Matrix Models and D-Branes in Twistor String Theory

Christian Sämann

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LMS Durham Symposium 2007

Based on:

- JHEP 0603 (2006) 002, O. Lechtenfeld and CS.
Well-known motivation for studying twistor strings:

- Alternative description of the AdS/CFT correspondence
- New tools for calculating gluon scattering amplitudes
- Alternative descriptions of supergravity

My motivation here:

- Description of super D-branes?
- Relationship between topological and physical D-branes?
- Rôle of Calabi-Yau supermanifolds in mirror symmetry?

⇒ Study variations of the usual twistor geometries and the associated Penrose-Ward transform.

Here: Full dimensional reductions yielding matrix models with interesting interpretations in terms of D-branes. The presented results are only a very preliminary step towards answering the above questions.
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1. Notation: Twistors and Penrose-Ward transform
2. Construction of the matrix models
3. D-Brane interpretation and completion for
   - ADHM construction
   - Nahm construction
4. Conclusions
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The twistor correspondence is a relation between subsets of twistor space and spacetime.

Incidence Relation: $\omega^\alpha = x^{\alpha \dot{\alpha}} \lambda_{\dot{\alpha}}$, Twistor: $Z^i = (\omega^\alpha, \lambda_{\dot{\alpha}}) \in \mathbb{C}P^3$

Twistor Correspondence

Point $x^{\alpha \dot{\alpha}}$ corresponds to sphere $\mathbb{C}P^1 \ni \lambda_{\dot{\alpha}}$

A twistor $Z^i$ is incident to a plane of points $x^{\alpha \dot{\alpha}} = x_0^{\alpha \dot{\alpha}} + \kappa^\alpha \lambda_{\dot{\alpha}}$.

Decompactification

$\mathbb{C}P^3$ is the twistor space of $S^4$ or $S_\infty^4$

$\mathbb{C}P^1$ take out $\infty$

$\mathbb{P}^3$ is the twistor space of $\mathbb{R}^4$ or $\mathbb{C}^4$

$\mathbb{C}P^1_\infty$ is described by $\lambda_{\dot{\alpha}} = 0$, therefore:

$\mathbb{P}^3 := \mathcal{O}(1) \oplus \mathcal{O}(1) \rightarrow \mathbb{C}P^1$

Homog. coords. $\lambda_{\dot{\alpha}}$ on $\mathbb{C}P^1$ and $\omega^\alpha$ in fibres

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Marrying **Twistor**- and **Calabi-Yau** geometry

... with **supermanifolds**: Witten, hep-th/0312171
Supertwistor Space

The supertwistor space $\mathcal{P}^{3|\mathcal{N}}$ is a holomorphic vector bundle of rank $3|4\mathcal{N}$ over $\mathbb{C}P^1$.

The Supertwistor Space $\mathcal{P}^{3|\mathcal{N}}$

Start from $\mathbb{C}P^{3|\mathcal{N}}$, take out $\mathbb{C}P^{1|\mathcal{N}}$ at infinity:

$$\mathcal{P}^{3|\mathcal{N}} := \mathbb{C}^2 \otimes \mathcal{O}(1) \oplus \mathbb{C}^{\mathcal{N}} \otimes \Pi \mathcal{O}(1) \to \mathbb{C}P^1$$

Incidence Relations

$$\omega^\alpha = x^\alpha \dot{\alpha} \lambda_{\dot{\alpha}}$$
$$\eta_i = \eta_i \dot{\alpha} \lambda_{\dot{\alpha}}$$

Double Fibration

First Chern Class of $\mathcal{P}^{3|4}$

$T\mathbb{C}P^1$ 2, $\mathcal{O}(1)$ 1, $\Pi \mathcal{O}(1)$ -1, in total: $c_1 = 0$.

Therefore, there exists a holomorphic measure $\Omega^{3,0|4,0}$. 
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$T\mathbb{C}P^1$ 2, $\mathcal{O}(1)$ 1, $\Pi \mathcal{O}(1)$ -1, in total: $c_1 = 0$.

Therefore, there exists a holomophic measure $\Omega^{3,0|4,0}$.

Double Fibration

$\mathbb{C}^4|2\mathcal{N} \times \mathbb{C}P^1$

$\mathcal{P}^{3|\mathcal{N}}$  $\mathbb{C}^4|2\mathcal{N}$
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Therefore, there exists a holomorphic measure $\Omega^{3,0|4,0}$. 
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**First Chern Class of $\mathcal{P}^{3|4}$**

$T\mathbb{C}P^1$ 2, $\mathcal{O}(1)$ 1, $\Pi \mathcal{O}(1)$ -1, in total: $c_1 = 0$.

Therefore, there exists a holomorphic measure $\Omega^{3,0|4,0}$. 

**Christian Sämann**

**Matrix Models and D-Branes in Twistor String Theory**
Outline of the Penrose-Ward Transform on $\mathcal{P}^{3|4}$

The PW-transform takes us from the topological B-model to SDYM theory.

**topological B-model on $\mathcal{P}^{3|4}$**

$\uparrow$

holomorphic Chern-Simons theory on $E \to \mathcal{P}^{3|4}$:

$$\int \Omega^{3,0|4,0} \wedge \text{tr} (A^{0,1} \wedge \bar{\partial} A^{0,1} + \frac{2}{3} A^{0,1} \wedge A^{0,1} \wedge A^{0,1})$$

with eom $\bar{\partial} A^{0,1} + A^{0,1} \wedge A^{0,1} = 0$

$\uparrow$

holomorphic vector bundles over $\mathcal{P}^{3|4}$

$\uparrow$

solutions to the $\mathcal{N} = 4$ SDYM equations on $\mathbb{C}^{4|8}$

Field contents: $(f_{\dot{\alpha}\dot{\beta}}, \chi^{\alpha i}, \phi^{[ij]}, \tilde{\chi}^{[ijk]}, G^{[ijkl]})$

$$f_{\dot{\alpha}\dot{\beta}} = 0 \ , \quad \nabla_{\dot{\alpha}} \tilde{\chi}^{\dot{\alpha}ijk} - [\chi_{\dot{\alpha}}, \phi^{jk}] = 0 \ ,$$

$$\nabla_{\dot{\alpha}} \chi^{\alpha i} = 0 \ , \quad \varepsilon^{\dot{\alpha}\dot{\gamma}} \nabla_{\dot{\alpha}} G^{[ijkl]} + ... = 0 \ ,$$

$$\Box \phi^{ij} + 2\{\chi^{\alpha i}, \chi^{j}_{\dot{\alpha}}\} = 0 \ ,$$
The PW-transform takes us from the topological B-model to SDYM theory.

**topological B-model on** $\mathcal{P}^{3|4}$

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Abstract of the Penrose-Ward Transform on $\mathcal{P}^{3|4}$

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\[ \Box \phi^{ij} + 2\{\chi^{\alpha i}, \chi^{j}_{\alpha}\} = 0 , \]
Introducing a **real structure**, the double fibration collapses:

\[
\mathbb{C}^4|2\mathcal{N} \times \mathbb{C} P^1 \xrightarrow{\mathcal{P}_3|\mathcal{N}} \mathbb{P}^{3|\mathcal{N}} \to \mathbb{R}_\tau^{4|2\mathcal{N}}
\]

\((\tau_{\pm 1} \text{ related to Kleinian and Euclidean metrics on } \mathbb{R}_\tau^{4|2\mathcal{N}}.)\)

Now: **Field expansion** of hCS gauge potential \(A^{0,1}\) available:

\[
A_\alpha = \lambda^\dot{\alpha} A_{\alpha\dot{\alpha}}(x) + \eta_i \chi_\alpha^i(x) + \gamma \frac{1}{2!} \eta_i \eta_j \hat{\lambda}^\dot{\alpha} \phi^{ij}_{\alpha\dot{\alpha}}(x) + \gamma^2 \frac{1}{3!} \eta_i \eta_j \eta_k \hat{\chi}^{ijk}_{\alpha\dot{\alpha}\dot{\beta}}(x) + \gamma^3 \frac{1}{4!} \eta_i \eta_j \eta_k \eta_l \hat{\lambda}^\dot{\alpha} \hat{\lambda}^\dot{\beta} \hat{\lambda}^\dot{\gamma} \hat{G}^{ijkl}_{\alpha\dot{\alpha}\dot{\beta}\dot{\gamma}}(x)
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\[
A_{\dot{\alpha}} = \gamma^2 \eta_i \eta_j \phi^{ij}(x) - \gamma^3 \eta_i \eta_j \eta_k \hat{\chi}^{ijk}_{\dot{\alpha}}(x) + 2\gamma^4 \eta_i \eta_j \eta_k \eta_l \hat{\lambda}^\dot{\alpha} \hat{\lambda}^\dot{\beta} \hat{G}^{ijkl}_{\dot{\alpha}\dot{\beta}}(x)
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Popov, CS, ATMP 9 (2005) 931

This field expansion makes the equivalence hCS\(\leftrightarrow\)SDYM manifest.
Introducing a real structure, the double fibration collapses:

\[ \mathbb{CP}^1 \times \mathbb{C}^4|2\mathcal{N} \]

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This field expansion makes the equivalence hCS\(\leftrightarrow\) SDYM manifest.
Matrix Models
Matrix models are obtained by dim. reduction or from spacetime noncommutativity.

Two ways of obtaining the matrix models:

- Dimensionally reducing the moduli space $\mathbb{R}^{4|8} \to \mathbb{R}^{0|8}$.
- Making the moduli space $\mathbb{R}^{4|8}$ noncommutative.
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Matrix Models via Dimensional Reduction

Full dimensional reduction yields equivalence between SDYM MM and hCS MQM.

Matrix Model from $\mathcal{N} = 4$ SDYM theory:

$$S := \text{tr} \left( G^{\dot{\alpha}\dot{\beta}} \left( -\frac{1}{2} \varepsilon^{\alpha\beta} [A_{\alpha\dot{\alpha}}, A_{\beta\dot{\beta}}] + \frac{\varepsilon}{2} \phi^{ij} [A_{\alpha\dot{\alpha}}, [A^{\alpha\dot{\alpha}}, \phi_{ij}]] + \ldots \right) \right)$$

Matrix Model from $\mathcal{N} = 4$ hCS theory (MQM):

$$S := \int_{\mathbb{C}P^1_{ch}} \Omega_{\text{red}} \wedge \text{tr} \varepsilon^{\alpha\beta} \chi_{\alpha} \left( \bar{\partial} \chi_{\beta} + [A_{0,1}, \chi_{\beta}] \right)$$

$$\Omega_{\text{red}} := \Omega^{3,0|4,0}_{\mathbb{C}P^1_{ch}} \quad \Omega_{\text{red}}^\pm = \pm d\lambda_\pm \wedge d\eta_1^\pm \ldots d\eta_4^\pm$$

Equivalence explicitly via:

$$\chi_{\alpha} = \chi^{\dot{\alpha}} A_{\alpha\dot{\alpha}} + \eta_i \chi^i_{\alpha} + \gamma \frac{1}{2!} \eta_i \eta_j \hat{\chi}^{\dot{\alpha}} \phi^{ij}_{\alpha\dot{\alpha}} + \gamma^2 \frac{1}{3!} \eta_i \eta_j \eta_k \hat{\chi}^{\dot{\alpha}} \hat{\chi}^{\dot{\beta}} \chi^{ijk}_{\alpha\dot{\alpha}\dot{\beta}} + \gamma^3 \frac{1}{4!} \eta_i \eta_j \eta_k \eta_l \hat{\chi}^{\dot{\alpha}} \hat{\chi}^{\dot{\beta}} \hat{\chi}^{\dot{\gamma}} G^{ijkl}_{\alpha\dot{\alpha}\dot{\beta}\dot{\gamma}}$$

$$A_{\dot{\chi}} = \gamma^2 \eta_i \eta_j \phi^{ij} - \gamma^3 \eta_i \eta_j \eta_k \hat{\chi}^{\dot{\alpha}} \chi^{ijk}_{\dot{\alpha}} + 2 \gamma^4 \eta_i \eta_j \eta_k \eta_l \hat{\chi}^{\dot{\alpha}} \hat{\chi}^{\dot{\beta}} \hat{\chi}^{\dot{\gamma}} G^{ijkl}_{\alpha\dot{\alpha}\dot{\beta}\dot{\gamma}}$$
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  $$A_{\chi} = \gamma^{2} \eta_{i} \eta_{j} \phi^{ij} - \gamma^{3} \eta_{i} \eta_{j} \eta_{k} \hat{\chi}_{\alpha}^{\dot{\alpha} \dot{\beta} \dot{\gamma}} + 2 \gamma^{4} \eta_{i} \eta_{j} \eta_{k} \eta_{l} \hat{\chi}_{\alpha}^{\dot{\alpha} \dot{\beta} \dot{\gamma} \dot{\delta}} G^{ijkl}_{\alpha\dot{\alpha} \dot{\beta} \dot{\gamma} \dot{\delta}}$$
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Christian Sämann
Matrix Models and D-Branes in Twistor String Theory
Noncommutativity on the moduli space

\[ [\hat{x}^{\alpha \dot{\alpha}}, \hat{x}^{\beta \dot{\beta}}] = i\theta^{\alpha \dot{\alpha} \beta \dot{\beta}} \]

with: \((\kappa = \pm 1)\)

\[ \theta^{1i2\dot{2}} = -\theta^{2\dot{2}1i} = -2i\kappa \varepsilon \theta \quad \text{and} \quad \theta^{1\dot{2}2i} = -\theta^{2i1\dot{2}} = 2i\varepsilon \theta \]

• representation space: two oscillator Fock space with \(|0, 0\rangle\)

\[ \hat{a}_1 \sim \hat{x}^{2\dot{1}} + \hat{x}^{1\dot{2}} \quad \text{and} \quad \hat{a}_2 \sim \hat{x}^{2\dot{2}} - \hat{x}^{1\dot{1}} \]

• derivatives become inner derivations of the above algebra:

\[ \frac{\partial}{\partial \hat{x}^{1\dot{1}}} f \sim [\hat{x}^{2\dot{2}}, f] , \quad \text{etc.} \]

• integral becomes trace:

\[ \int d^4x \ f \mapsto (2\pi \theta)^2 \text{tr}_{ \mathcal{H} } \hat{f} \]
Functions on the noncommutative moduli space are infinite-dimensional matrices.

**Noncommutativity on the moduli space**

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\theta^{1i22} = -\theta^{221i} = -2i\kappa \varepsilon \theta \quad \text{and} \quad \theta^{122i} = -\theta^{2i12} = 2i\varepsilon \theta
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Matrix Models from Noncommutativity

Sections $\omega$ of the bundle defining supertwistor space are now matrix valued.

Noncommutativity on the twistor space

Induced algebra:

\[
\begin{align*}
[\hat{\omega}^1_\pm, \hat{\omega}^2_\pm] &= 2(\kappa - 1)\varepsilon\lambda_\pm \theta, \\
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[\hat{\omega}^2_\pm, \hat{\omega}^2_\pm] &= 2(1 - \varepsilon\kappa \lambda_+ \bar{\lambda}_+) \theta, \\
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[\hat{\omega}^2_\pm, \hat{\omega}^2_\pm] &= 2(\lambda_- \bar{\lambda}_- - \varepsilon\kappa) \theta,
\end{align*}
\]

Matrix Models

All operators can be seen as infinite dimensional matrices.

$\Rightarrow$ Matrix models from SDYM and hCS theory explicit equivalence again via field expansion.

Large $N$ limit

$N$: rank of gauge group, limit $N \to \infty$: all MMs equivalent.
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[\hat{\omega}_2^\pm, \hat{\omega}_2^\pm] &= 2(1 - \varepsilon\kappa\lambda_+\bar{\lambda}_+)\theta, & [\hat{\omega}_2^\pm, \hat{\omega}_2^\pm] &= 2(\lambda_+\bar{\lambda}_+ - \varepsilon\kappa)\theta,
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$$

$$
[\hat{\omega}^1_{+}, \hat{\omega}^1_{+}] = 2(\kappa\varepsilon - \lambda_+\bar{\lambda}_+)\theta , \quad [\hat{\omega}^1_{-}, \hat{\omega}^1_{-}] = 2(\kappa\varepsilon\lambda_-\bar{\lambda}_- - 1)\theta ,
$$

$$
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$$

### Matrix Models

All operators can be seen as infinite dimensional matrices.

$\Rightarrow$ Matrix models from SDYM and hCS theory explicit equivalence again via field expansion.

### Large $N$ limit

$N$: rank of gauge group, limit $N \to \infty$: all MMs equivalent
### B-Type Topological Branes

- **D(-1)-, D1-, D3-, and D5-branes**
- Stack of $N$ D-branes comes with rank $N$ vector bundle
- Effective action: $\text{GL}(N, \mathbb{C})$ holomorphic Chern-Simons theory
- I.e. $F^{0,2} = F^{2,0} = 0$ (stability missing: $k^{d+1} \wedge F^{1,1} = \gamma k^d$)

**hCS MM**: Stack of $n$ D1|4-branes wrapping $\mathbb{C}P^{1|4} \hookrightarrow \mathcal{P}^{3|4}$.

Expand Higgs-fields $\mathcal{X}_\alpha = \mathcal{X}_\alpha^0 + \mathcal{X}_\alpha^i \eta_i + \mathcal{X}_\alpha^{ij} \eta_i \eta_j + \ldots$

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\begin{align*}
[\mathcal{X}_1^0, \mathcal{X}_2^0] &= 0, \\
[\mathcal{X}_1^i, \mathcal{X}_2^0] + [\mathcal{X}_1^0, \mathcal{X}_2^i] &= 0, \\
\{\mathcal{X}_1^i, \mathcal{X}_2^j\} - \{\mathcal{X}_1^j, \mathcal{X}_2^i\} + [\mathcal{X}_1^{ij}, \mathcal{X}_2^0] + [\mathcal{X}_1^0, \mathcal{X}_2^{ij}] &= 0,
\end{align*}
$$

Bodies $\mathcal{X}_\alpha^0$ can be diagonalized: positions of the D1|4-branes.

Fermionic directions are "smeared out" even classically.
D-Brane Interpretation
There is an obvious interpretation of the hCS MM in terms of topological B-branes.

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D-Branes in Type IIB String Theory
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- curved spaces: $F^{0,2} = F^{2,0} = 0$ and $k^{d+1} \wedge F^{1,1} = \gamma k^d$
- arising Higgs fields: normal fluctuations of D-branes
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Bound state of $\text{D3-D}(-1)$-branes ($\text{D9-D5-branes + dim. reduction}$)

**Perspective of D3-brane**

D3-D3-strings + BPS condition: SDYM equations

D(-1)-brane: instanton, nontrivial $ch_2$

**Perspective of D(-1)-brane**

D(-1)-D(-1)-strings:

$\mathcal{N} = (0, 1)$ hypmult., adj. $(A_{\alpha \dot{\alpha}}, \chi^i_{\alpha})$

D(-1)-D3-strings:

$\mathcal{N} = (0, 1)$ hypmult., bifund. $(w_\alpha, \psi^i)$

$D$-flatness condition/ADHM eqns.:

$$\frac{i}{16\pi^2} \bar{\sigma}^{\dot{\alpha}}_{\dot{\beta}} (\bar{w}^{\dot{\beta}} w_\alpha + \bar{A}^{\dot{\alpha} \dot{\beta}} A_{\alpha \dot{\alpha}}) = 0$$

Witten, hep-th/9510135, Douglas, hep-th/9512077, ...
ADHM Construction and D-Brane Bound States

There is a nice interpretation of the ADHM construction in terms of D-branes. Bound state of D3-D(-1)-branes (D9-D5-branes + dim. reduction)

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The SDYM Matrix Model is almost equivalent to the ADHM equations.

**Perspective of D(-1)-branes**

- Supersymmetrically extend ADHM eqns.:
  \[ A_{\alpha\dot{\alpha}} \rightarrow A_{\alpha\dot{\alpha}} + \eta_i^{i\dot{i}} \chi_{i\dot{\alpha}} \quad \text{and} \quad w_{\dot{\alpha}} \rightarrow w_{\dot{\alpha}} + \eta_{\dot{\alpha}}^{i\dot{i}} \psi_i \]

- Drop the D(-1)-D3-strings, i.e. \( w_{\dot{\alpha}} \equiv 0 \)

- \( \Rightarrow \) SDYM MM equations

- How to obtain the full picture?

- Incorporate D(-1)-D3-strings in MM in hCS: D1-D5-strings.
ADHM and the SDYM Matrix Model

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Perspective of $\text{D}(-1)$-branes

Supersymmetrically extend ADHM eqns.:

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Drop the $\text{D}(-1)$-$\text{D}3$-strings, i.e. $w_{\dot{\alpha}} \neq 0$

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The hCS MM can be extended to be equivalent to the ADHM equations.

Extended action

\[ S_{\text{ext}} = S_{\text{hCS MM}} + \int_{\mathbb{C}P_1} \Omega_{\text{red}} \wedge \text{tr} \left( \beta \bar{\partial} \alpha + \beta A^{0,1}_{\mathbb{C}P_1} \alpha \right) \]

\( \alpha = \beta^* \), sections of \( \mathcal{O}(1) \), fund. and antifund. of \( \text{GL}(N, \mathbb{C}) \)
(\( \alpha \) and \( \beta \) bosons not fermions as in Witten, hep-th/0312171)

Equations of motion:

\[ \bar{\partial} \chi_\alpha + [A^{0,1}_{\mathbb{C}P_1}, \chi_\alpha] = 0 \]
\[ [\chi_1, \chi_2] + \alpha \beta = 0 \]
\[ \bar{\partial} \alpha + A^{0,1}_{\mathbb{C}P_1} \alpha = 0 \text{ and } \bar{\partial} \beta + \beta A^{0,1}_{\mathbb{C}P_1} = 0 \]
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Extended action

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\[ \bar{\partial}x_{\alpha} + [A_{\mathbb{C}P^{1}}^{0,1}, x_{\alpha}] = 0 \]

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ADHM and the Extended Matrix Models

The hCS MM can be extended to be equivalent to the ADHM equations.

\[ S_{\text{ext}} = S_{\text{hCS MM}} + \int_{\mathbb{C}P^1_{\text{ch}}} \Omega_{\text{red}} \wedge \text{tr} (\beta \bar{\alpha} + \beta A_{\mathbb{C}P^1}^{0,1} \alpha) \]

\( \alpha = \beta^* \), sections of \( \mathcal{O}(1) \), fund. and antifund. of \( \text{GL}(N, \mathbb{C}) \)  
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Extended action

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Again, the equivalence can be made manifest by a field expansion.

Extended Penrose-Ward transform explicitly

\[
\begin{align*}
\beta &= \lambda^{\dot{\alpha}} w_{\dot{\alpha}} + \psi^i \eta_i + \gamma \frac{1}{2!} \eta_i \eta_j \hat{\lambda}^{\dot{\alpha}} \rho^{ij}_{\dot{\alpha}} + \gamma^2 \frac{1}{3!} \eta_i \eta_j \eta_k \hat{\lambda}^{\dot{\alpha}} \hat{\lambda}^{\dot{\beta}} \sigma^{ijk}_{\dot{\alpha}\dot{\beta}} + \ldots \\
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\]

Truncate the SDYM field content \((\phi^{ij}, \tilde{\chi}^{ijk}_{\dot{\alpha}}, G^{ijkl}_{\dot{\alpha}\dot{\beta}} = 0)\):

- Higher fields in extension also vanish
- This expansion and the hCS MM equations yield the full ADHM-equations.

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Reduction of SDYM eqns. $\mathbb{R}^4 \rightarrow \mathbb{R}^3$: Bogomolny monopole eqns.

(static) pair of D3 branes with D1-branes in normal directions

**Perspective of D3-brane**
- static D3-D3-strings + BPS cond.:
  - Bogomolny equations
  - (three-dimensional SDYM)
  - D1-branes: monopoles

**Perspective of D1-brane**
- D1-D1-strings: Nahm equations (one-dimensional SDYM)
- D1-D3-strings: Nahm boundary conditions

Diaconescu, hep-th/9608163
Dimensional Reductions and the Nahm equations

Also for the Nahm Equations, there is a nice interpretation in terms of D-branes.

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For treating the Nahm eqns., one has to change slightly the geometry of twistor space.

Recall
All our MM considerations are based upon $P^3|\cdots = \mathcal{O}(1) \oplus \mathcal{O}(1) \oplus \cdots \rightarrow \mathbb{C}P^1$ and its dim. red. $\mathbb{C}P^1|4$.

The twistor space for the Bogomolny equations is $\mathcal{O}(2) \rightarrow \mathbb{C}P^1$.

New Calabi-Yau supermanifold
Start from $Q^3|4 = \mathcal{O}(2) \oplus \mathcal{O}(0) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1)$
Restrict sections $\hat{Q}^3|4$: $w^1 = y^{\dot{\alpha}\dot{\beta}} \lambda_{\dot{\alpha}} \lambda_{\dot{\beta}}$, $w^2 = y^{i\bar{j}}$

Dimensional reductions
$\hat{Q}^3|4 \rightarrow \begin{cases} P^2|4 := \mathcal{O}(2) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \\ \hat{Q}^2|4 := \mathcal{O}(0) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \\ CP^1|4 := \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \end{cases}$
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Dimensional Reductions and the Nahm equations

Different dimensional reductions yield the various field theories in the Nahm construction.

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Upon imposing a reality condition, hCS theory turns into partially hCS theory (\(\rightarrow\) CR manifolds, etc.): Equiv. to Bogomolny eqns.

Popov, CS, Wolf, JHEP 10 (2005) 058

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D-Brane correspondences
We find a list of correspondences between topological and physical D-branes.

Summing up, we have

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\text{D}5|4\text{-branes in } \mathcal{P}^{3|4} \leftrightarrow \text{D}3|8\text{-branes in } \mathbb{R}^{4|8}
\]

D3|4-branes wr. \( \mathcal{P}^{2|4} \) in \( \mathcal{P}^{3|4} \) or \( \hat{\mathcal{Q}}^{3|4} \) ↔ static D3|8-branes in \( \mathbb{R}^{4|8} \)

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D1|4-branes in \( \mathcal{P}^{3|4} \) ↔ D(-1|8)-branes in \( \mathbb{R}^{4|8} \).

straightforward: add diagonal line bundle \( \mathcal{D}^{2|4} \), defined by \( \omega^1 = \omega^2 \)

D3|4-branes wrapping \( \mathcal{D}^{2|4} \) in \( \mathcal{P}^{3|4} \) ↔ D1|8-branes in \( \mathbb{R}^{4|8} \).

Note:

- Branes extend only into chiral fermionic dimensions
- Branes appear in bound state configurations.
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\text{D}3|4\text{-branes wrapping } \mathcal{D}^2|4 \text{ in } \mathcal{P}^3|4 & \leftrightarrow \text{D}1|8\text{-branes in } \mathbb{R}^4|8
\end{align*}

Note:
- Branes extend only into chiral fermionic dimensions
- Branes appear in bound state configurations.
D-Brane correspondences
We find a list of correspondences between topological and physical D-branes.

Summing up, we have

\[ \text{D}5|4\text{-branes in } \mathcal{P}^3|4 \leftrightarrow \text{D}3|8\text{-branes in } \mathbb{R}^4|8 \]

\[ \text{D}3|4\text{-branes wr. } \mathcal{P}^2|4 \text{ in } \mathcal{P}^3|4 \text{ or } \hat{\mathcal{Q}}^3|4 \leftrightarrow \text{static D}3|8\text{-branes in } \mathbb{R}^4|8 \]

\[ \text{D}3|4\text{-branes wr. } \hat{\mathcal{Q}}^2|4 \text{ in } \hat{\mathcal{Q}}^3|4 \leftrightarrow \text{static D}1|8\text{-branes in } \mathbb{R}^4|8 \]

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\[ D_1|4 \text{-branes in } \mathcal{P}^{3|4} \leftrightarrow D(-1|8) \text{-branes in } \mathbb{R}^{4|8} \]

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D-brane configuration equivalences
We had topological-physical D-brane equivalences for ADHM and Nahm construction.

But: There are many more.
Conclusions
Summary and Outlook

Done:
- Definition of twistor matrix models
- Extension of the matrix models to full ADHM-equations
- full Nahm-equations
- Map between topological and physical D-brane bound states

Future Directions:
- Study Nahm equations more closely
- Study mirror configurations?
- Generalize to full Yang-Mills theory
- Carry over results from topological strings to physical ones (e.g. Derived Categories).
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Matrix Models and D-Branes in Twistor String Theory

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- JHEP 0603 (2006) 002, O. Lechtenfeld and CS.