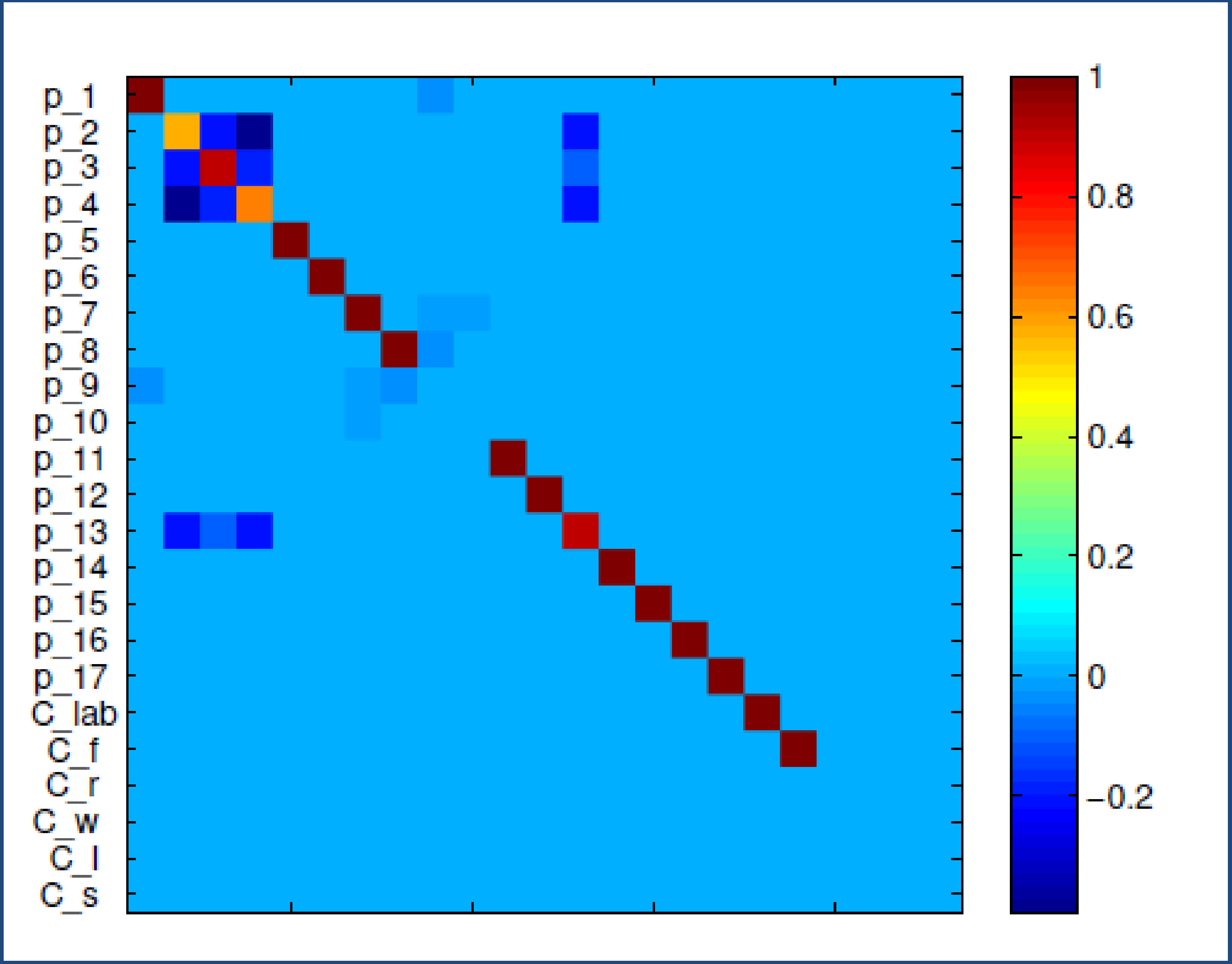
Model Resolution: LAI



Reduced Model



EDCs



p_1
p_2
p_3
p_4
p_5
p_6
p_7
p_8
p_9
p_10
p_11
p_12
p_13
p_14
p_15
p_16
p_17
C_lab
C_f
C_r
C_w
C_l
C_s
1
0.8
0.6
0.4
0.2
0
−0.2

Constraining DALEC2 using multiple data streams and ecological constraints: analysis and application

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Future Work

- Sensitivity analysis of TRIFFID/JULES (Met O model)
- Sensitivity analysis of visible radiance, near infrared radiance and vegetation index, to the model parameters of the two-stream Sellers approximation of radiative transfer (with NPL)