Breathers on Quantum Lattices

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Outline

- Introduction- Solitons and Breathers

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- The DNLS equation

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  - Non uniform lattices $N = 2$
  - Breather-Breather collisions

Solitons and Breathers in Lattices

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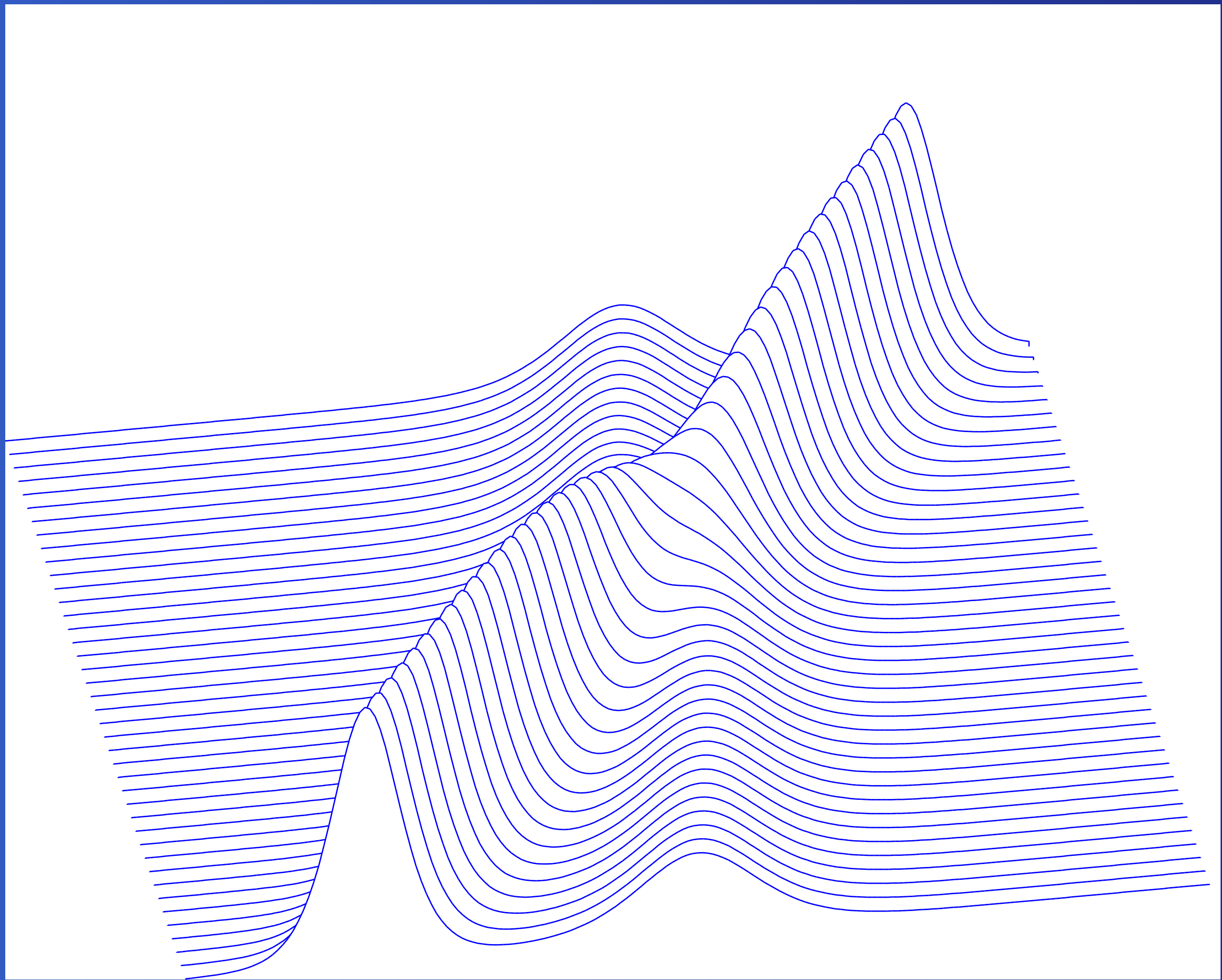
Solitons and Breathers in Lattices

- **Soliton**. Strongly localized package (lump) of energy, can move large distances with no distortion, very stable even under collisions or perturbations.
- **Breather**. A more complicated form of nonlinear wave which can often occur in discrete systems. It looks like a soliton modulated by an internal carrier wave.

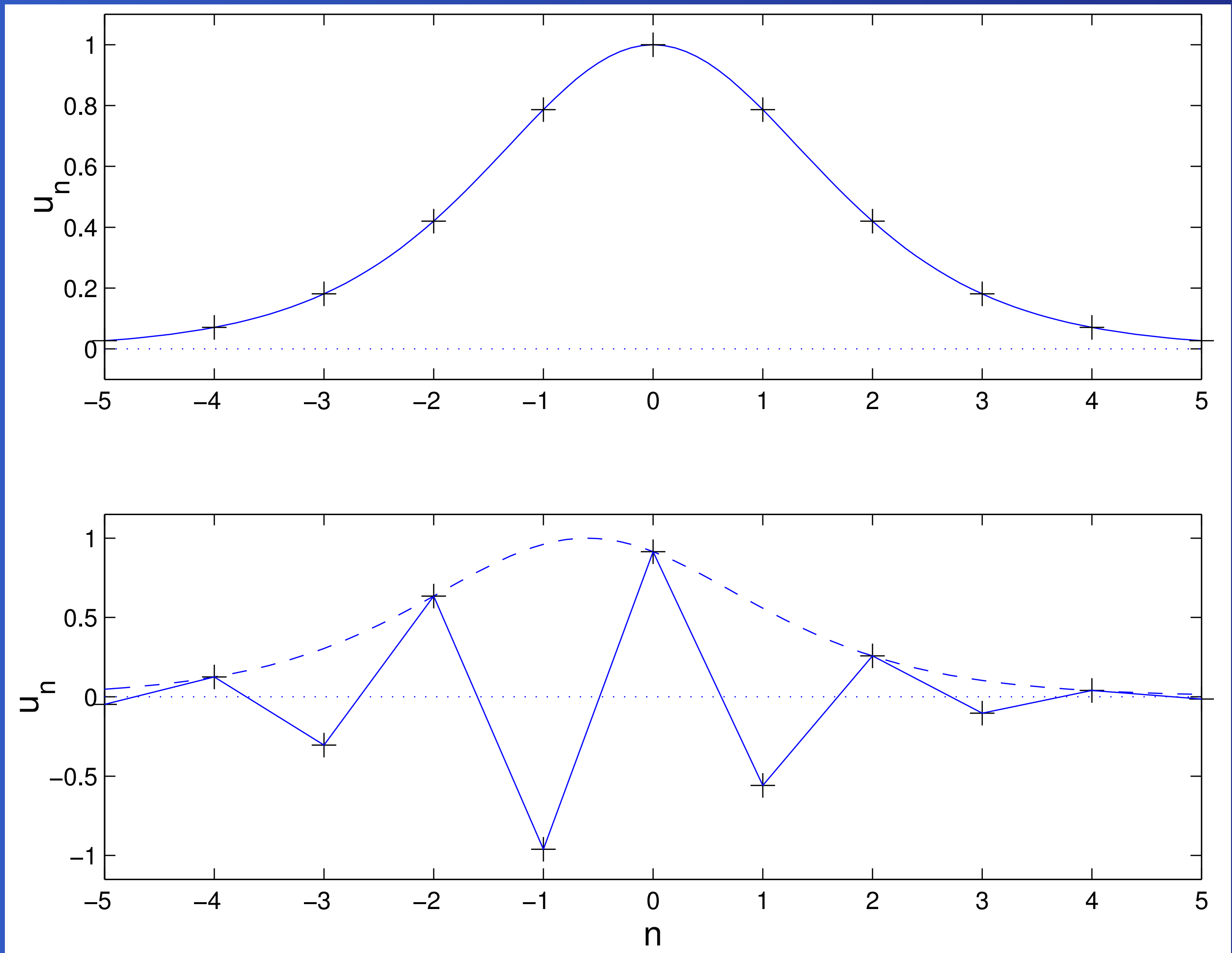
soliton collision

Start animation

soliton collision 2



Solitons and Breathers in Lattices



Breathers, classical DNLS

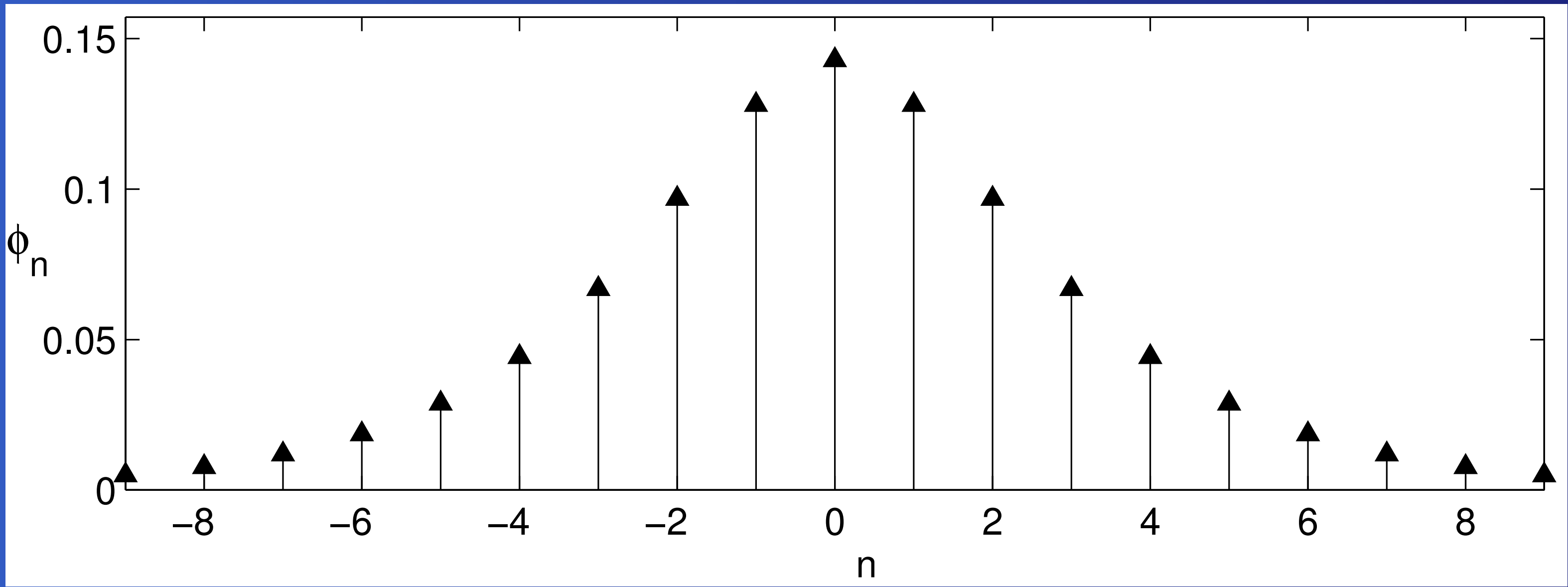
For simplicity we focus on one model, the Discrete Nonlinear Schrödinger (DNLS) equation.

$$\mathrm{i}\frac{dA_j}{dt} + (A_{j-1} - 2A_j + A_{j+1}) + \gamma |A_j|^2 A_j = 0,$$

where $A_j(t)$ is the *complex* oscillator amplitude at the jth lattice site. DNLS Hamiltonian:

$$H = \sum_{j=1}^{f} \left[\frac{\gamma}{2} |A_j|^4 - A_j^* (A_{j-1} + A_{j+1}) \right]$$

Breathers, DNLS equation



This is a stationary breather on a larger lattice. The amplitude goes to zero exponentially as $|n| \longrightarrow \infty$.

Breathers, DNLS equation

Simulations:

- Stationary breather
- Mobile breather
- Colliding breathers

Exact breather solutions?

Exact Breathers, DNLS equation?

In 1991, Henrik Feddersen (Springer Lect. Notes. in Phys., **393**, 159) made a numerical study of the DNLS equation using the ansatz

$$A_n(t) = \phi(n - ct)e^{i(kn-\omega t)}.$$

He found branches of localized solutions to high accuracy, but the *existence* of such solutions is still an open question.

Quantum breathers

Quantum DNLS (boson Hubbard) Hamiltonian in 1D, nearest neigbour interactions:

$$\hat{H} = -\frac{\gamma}{2}\sum_{j=1}^{f} b_j^\dagger b_j^\dagger b_j b_j - \epsilon \sum_j b_j^\dagger b_{j+1}$$

$\hat{H}$ conserves the *number* of quanta

$$\hat{N} = \sum_{j=1}^{f} b_j^\dagger b_j \ ,$$

Quantum wavefunctions

The operators b_j, $b_j^\dagger$ acts on *number states*
$|\psi_n >= |n_1 > |n_2 > \ldots |n_f >= [n_1, n_2, \ldots, n_f]$,
where $N = \sum n_i$.

Example: [2,2,0,0,0,1] means 2 quanta on site 1, 2 quanta on site 2, 1 quanta on site 6, on a lattice with 6 sites.

Raising/Lowering operators satisfy

$$b_j|n_j >= \sqrt{n_j}|n_j - 1 >, \quad b_j|0 >= 0,$$

$$b_j^\dagger|n_j >= \sqrt{n_j + 1}|n_j + 1 >.$$

General wave function is $|\Psi_N >= \sum_n c_n|\psi_n >$.

Conserved number of quanta

We can block-diagonalize the Hamiltonian matrix $H = \left\langle \Psi \middle| \hat{H} \middle| \Psi \right\rangle$ as

$$H = \begin{pmatrix} H_1 & 0 & & \\ 0 & H_2 & 0 & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \ddots \end{pmatrix}$$

where each H_N is the Hamiltonian for N quanta.

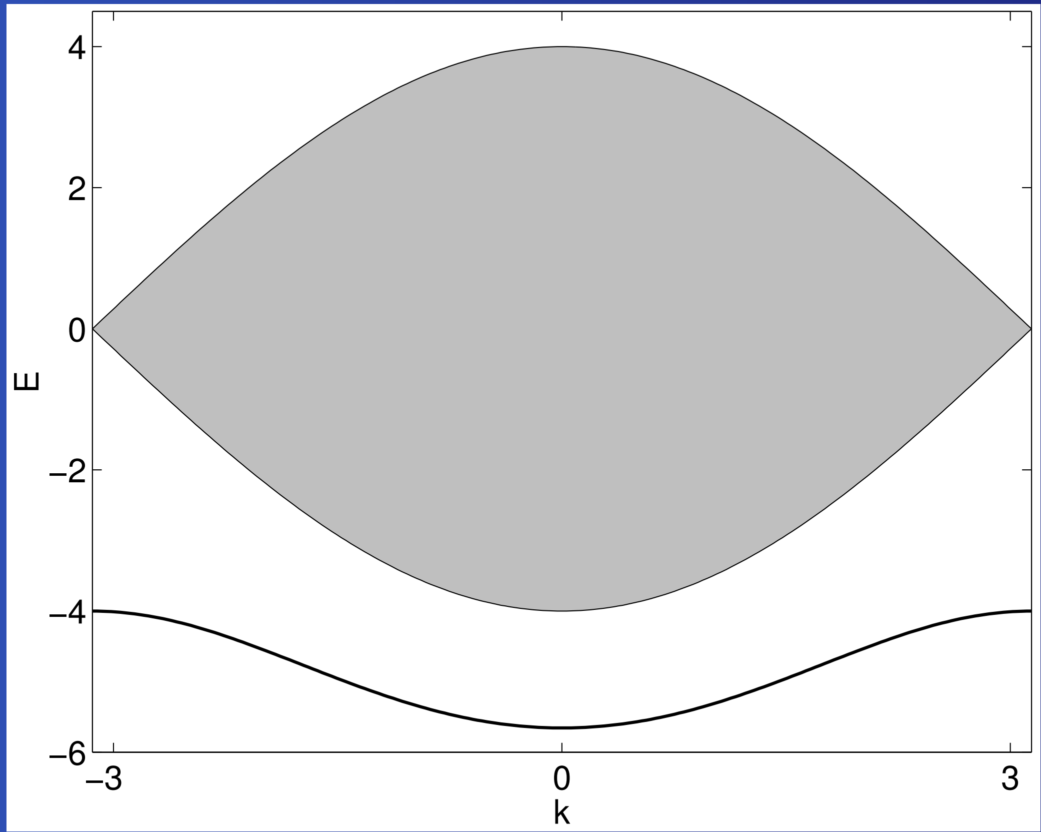
Rotational symmetry

If model is rotationally invariant (translations + periodic b.c.'s) we can further block-diagonalize H_N using eigenfunctions of the translation operator $\hat{T}$, giving states with fixed momentum k

$$H_N = \begin{pmatrix} H_{N,k_1} & 0 & & \\ 0 & H_{N,k_2} & 0 & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \ddots \end{pmatrix}$$

N=2 Case In this case each H_{2,k_p} is tridiagonal.

1D Quantum Breather – 2 quanta



Eigenvalues $E(k)$ for QDNLS. The lower band is the “breather” band.

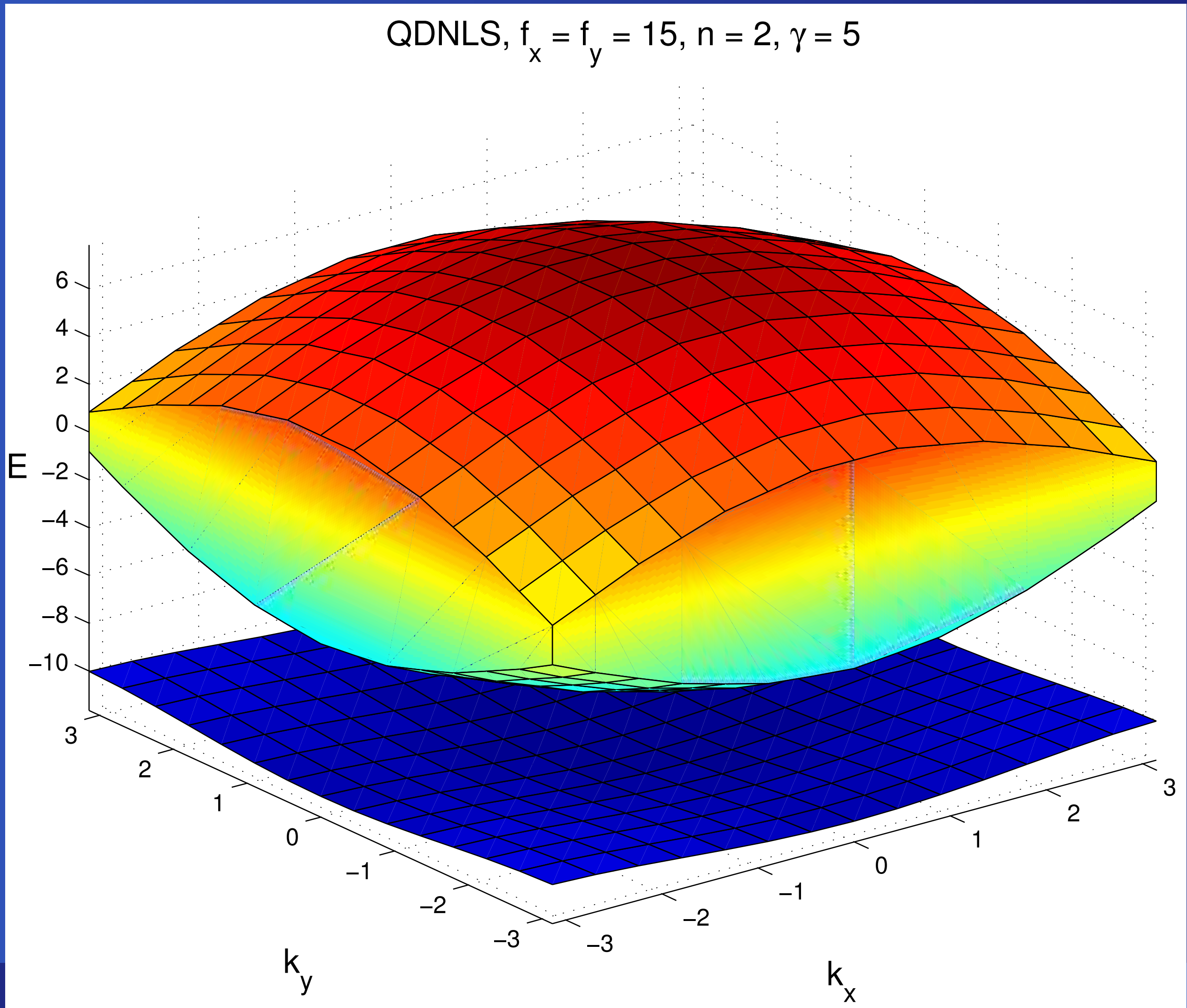
Quantum Breather? – 2 quanta

The "breather" band has wave function

$$|\Psi_n> = [2,0,0,\dots] + [0,2,0,\dots] + [0,0,2,\dots] + \\ + \dots + O(\epsilon/\gamma)\left([1,1,0,\dots] + \dots\right)$$

So for large γ the quanta are *localized* (both on the same site), but occur at *all* sites with *equal* probability! Localized breathers in the *classical* sense are not eigenstates, but decay slowly.

2D Quan. Breather Bands – 2 quanta



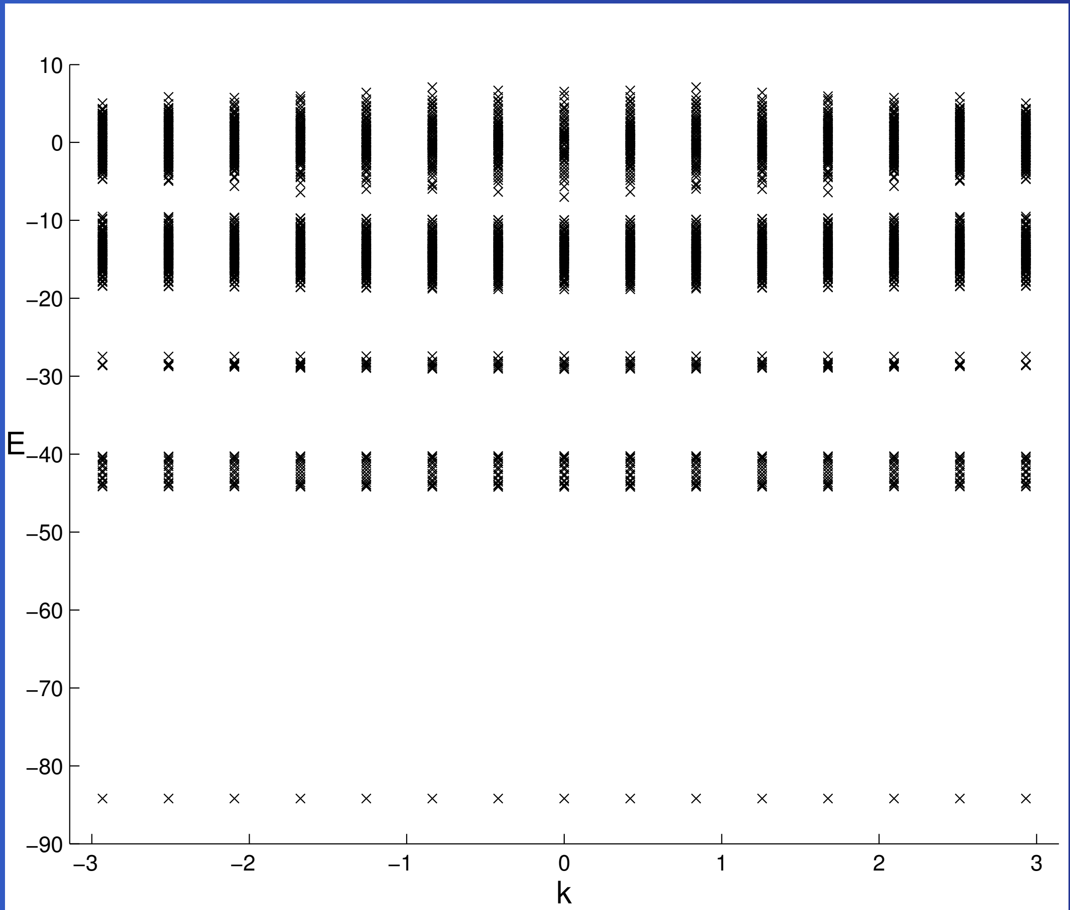
1D periodic lattice, $N > 2$

Breather-breather collisions: we use the periodic QDNLS Hamiltonian:

$$\hat{H} = -\frac{1}{2}\gamma \sum_{j=1}^{f} b_j^\dagger b_j^\dagger b_j b_j - \epsilon \sum_j \left(b_j^\dagger b_{j+1} + b_j^\dagger b_{j-1} \right)$$

But now we work with $N = 4$ quanta and get the spectrum shown on the next slide

1D periodic lattice, $N = 4$



$[1111]$

$[211]$

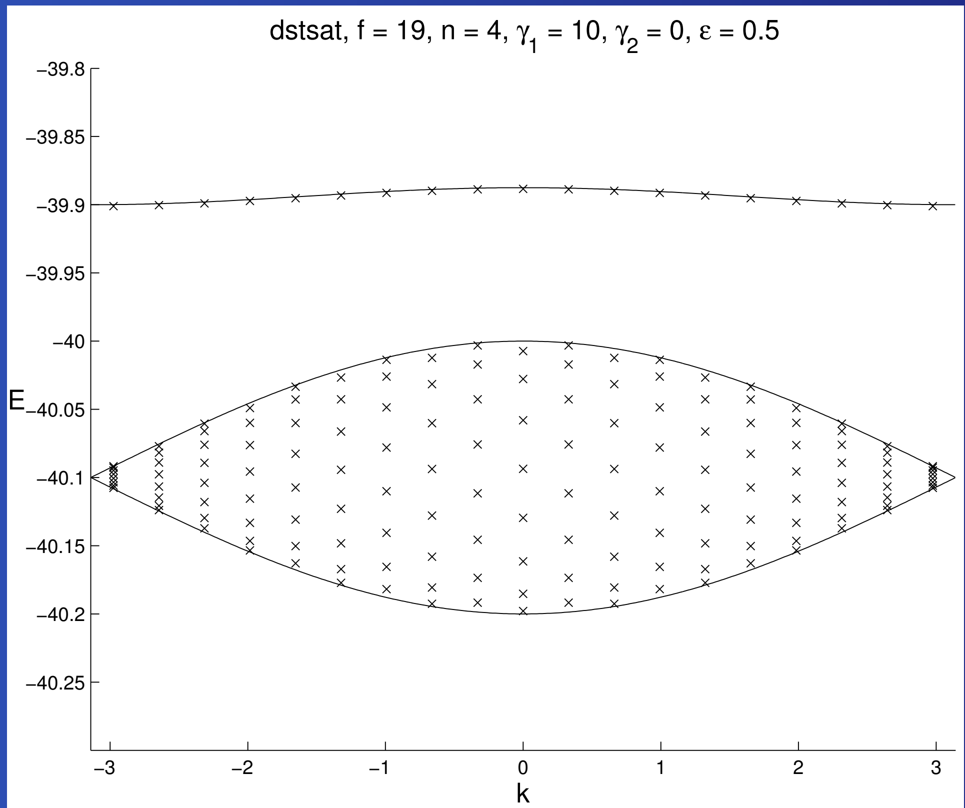
$[22]$

$[31]$

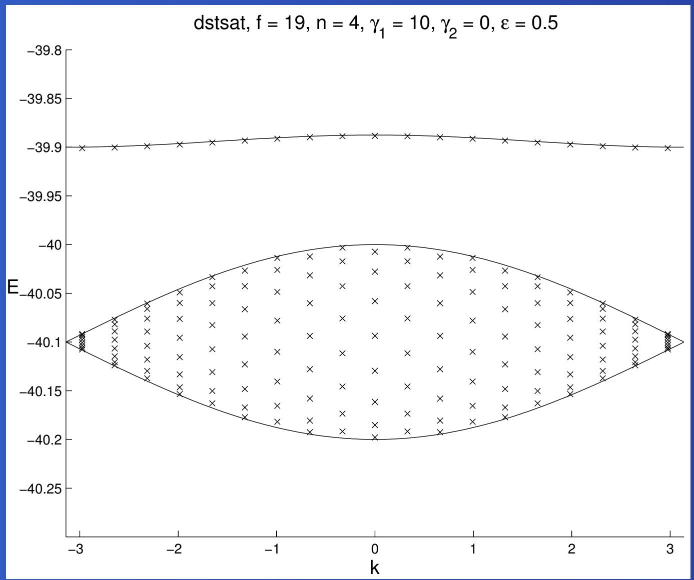
$[4]$

Breather-breather collisions

Enlarging the $[22]$ band shows interesting fine structure



Breather-breather collisions



These are the breather-breather collision bands! The top line is essentially the $[\ldots 0220 \ldots]$ band, the rest a mix of $[\ldots 02020 \ldots]$, $[\ldots 020020 \ldots]$, etc., terms. Solid lines are from 2nd order perturbation theory.

Modified QDNLS

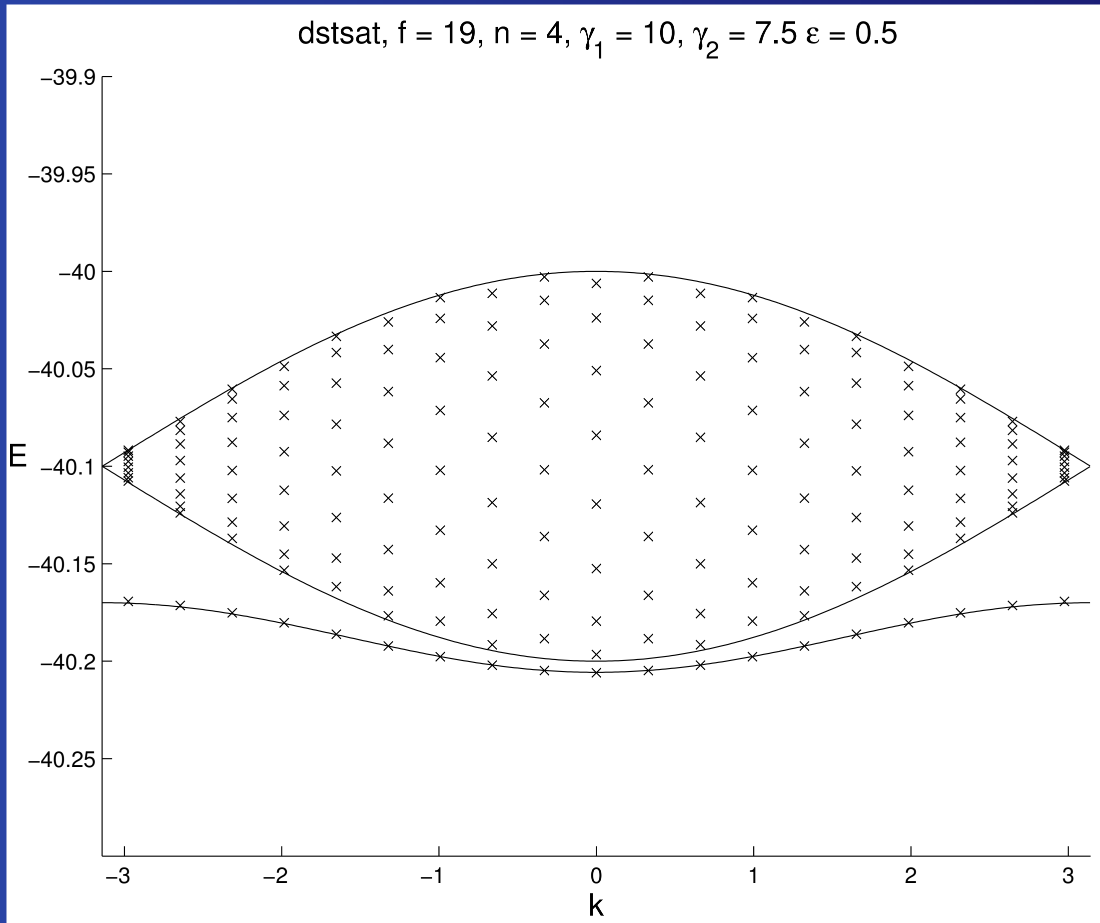
If we want the breather-breather collision bands to be the ground states of the system, we use the modified QDNLS Hamiltonian:

$$\hat{H} = -\frac{1}{2}\gamma_1 \sum_{j=1}^{f} b_j^\dagger b_j^\dagger b_j b_j + \frac{1}{2}\gamma_2 \sum_{j=1}^{f} b_j^\dagger b_j^\dagger b_j^\dagger b_j b_j b_j - \\ - \epsilon \sum_j \left(b_j^\dagger b_{j+1} + b_j^\dagger b_{j-1} \right)$$

This is equivalent to adding a saturation term in nonlinear optics.

Modified QDNLS

By appropriate choice of γ_1 and γ_2 the $[\ldots 0220 \ldots]$ band can lie below the $[\ldots 02020 \ldots]$, $[\ldots 020020 \ldots], \ldots$ continuum.



1D twisted lattice

Quantum DNLS Hamiltonian:

$$\hat{H} = -\frac{1}{2}\gamma \sum_{j=1}^{f} b_j^\dagger b_j^\dagger b_j b_j - \sum_j \left(b_j^\dagger b_{j+1} + b_j^\dagger b_{j-1} \right)$$
$$- \beta \left(b_m^\dagger b_n + b_n^\dagger b_m \right)$$

for some m, n such that $|m - n| >> 1$. (Long range interaction).