MULTILOOP SUPERSTRING AMPLITUDES USING THE PURE SPINOR FORMALISM

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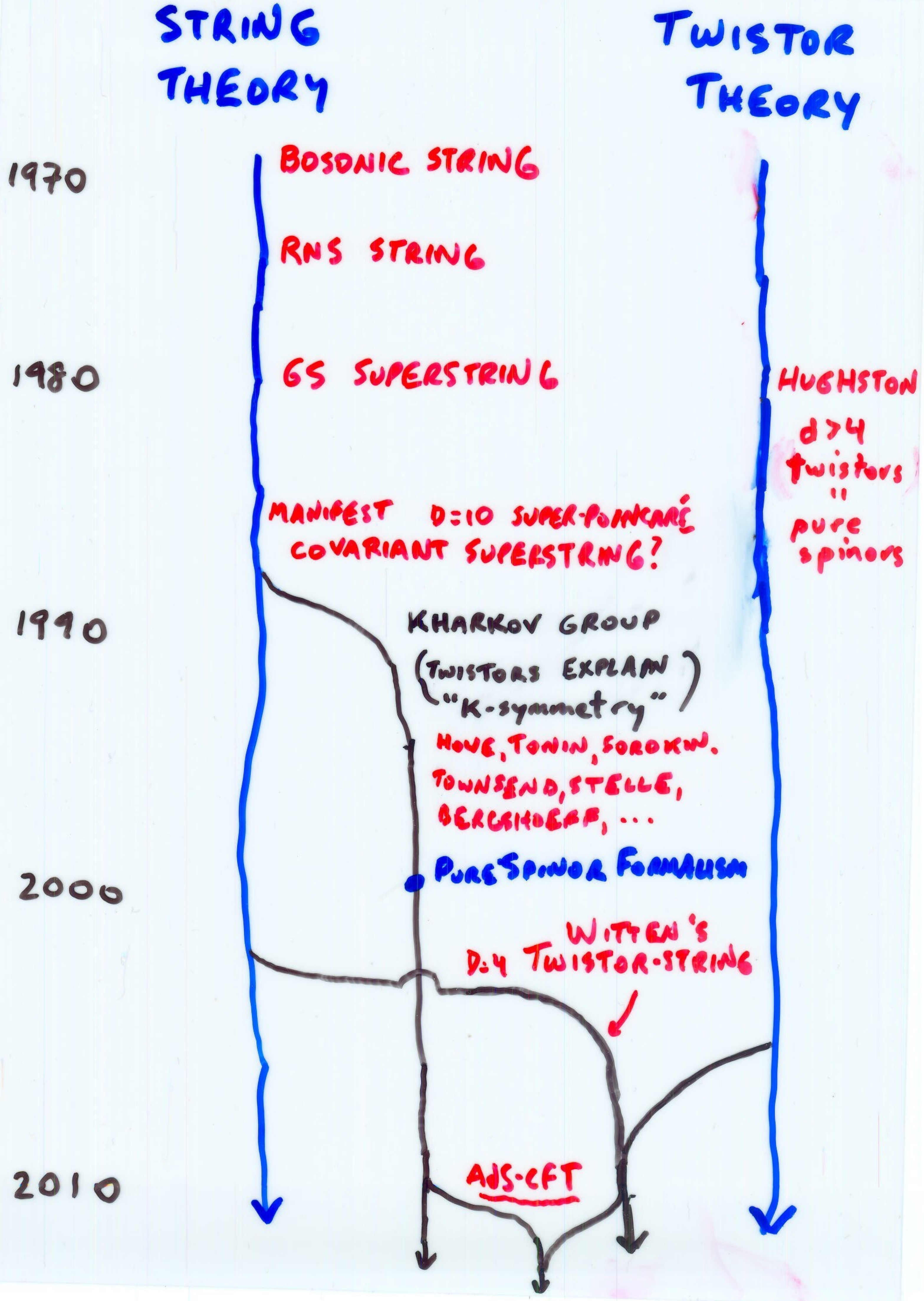
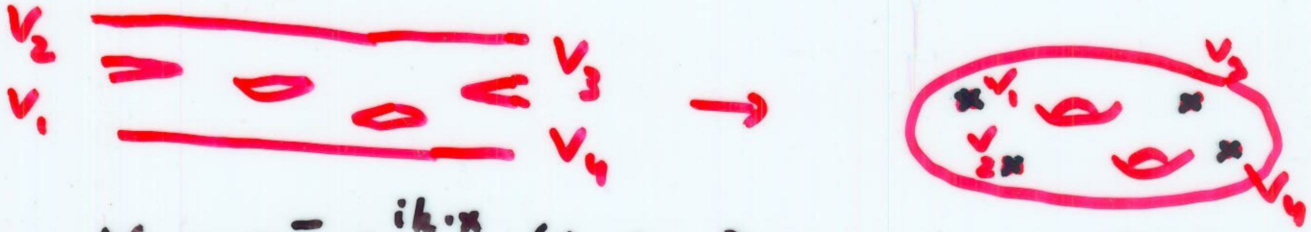

STRING THEORY
TWISTOR THEORY
1970
BOSONIC STRING
RNS STRING
1980
GS SUPERSTRING
HUGHSTON
d>4 twistors
"
pure spinors
MANIFEST D=10 SUPER-POINCARÉ COVARIANT SUPERSTRING?
1990
KHARKOV GROUP
(TWISTORS EXPLAIN "K-symmetry")
HOWE, TONIN, SOROKIN, TOWNSEND, STELLE, BERGSHOEFF, ...
PURE SPINOR FORMALISM
2000
WITTEN'S D=4 TWISTOR-STRING
2010
AdS-CFT

In bosonic string theory, g-loop scattering amp's are computed as d=2 correlation functions on genus g:

$$\mathcal{A}_{g,n} = \int \mathcal{D}S_{g,n} \langle V_1(z_1,\bar{z}_1) \ldots V_n(z_n,\bar{z}_n) \rangle$$



$V_t = c\bar{c}\, e^{ik\cdot x}$ (tachyon), $V_g = c\bar{c}\, \partial X^m \bar{\partial} X^n h_{mn} e^{ik\cdot x}$, etc.

V is in cohomology of $Q + \bar{Q}$

$$\langle V_1 \ldots V_n \rangle = \int \mathcal{D}^{26}x\, \mathcal{D}^2 b\, \mathcal{D}^2 c\; e^{\int dz\, d\bar{z}\, (\partial x\cdot\bar{\partial}x + b\bar{\partial}c + \bar{b}\partial\bar{c})}\, V_1 \ldots V_n$$

After gauge-fixing d=2 reparam. inv,

$$\mathcal{A}_{g,n} = \int d^2\tau_1 \ldots d^2\tau_{3g-3} \int d^2z_1 \ldots \int d^2 z_n \left\langle \left(\int b\right)^{3g-3+n} \left(\int \bar{b}\right)^{3g-3+n} V_1 \ldots V_n \right\rangle$$

$$= \int d^2\tau_1 \ldots d^2\tau_{3g-3} \int d^2z_1 \ldots \int d^2 z_n \left\langle \left(\int b\right)^{3g-3} \left(\int \bar{b}\right)^{3g-3} U_1 \ldots U_n \right\rangle$$

$U_t = e^{ik\cdot x}$, $U_g = \partial X^m \bar{\partial} X^n h_{mn} e^{ik\cdot x}$, ...

In $N=2$ $\hat{c}=3$ "topological" string theory, use similar rules to compute amplitudes:

$$[T, G^+, G^-, J]$$

Replace $Q : \oint dz\, G^+$ $\Rightarrow V$ = chiral primaries

$b : G^-$

$T_{matter} + T_{ghost} : T + \frac{1}{2}\partial J$

$\oint dz\,(bc) = \oint dz\, J$

$\hat{c}=3 \Rightarrow \langle (\int b)^{3g-3} (\int \bar{b})^{3g-3} U_1 \ldots U_n \rangle$ can be nonzero

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Usual description of superstring with RNS formalism is more complicated. Have $N=1$ superconf. invariance which requires picture-changing operators and summing over spin structures. Explicit computations up to genus 2 with 4 external bosonic states.

Spacetime susy is <u>not</u> <u>manifest</u>

$\Rightarrow$ difficult to prove vanishing theorems needed for finiteness, dualities, ...

Using description of superstring with "pure spinor formalism", spacetime susy is manifest.

RNS: x^m, ψ^m; b, c, β, γ

$\theta^\alpha = e^{1/2 \int (\beta\gamma + \psi\psi)}$ is "composite" operator

Pure spinor: $x^m, \theta^\alpha, p_\alpha$; $\lambda^\alpha, \omega_\alpha$

$b = \partial x^m (\gamma_m p) + \ldots$ is composite operator

λ^α is $d=10$ pure spinor $\Leftrightarrow$ $\lambda \gamma^m \lambda = 0$

$Q = \oint dz\, \lambda^\alpha d_\alpha$ defines physical states

Rules for amplitudes comes from interpreting formalism as $\hat{c}=3$ $N=2$ topological string.

Applications:

Explicit tree, one-loop, two-loop amp's with manifest $d=10$ super-Poincaré invariance.

New vanishing theorems for $\partial^n R^4$ terms at g loops.

Review of "Minimal" Pure Spinor formalism

Worldsheet action (ignoring right-movers): $S = \int d^2z \left[\frac{1}{2}\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha - w_\alpha \bar{\partial}\lambda^\alpha \right]$

$m = 0, \ldots, 9 \quad \alpha = 1, \ldots, 16 \qquad \gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\gamma} = 2\eta^{mn}\delta^\gamma_\alpha$

λ^α satisfies pure spinor constraint $\boxed{\lambda\gamma^m\lambda = 0}$

w_α defined up to $\delta w_\alpha = \Lambda_m(\gamma^m\lambda)_\alpha \qquad \lambda \in \mathbb{C}^* \times \frac{SO(10)}{U(5)}$

$\Rightarrow$ w_α only appears in gauge-inv. combinations

$$N_{mn} = \frac{1}{2} w\gamma_{mn}\lambda \ , \ J_\lambda = w_\alpha\lambda^\alpha \ , \ T_\lambda = w_\alpha \partial\lambda^\alpha$$

OPE's: $N_{mn}(z)\lambda^\alpha(0) \to \frac{1}{z}(\gamma_{mn}\lambda)^\alpha$, $N_{mn}(z)N_{pq}(0) \to \frac{-3\eta\eta}{z^2} + \frac{\eta N}{z}$

$J_\lambda(z)\lambda^\alpha(0) \to \frac{1}{z}\lambda^\alpha$, $J_\lambda(z)T_\lambda(0) \to \frac{-8}{z^3} + \frac{J_\lambda}{z^2}$

$T_\lambda(z)\lambda^\alpha(0) \to \frac{1}{z}\partial\lambda^\alpha$, $T_\lambda(z)T_\lambda(0) \to \frac{22}{2z^4} + \frac{2T_\lambda}{z^2} + \frac{\partial T_\lambda}{z}$

$\Rightarrow$ -3 level for Lorentz current $\Rightarrow$ $k = \underset{(\theta^\alpha, p_\alpha)}{+4} \ \underset{(\lambda^\alpha, w_\alpha)}{-3} = \underset{\psi^m}{+1}$

$+22$ central charge $\Rightarrow$ $c = \underset{x^m}{+10} \ \underset{(\theta^\alpha, p_\alpha)}{-32} \ \underset{(\lambda^\alpha, w_\alpha)}{+22} = 0$

-8 ghost-number anomaly? Will be explained later

$x^m(z)x^n(0) \to -\eta^{mn}\log|z|^2$

$\theta^\alpha(z)p_\beta(0) \to \frac{1}{z}\delta^\alpha_\beta$

$\Rightarrow$ All OPE's are manifestly Lorentz covariant

Physical states defined as states in cohomology of "BRST" operator $\boxed{Q = \int dz\, \lambda^\alpha d_\alpha}$

$$d_\alpha = p_\alpha - \frac{1}{2}(\gamma^m\theta)_\alpha \partial x_m - \frac{1}{8}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta)$$

is supersymmetric Green-Schwarz constraint

OPE's:
(Siegel '86)

$$d_\alpha(z)\, d_\beta(0) \to -\frac{\gamma^m_{\alpha\beta}\Pi_m}{z} \qquad \Pi_m = \partial x_m + \frac{1}{2}\theta\gamma_m\partial\theta$$

$$d_\alpha(z)\, \Pi^m(0) \to \frac{1}{z}(\gamma^m\partial\theta)_\alpha$$

$$d_\alpha(z)\, f(x(0),\theta(0)) \to \frac{1}{z} D_\alpha f(x,\theta) \qquad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}(\gamma^m\theta)_\alpha\partial_m$$

$Q\theta^\alpha = \lambda^\alpha$, $Qx^m = \frac{1}{2}\lambda\gamma^m\theta$ resembles κ-transformation

$\lambda\gamma^m\lambda = 0 \Rightarrow$ BRST transformation is nilpotent

Cohomology of Q

g.n. = 0	Ghost	1	1
g.n. = +1	Field	$\lambda^\alpha A_\alpha(x,\theta,p,N)$	$c\,V(x)$
g.n. = +2	Antifield	$\lambda^\alpha\lambda^\beta A^*_{\alpha\beta}(x,\theta,p,N)$	$c\partial c\, V^*(x)$
g.n. = +3	Antighost	$(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)$	$c\partial c\partial^2 c$

Massless open string states:

Massless $\Rightarrow$ $V = \lambda^\alpha A_\alpha(x,\theta)$ only depends on (x,θ) zero modes

$$QV = \lambda^\alpha \lambda^\beta D_\beta A_\alpha = \frac{1}{3840}(\lambda\gamma^{m_1\dots m_5}\lambda)(D\gamma_{m_1\dots m_5}A)$$

$$\delta V = Q\Omega(x,\theta) = \lambda^\alpha D_\alpha \Omega$$

Cohom. of $Q \Rightarrow$ $(D\gamma_{m_1\dots m_5}A) = 0$ and $\delta A_\alpha = D_\alpha \Omega$

$\Rightarrow$ eq. of motion and gauge inv. of

Spinor super-Yang-Mills gauge superfield $A_\alpha(x,\theta)$

$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$ and $\nabla_m = \partial_m + A_m(x,\theta)$ are sYM cov. derivatives

Bianchi identity $\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta}\nabla_m \Rightarrow (\gamma_{m_1\dots m_5})^{\alpha\beta}\nabla_\alpha\nabla_\beta = 0$

In components, $A_\alpha(x,\theta) = a_m(x)(\gamma^m\theta)_\alpha + \chi^\beta(x)(\gamma^m\theta)_\beta(\gamma_m\theta)_\alpha + \dots$

where $\partial^n\partial_{[m}a_{n]} = 0$ and $(\partial\!\!\!/\chi)_\beta = 0$

and $\dots$ involves derivatives of a_m and χ^α.

$\Rightarrow$ $V = \lambda^\alpha A_\alpha(x,\theta)$ describes linearized on-shell super-YM

Integrated vertex op. $\int dz\, U$ satisfies $QU = \partial V$

$$\Rightarrow \int dz\, U = \int dz \left(A_\alpha(x,\theta)\partial\theta^\alpha + A_m(x,\theta)\Pi^m + W^\alpha(x,\theta)d_\alpha + F_{mn}(x,\theta)N^{mn}\right)$$

$W^\alpha = \chi^\alpha + \dots$ and $F_{mn} = \partial_{[m}a_{n]} + \dots$ are sYM superfield strengths

RNS: $\int dz\, U = \int dz(a_m\partial x^m + f_{mn}\psi^m\psi^n)$

Non-minimal Pure Spinor Formalism

Add new left-moving worldsheet variables

$(\bar\lambda_\alpha, \bar w^\alpha)$ 22 bosonic $\boxed{\bar\lambda\gamma^m\bar\lambda = 0}$

(r_α, s^α) 22 fermionic $\boxed{\bar\lambda\gamma^m r = 0}$

$$S = \int d^2z \left(\tfrac{1}{2}\partial x^m \bar\partial x_m + p_\alpha \bar\partial\theta^\alpha - w_\alpha \bar\partial\lambda^\alpha - \bar w^\alpha \bar\partial \bar\lambda_\alpha + s^\alpha \bar\partial r_\alpha\right)$$

$$Q = \int dz \left(\lambda^\alpha d_\alpha + \bar w^\alpha r_\alpha\right)$$

$(\lambda, \bar\lambda) \in \mathbb{R}^+ \times \frac{SO(10)}{SU(5)}$

This non-minimal BRST operator was originally suggested by Nekrasov based on $N=(0,2)$ models.

New "non-minimal" variables do not affect cohomology and will allow functional integration without picture-changing operators.

Similar to "Big Picture" approach in RNS (Siegel, '91)

In non-minimal formalism, g.n. anomaly $= +3 = -8 + 11$ and formalism will be interpreted as a critical $N=2$ topological string.

$J = w_\alpha\lambda^\alpha - \bar w^\alpha \bar\lambda_\alpha$ is ghost-number current

b ghost

In minimal formalism, cannot define b satisfying $\{Q, b\} = T$ since no gauge-invariant states with negative ghost number.

But can define G^α, $H^{[\alpha\beta]}$, $K^{[\alpha\beta\gamma]}$, $L^{[\alpha\beta\gamma\delta]}$ where

$$\{Q, G^\alpha\} = \lambda^\alpha T \qquad G^\alpha = \frac{1}{2}\Pi^m(\gamma_m d)^\alpha - \frac{1}{4}N_{mn}(\gamma^{mn}\partial\theta)^\alpha - \frac{1}{4}J_\lambda \partial\theta^\alpha - \frac{1}{4}\partial^2\theta^\alpha$$

$$[Q, H^{[\alpha\beta]}] = \lambda^{[\alpha} G^{\beta]} \qquad H^{[\alpha\beta]} = \frac{1}{192}\gamma^{\alpha\beta}_{mnp}(d\gamma^{mnp}d + 24 N^{mn}\Pi^p)$$

$$\{Q, K^{[\alpha\beta\gamma]}\} = \lambda^{[\alpha} H^{\beta\gamma]} \qquad K^{[\alpha\beta\gamma]} = \frac{1}{96}\gamma^{[\alpha\beta}_{mnp}(\gamma^m d)^{\gamma]} N^{np}$$

$$[Q, L^{[\alpha\beta\gamma\delta]}] = \lambda^{[\alpha} K^{\beta\gamma\delta]} \qquad L^{[\alpha\beta\gamma\delta]} = \frac{1}{3072}\gamma^{[\alpha\beta}_{mnp}(\gamma^{pqr})^{\gamma\delta]} N^{mn} N_{qr}$$

$$0 = \lambda^{[\alpha} L^{\beta\gamma\delta\rho]}$$

Using these operators, can construct

$$b = s^\alpha \partial\bar\lambda_\alpha + \frac{\bar\lambda_\alpha G^\alpha}{\lambda\bar\lambda} + \frac{\bar\lambda_\alpha r_\beta H^{[\alpha\beta]}}{(\lambda\bar\lambda)^2} - \frac{\bar\lambda_\alpha r_\beta r_\gamma K^{[\alpha\beta\gamma]}}{(\bar\lambda\lambda)^3} - \frac{\bar\lambda_\alpha r_\beta r_\gamma r_\delta L^{[\alpha\beta\gamma\delta]}}{(\lambda\bar\lambda)^4}$$

such that $\left\{ \int dz\,(\lambda^\alpha d_\alpha + \bar\omega^\alpha r_\alpha),\ b \right\} = T$.

Can verify that $b(y)\,b(z) \to 0$ as $y \to z$.

$\Rightarrow$ $(J_{ghost}, j_{BRST}, b, T)$ generate $\hat{c} = 3$ $N=2$ algebra

Scattering amplitudes using "non-minimal" formalism

To functionally integrate over worldsheet variables, use OPE's to integrate over non-zero modes.

For tree amplitudes, need to perform integration over worldsheet zero modes

$$\int [d\lambda][d\bar{\lambda}][dr] \int d^{16}\theta$$

$$[d\lambda] = \frac{(T^{-1})^{\beta\gamma\delta}_{\alpha_1 \dots \alpha_{11}} d\lambda^{\alpha_1} \wedge \dots \wedge d\lambda^{\alpha_{11}}}{\lambda^\beta \lambda^\gamma \lambda^\delta} \qquad \left(\begin{matrix}\text{no sum} \\ \text{over } \beta\gamma\delta\end{matrix}\right)$$

$T^{\alpha_1 \dots \alpha_{11}}_{\beta\gamma\delta}$ defined by

$$(\theta\gamma_{mnp}\theta)(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta) = \epsilon_{\alpha_1 \dots \alpha_{16}} T^{\alpha_1 \dots \alpha_{11}}_{\beta\gamma\delta} \lambda^\beta \lambda^\gamma \lambda^\delta \theta^{\alpha_{12}} \dots \theta^{\alpha_{16}}$$

$\lambda\gamma^m\lambda = 0 \Rightarrow [d\lambda]$ independent of choice of β, γ, δ

$$[d\bar{\lambda}] = \frac{T^{\alpha_1 \dots \alpha_{11}}_{\beta\gamma\delta} d\bar{\lambda}_{\alpha_1} \wedge \dots \wedge d\bar{\lambda}_{\alpha_{11}}}{\bar{\lambda}_\beta \bar{\lambda}_\gamma \bar{\lambda}_\delta}$$

$$[dr] = (T^{-1})^{\beta\gamma\delta}_{\alpha_1 \dots \alpha_{11}} \bar{\lambda}_\beta \bar{\lambda}_\gamma \bar{\lambda}_\delta \left(\frac{\partial}{\partial r_{\alpha_1}}\right) \dots \left(\frac{\partial}{\partial r_{\alpha_{11}}}\right)$$

$(\lambda^\alpha, \bar\lambda_\alpha)$ parameterizes (after Wick rotation) the 22-dimensional space $\mathbb{R} \times \frac{SO(10)}{SU(5)}$.

Infinity from integration over non-compact $\mathbb{R}$ is cancelled by zero from integration over $(\theta^\alpha, r_\alpha)$.

To regularize, insert

$$\mathcal{N}_\rho = \exp(\rho \{Q, \chi\}) \qquad \boxed{\chi = -\bar\lambda_\alpha \theta^\alpha}$$

$$= \exp(-\rho(\lambda^\alpha \bar\lambda_\alpha + r_\alpha \theta^\alpha))$$

On-shell amplitude indep. of ρ since $\mathcal{N}_\rho = 1 + \{Q, \Omega_\rho\}$.

$\mathcal{N}_\rho \to \delta(\lambda^\alpha \bar\lambda_\alpha)$ when $\rho \to \infty$, $\mathcal{N}_\rho \to 1$ when $\rho \to 0$.

Using $N=2$ topological rules, tree amplitude is

$$A_{tree} = \langle \mathcal{N}\, V_1 \ldots V_N \int dz_4\, b(z_4) \ldots \int dz_N\, b(z_N) \rangle$$

$$= \langle \mathcal{N}\, V_1 V_2 V_3 \int dz_4\, U_4 \ldots \int dz_N\, U_N \rangle$$

After using OPE's to integrate over non-zero modes,

$$A_{tree} = \int dz_4 \ldots \int dz_N \int [d\lambda][d\bar\lambda][dr]\, d^{16}\theta\ \mathcal{N}\, \lambda^\alpha \lambda^\beta \lambda^\gamma f_{\alpha\beta\gamma}(\theta, k_r, z_r)$$

$$= \int dz_4 \ldots \int dz_N \int d^{16}\theta\, (T^{-1})^{\beta\gamma\delta}_{\alpha_1 \ldots \alpha_{11}}\, \theta^{\alpha_1} \ldots \theta^{\alpha_{11}} f_{\beta\gamma\delta}(\theta, k_r, z_r)$$

$=$ result using minimal formalism

For g-loop amplitudes, need to integrate over worldsheet zero modes

$$\int [d\lambda][d\bar\lambda][dr]\, d^{16}\theta \prod_{I=1}^{g} [dN\, dJ_\lambda]_I\, [d\bar N\, d\bar J_{\bar\lambda}]_I\, [dS_{mn}\, dS]_I\, d^{16} d_I$$

$$[dN\, dJ_\lambda] = \frac{(M^{-1})^{\alpha_1 \dots \alpha_8}_{m_1 n_1 \dots m_{10} n_{10}}\, dN^{m_1 n_1} \wedge \dots \wedge dN^{m_{10} n_{10}} \wedge dJ_\lambda}{\lambda^{\alpha_1} \dots \lambda^{\alpha_8}}$$

$$[dS_{mn}\, dS] = (M^{-1})^{\alpha_1 \dots \alpha_8}_{m_1 n_1 \dots m_{10} n_{10}}\, \bar\lambda_{\alpha_1} \dots \bar\lambda_{\alpha_8}\, \frac{\partial}{\partial S_{m_1 n_1}} \cdots \frac{\partial}{\partial S_{m_{10} n_{10}}} \frac{\partial}{\partial S}$$

$(M^{-1})^{\alpha_1 \dots \alpha_8}_{m_1 n_1 \dots m_{10} n_{10}}$ defined by $\bar\lambda_{\alpha_1} \dots \bar\lambda_{\alpha_8} (M^{-1})^{\alpha_1 \dots \alpha_8}_{m_1 n_1 \dots m_{10} n_{10}} =$

$$(\bar\lambda \gamma_{m_1 n_1 m_2 m_3 m_4} \bar\lambda)(\bar\lambda \gamma_{m_5 n_5 n_2 m_6 m_7} \bar\lambda)(\bar\lambda \gamma_{m_8 n_8 n_3 n_6 m_9} \bar\lambda)(\bar\lambda \gamma_{m_{10} n_{10} n_4 n_7 n_9} \bar\lambda)$$

To regularize, insert

$$\boxed{\chi = -\bar\lambda_\alpha \theta^\alpha - \sum_{I=1}^{g} N^I_{mn} S^{mn\,I} + J^I_\lambda S^I}$$

$$\mathcal{N}_\rho = \exp(\rho \{Q, \chi\})$$

$$= \exp\Big[-\rho \Big(\lambda^\alpha \bar\lambda_\alpha + r_\alpha \theta^\alpha + \sum_{I=1}^{g} N^I_{mn} \bar N^{mn\,I} + J^I_\lambda \bar J^I_{\bar\lambda} + S^I_{mn} (\lambda \gamma^{mn} d_I) + S^I (\lambda d_I)\Big)\Big]$$

Using N=2 topological rules,

$$A_{g\text{-loop}} = \int d^{3g-3}\tau \left\langle \mathcal{N} \int dz_1 U_1 \dots \int dz_N U_N \prod_{j=1}^{3g-3} \int dw_j\, b(w_j) \right\rangle.$$

Agrees with RNS result for 4-point one and two loop.

(NB + Carlos Mafra) [redacted]

Results:

$$A^{open}_{0,3} \sim \int d^{16}\theta \, (T^{-1}\theta^{11})^{\alpha\beta\gamma} A_\alpha(\theta) A_\beta(\theta) A_\gamma(\theta)$$

$$\sim f_{mn} a^m a^n + a_m (\chi \gamma^m \chi)$$

$$A^{open}_{1,4} \sim \int d^{16}\theta \, (T^{-1}\theta^{11})^{\alpha\beta\gamma} A_\alpha \gamma^{mnpqr}_{\beta\gamma} (W \gamma_{mnp} W) F_{qr}$$

$$\sim t^8 f^4 + \text{susy completion}$$

$$A^{open}_{2,4} \sim \int d^{16}\theta \, (T^{-1}\theta^{11})^{\alpha\beta\gamma} \gamma^{mnpqr}_{\alpha\beta} (\gamma^s W)_\gamma F_{mn} F_{pq} F_{rs}$$

$$\sim \partial^2 (t^8 f^4) + \text{susy completion}$$

Supergravity amplitudes are obtained from "left-right" product of super-YM amp's

$$A^{closed}_{0,3} \sim \int d^{16}\theta \int d^{16}\bar\theta \, (T^{-1}\theta^{11})^{\alpha\gamma\rho} (T^{-1}\bar\theta^{11})^{\beta\delta\sigma} A_{\alpha\beta} A_{\gamma\delta} A_{\rho\sigma}$$

$$A^{closed}_{1,4} \sim t^8 t^8 R^4 + \text{susy completion}$$

$$A^{closed}_{2,4} \sim \partial^4 (t^8 t^8 R^4) + \text{susy completion}$$

Vanishing Theorems

$$A^{\text{closed}}_{2,4} \sim \int d^6 z \, | \langle \eta \, (W^\alpha d_\alpha)^4 \, b^3 \rangle |^2$$

$$\sim \int d^{16}\theta \int d^{16}\bar{\theta} \; | \theta^8 \, W^4 |^2 \sim \partial^4 (t^8 t^8 R^4)$$

$$A^{\text{closed}}_{3,4} \sim \int d^{12} z \, | \langle \eta \, (W^\alpha d_\alpha)^4 \, b^6 \rangle |^2$$

$$\sim \int d^{16}\theta \int d^{16}\bar{\theta} \; | \theta^6 \, W^4 |^2 \sim \partial^6 (t^8 t^8 R^4)$$

$\vdots$

$$A^{\text{closed}}_{6,4} \sim \int d^{30} z \, | \langle \eta \, (W^\alpha d_\alpha)^4 \, b^{15} \rangle |^2$$

$$\sim \int d^{16}\theta \int d^{16}\bar{\theta} \; | W^4 |^2 \sim \partial^{12} (t^8 t^8 R^4)$$

$\Rightarrow$ For 4-pt g-loop amplitudes where $2 \leq g \leq 6$, low-energy effective action starts at

$$\int d^{10}x \sqrt{g} \left(\partial^{2g} (t_8 t_8 R^4) + \ldots \right).$$

Vanishing theorem is not expected to continue above $g=7$ (e.g. $A^{\text{closed}}_{7,4} \sim \partial^{12}(t_8 t_8 R^4)$).

Green, Russo, Vanhove

$\Longrightarrow$ $N=8$ $d=4$ sugra finite up to 8 loops.

Open questions:

1) $b = s^{\alpha}\bar{\partial}\lambda_{\alpha} + \frac{\bar{\lambda}G}{\lambda\bar{\lambda}} + \ldots - \frac{\bar{\lambda}rrrNN}{(\lambda\bar{\lambda})^4}$ has poles when $(\lambda\bar{\lambda}) = 0$. If product of b ghosts contributes divergence worse than $(\lambda\bar{\lambda})^{-10}$, functional integral $\int d^{11}\lambda\, d^{11}\bar{\lambda}$ needs to be regularized. NB + Nekrasov, hep-th/0609012

Regularization affects high-energy contributions to 4-pt. 3-loop computation, but does not affect low-energy contributions computed here.

2) Unitarity of amplitude prescription has not yet been proven. Can probably be proven by showing equivalence to light-cone GS prescription.

3) Computation of coefficients of low-energy amp's would be useful for verifying duality conjectures.