Giant gravitons: a collective coordinate approach

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Based mostly on arXiv:1301.3519 + work in progress with E. Dzienkowski

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Remarks on AdS/CFT

AdS/CFT is a remarkable duality between ordinary (even perturbative) field theories and a theory of quantum gravity (and strings, etc) with specified boundary conditions.

AdS/CFT is a quantum equivalence

Everything that happens in field theory (the boundary) has a counterpart in gravity (the bulk).

Everything that happens in the bulk has a counterpart in the boundary

Coordinate choice

Global coordinates in bulk correspond to radial quantization in Euclidean field theory, or quantizing on a sphere times time.

Choosing Euclidean versus Lorentzian time in radial quantization of CFT implements the Operator-State correspondence

$$
\begin{aligned}
ds^2 &= r^2(dr^2/r^2 + d\Omega_3^2) \\
&\simeq (d\tau^2 + d\Omega_3^2) \\
&\simeq (-dt^2 + d\Omega_3^2)
\end{aligned}
$$

$$\mathcal{O}(0) \simeq \mathcal{O}|0\rangle_{R.Q.} \simeq |\mathcal{O}\rangle$$

For this talk

$AdS_5 \times S^5$	dual to	N=4 SYM
(Orbifolds of)		(Orbifolds of)

Technical remarks

- I will blur the lines between operators and states.
- One loop anomalous dimension computations are equivalent to second order time independent perturbation theory on a sphere times time (count powers of g).
- States on a Hilbert space can be added with coefficients. For operators one can take sums of operators of different dimension (not a problem, though we usually don’t do this).

Plan of the (rest of the) talk

- Giant gravitons
- Giant graviton states and collective coordinates
- Strings stretched between giants and emergent gauge symmetry
- Masses of open strings
- Emergent Lorentzian field theory
- Spring field theory
- Conclusion/Outlook

GIANT GRAVITONS

Gravitons: half BPS states of AdS

Point particles moving on a diameter of sphere and sitting at origin of AdS

Preserve SO(4)x SO(4) symmetry

There are also D-brane (D3-branes) states that respect the same symmetry and leave half the SUSY invariant.

SO(4) x SO(4) invariance implies

Branes wrap a 3-sphere of 5-sphere at origin of AdS (moving in time)

OR

Branes wrap a 3-sphere of AdS, at a point on diameter of 5- sphere

Solution

$$(x^1)^2 + (x^2)^2 + r_{S^3}^2 = 1$$

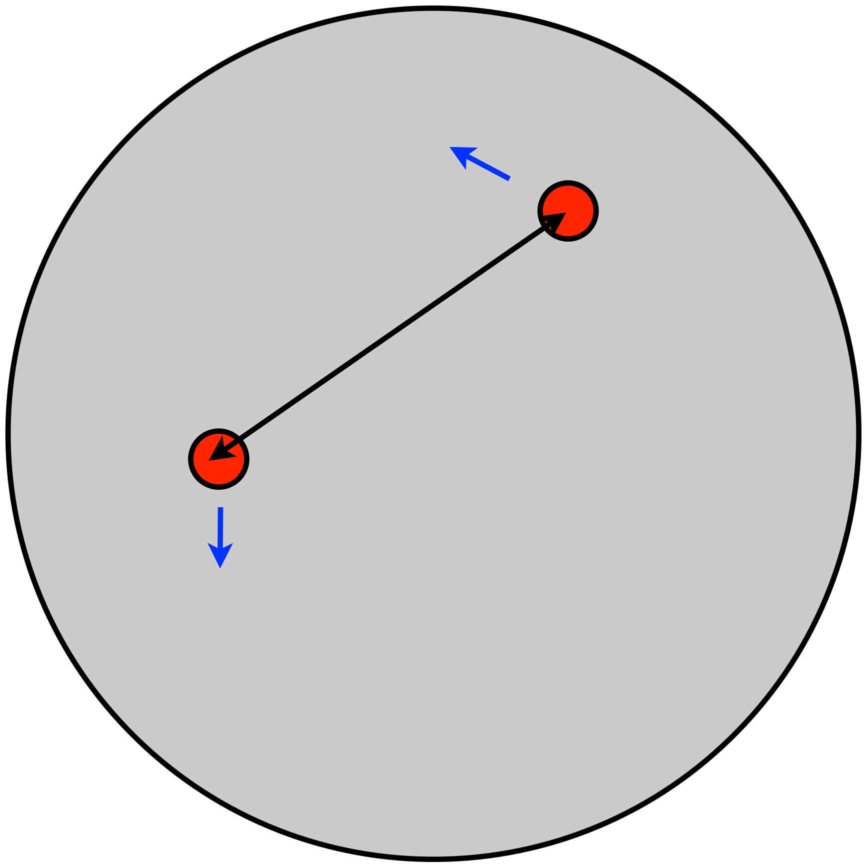
solving equations of motion gives

$$x^1 + ix^2 = z = \exp(it)$$

Picture as a point on disk moving with angular velocity one

The one at z=0 has maximum angular momentum

McGreevy, Susskind, Toumbas, hep-th/000307



They are D-branes

Can attach strings

Gauge symmetry
on worldvolume

Gauss' law
Strings in =
Strings out

Mass of strings should be roughly a distance:
depends on geometric position of branes

In gravity, D-branes are localized, but if they have a fixed R-charge in the quantum theory, they are delocalized in the angle variable of z

This is, they correspond to a oscillating wave function on the angle of z (zero mode)

To find masses of strings the branes must also be localized on angles, so they require uncertainty in angular momentum.

Giant graviton states and their collective coordinates.

To preserve SO(4)xSO(4) invariance, gravitons need to look like

$$Tr(Z^n)$$

Where Z is a complex scalar of the N=4 SYM multiplet.

Giant graviton states:

$$\det{}_\ell Z = \frac{1}{N!}\binom{N}{\ell}\epsilon_{i_1,\ldots,i_\ell,i_{\ell+1}\ldots,i_N}\epsilon^{j_1,\ldots,j_\ell,i_{\ell+1}\ldots,i_N} Z^{i_1}_{j_1}\ldots Z^{i_\ell}_{j_\ell}$$

Subdeterminant operators

Balasubramanian, Berkooz, Naqvi, Strassler, hep-th/0107119

Complete basis of all half BPS operators in terms of Young Tableaux,

Corley, Jevicki, Ramgoolam, hep-th/0111222

Interpretation

A giant graviton with fixed R-charge is a quantum state that is delocalized in dual variable to R-charge

To build localized states in dual variable we need to introduce a collective coordinate that localizes on the zero mode: need to introduce uncertainty in R-charge

Collective coordinate for giant gravitons

Consider

$$\det(Z-\lambda)=\sum_{\ell=0}^{N}(-\lambda)^{N-\ell}\,\det{}_{\ell}(Z)$$

This is a linear combination of states with different R-charge, depends on a parameter, candidate for localized giant gravitons in angle direction

Can compute norm of state

$$\langle \det(\bar{Z} - \tilde{\lambda}^*) \det(Z - \lambda) \rangle = \sum_{\ell=0}^{N} (\lambda \tilde{\lambda}^*)^{N-\ell} \frac{N!}{(N-\ell)!} = N! \sum_{\ell=0}^{N} (\lambda \tilde{\lambda}^*)^{\ell} \frac{1}{(\ell)!}$$

can be well approximated by

$$\langle \det(\bar{Z} - \tilde{\lambda}^*) \det(Z - \lambda) \rangle \simeq N! \exp(\lambda \tilde{\lambda}^*)$$

For

$$|\lambda| < \sqrt{N}$$

The parameter belongs to a disk

Consider a harmonic oscillator and coherent states

$$|\alpha\rangle = \exp(\alpha a^\dagger)$$

Then

$$\begin{aligned}
\langle\beta|\alpha\rangle &= \langle 0|\exp(\beta^* a)\exp(\alpha a^\dagger)|0\rangle \\
&= \exp(\alpha\beta^*)\langle 0|\exp(\alpha a^\dagger)\exp(\beta^* a)|0\rangle \\
&= \exp(\alpha\beta^*)
\end{aligned}$$

This means that our parameter can be interpreted as a parameter for a coherent state of a harmonic oscillator.

Can compute an effective action

$$S_{eff} = \int dt\,[\langle\lambda|i\partial_t|\lambda\rangle - \langle\lambda|H|\lambda\rangle]$$

The first term is a Berry phase

We get an inverted harmonic oscillator in a first order formulation.

$$S_{eff} = \int dt \left[\frac{i}{2}(\lambda^* \dot{\lambda} - \dot{\lambda}^* \lambda) - (N - \lambda\lambda^*) \right]$$

Approximation breaks down exactly when Energy goes to 0

Solution to equations of motion is that the parameter goes around in a circle with angular velocity one.

This is very similar to what happens in gravity

If we rescale the disk to be of radius one, we get

$$S_{eff} = N \int dt \left[\frac{i}{2}(\xi^* \dot{\xi} - \dot{\xi}^* \xi) - (1 - \xi \xi^*) \right]$$

The factor of N in planar counting suggests that this object can be interpreted as a D-brane

Matches exactly with the fermion droplet picture of half BPS states

D. B. hep-th/0403110
Lin, Lunin, Maldacena, hep-th/0409174

Attaching strings

The relevant operators for maximal giant are

$$\epsilon\epsilon(Z, \ldots Z, W^1, \ldots W^k)$$

Balasubramanian, Huang, Levi and Naqvi, hep-th/0204196

These can be obtained from expanding

$$\det(Z + \sum \xi_i W^i)$$

And taking derivatives with respect to parameters

Consider the identity

$$\det(Z + \sum \xi_i W^i) = \det(Z) \exp(\mathrm{Tr} \log(1 + Z^{-a} \sum_i \xi_i W^i Z^{-b}))$$

With a+b=1 arbitrary (think of it as a gauge redundancy)

We can expand the exponential and the log in Taylor series in the parameters

We see that we generate traces with inverse powers of Z between the "words" W

Main idea: replace Z by Z-λ in the expansion, so we generate inverse powers of Z-λ inside traces.

Intuition

$$\det(Z-\lambda)\mathrm{Tr}\left(\frac{1}{Z-\lambda}W\right)$$

Poles that are generated in eigenvalues of Z are cancelled by zeros in determinant: it is polynomial.

Multi-trace structure for many W is exactly such that would be double poles in eigenvalues of Z are absent.

In this case these are just strings that go from a giant to itself.

For many giants:

$$\prod_c \det(Z-\lambda_c)$$

We dress each word with prefactors and postfactors associated to which giant graviton they end on

$$W^i_{cd} \simeq \frac{1}{(Z-\lambda_c)^{a_c}} W^i \frac{1}{(Z-\lambda_d)^{b_d}}$$

To cancel brach cuts, we need pairings between pre-factors and post-factors.

$$\mathrm{Tr}(W(Z-\lambda_c)^{a_c+b_c} W(Z-\lambda_d)(Z-\lambda_d)^{a_d+b_d} \dots)$$

This implements Gauss' law.

These are all of the operators.

Suppose you want to be perverse and consider multi-pronged objects

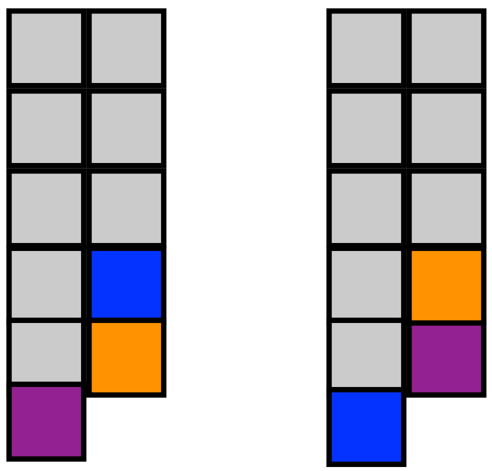
$$(Z-\lambda_c)^{a_c}(Z-\lambda_d)^{a_d}W\ldots$$

Can use partial fractions to show that it is a linear combination of stuff we already had

$$\frac{1}{(Z-\lambda_c)(Z-\lambda_d)} \simeq \frac{1}{(\lambda_c-\lambda_d)}\left(\frac{1}{Z-\lambda_c}-\frac{1}{Z-\lambda_d}\right)$$

Multi-trace structure should (uniquely) cancel all possible double poles

Contrast this to intertwined Young Tableauxs



Balasubramanian, Berenstein, Feng and Huang
hep-th/0411205

Possible reorderings on both columns and rows? Showing counting (really) works requires heavy lifting in combinatorics

de Mello Koch and Ramgoolam,arXiv:1204.2153

One loop anomalous dimensions = masses of strings

Want to compute effective Hamiltonian of strings stretched between two giants.

$$\det(Z-\lambda_1)\det(Z-\lambda_2)\mathrm{Tr}((Z-\lambda_1)^{-1}Y(Z-\lambda_2)^{-1}X)$$

I have not done full combinatorics of 2 giants on same group

Can work on an orbifold and recycle combinatorics of single giant.

$$\det(Z-\lambda)\det(\tilde{Z}-\tilde{\lambda})\mathrm{Tr}((Z-\lambda)^{-1}Y(\tilde{Z}-\tilde{\lambda})^{-1}X)$$

For example, generating series is

$$\langle \det(\bar{Z}-\gamma^*)\mathrm{tr}(\bar{Y}(\bar{Z}-\gamma^*)^{-1})\det(Z-\lambda)\mathrm{tr}(Y(Z-\lambda)^{-1}\rangle$$

$$=(N-\gamma^*\lambda)\exp(\gamma^*\lambda)$$

Setting both paramters equal gives norm of state

With a little work one shows that

$$\langle \lambda, \tilde{\lambda}, Y_{12}, X_{21} | H_{1-loop} | \lambda, \tilde{\lambda}, Y_{12}, X_{21} \rangle$$

involves expressions of the form

$$H|\lambda, \tilde{\lambda}, Y_{12}, X_{21}\rangle \simeq Z\partial_Z|\lambda, \tilde{\lambda}, Y_{12}, X_{21}\rangle$$

$$\simeq \left[\mathrm{tr}\left(\frac{1}{(Z-\lambda)^2} ZY \frac{1}{\tilde{Z}-\tilde{\lambda}} X \right) + \ldots \right] \det\det$$

Now use

$$Z = Z - \lambda + \lambda$$

What do we get?

Single pole terms are subleading (order 1 in planar diagrams)

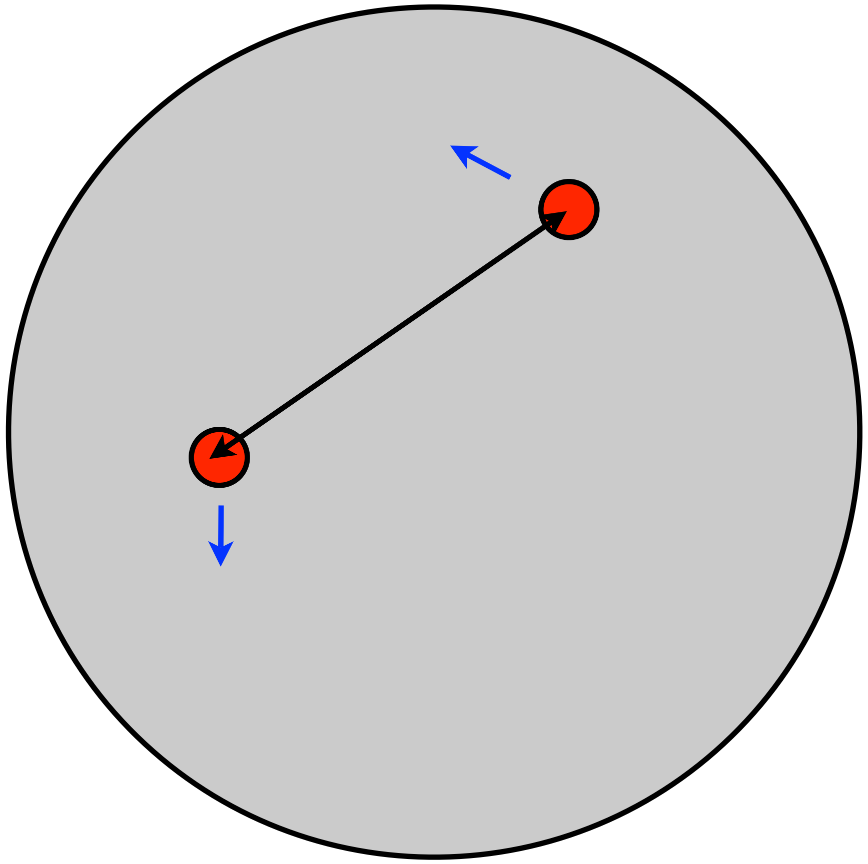
Double pole terms go as

$$(\lambda - \tilde{\lambda})[\partial_\lambda - \partial_{\tilde{\lambda}}]\text{Generating series for overlap}$$

Leading order we get for each Y, and similar for X

$$g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$

In pictures



$$m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$

$$E \simeq m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$
$$\simeq g_{YM}^2 N |\xi - \tilde{\xi}|^2$$

Result is local in collective coordinates (terms that could change collective parameters are exponentially suppressed)

Mass proportional to distance is interpreted as Higgs mechanism for emergent gauge theory.

Can do better

$$Y \to Y^n$$

Full calculation produces a spin chain of Z intertwined in between the Y, and for ground state of spin chain

$$E_n \simeq n + n^{-1} g_{YM}^2 |\lambda - \tilde{\lambda}|^2 \simeq \sqrt{n^2 + g_{YM}^2 |\lambda - \tilde{\lambda}|^2}$$

Starts showing an emergent Lorentz invariance for massive W particles in the worldsheet fluctuations of giant graviton.

With unbroken SUSY of background, one must get spectrum of N=4 SYM

If one can show string splitting and joining is local on sphere, this would imply a low energy emergent N=4 SYM on giant graviton worldvolume.

Spring field theory

de Mello-Koch et al. (Many papers)

After doing a lot of combinatorics with Young Tableaux and diagonalizing operators for strings attached to giants, they obtained an effective theory of harmonic oscillators for giant graviton motion.

Show point of view of this setup.

Assume all strings between branes give a contribution to 1-loop energy proportional to

$$\sum g_{YM}^2 |\lambda_c - \lambda_d|^2$$

Remember classical action for collective coordinates

$$\int dt \; i\bar{\lambda}\dot{\lambda} - (N - \lambda\bar{\lambda})$$

Solutions do not change the norm of the relative positions.

In that situation the strings between branes are adiabatic (energy changes very slowly, but these are fast degrees of freedom), so we can use a Born-Oppenheimer type calculation and just solve for the expectation value of energy of these modes.

Effective action is then

$$\int dt\ i\bar{\lambda}\dot{\lambda} - (N - \lambda\bar{\lambda}) - \sum_{strings} g_{YM}^2 |\lambda_c - \lambda_d|^2$$

Interpretation

Motion of collective coordinates is determined by a quadratic action in a first order formulation of harmonic oscillators: a set of springs

Corrections to naive motion are suppressed by

$$g_{YM}^2 \simeq 1/N$$

One computes backreaction of D-branes to the presence of strings, proportional to occupation numbers.

We can play with these

$$nY \rightarrow series$$

$$Y^n \rightarrow parallel$$

If strings move fast, there is a lot less backreaction of D-branes, purely kinematical effect

Conclusion/outlook

- Collective coordinate method for giant gravitons
- One can do some approximations at the beginning of calculation to generate very friendly generating series.
- Collective coordinate has a natural size as a function of N
- This makes planar versus non-planar counting rather simple, even in the presence of giant gravitons.
- The collective coordinate resolves the problem of delocalized branes (zero modes are resolved with a local coordinate)

Conclusion/outlook 2

- Stretching strings shows emergent gauge symmetry and that modes fill matrix degrees of freedom between D- brane simply.
- Final answer for 1-loop string masses in leading order is really simple
- Can work it for longer spin chains
- Hint of Lorentz symmetry (trades SO(4) R-charge and SO(4) AdS symmetry)
- Gives a really simple derivation of Spring field theory.
- What makes it work is that the string mode energies are local in collective coordinates.

Future

- Fill some technical gaps to get an arbitrary number of giants/strings: one needs to make it completely systematic
- Understand Lorentz invariance to higher loop orders
- Understand non-abelian enhanced gauge symmetry
- Show that string splitting/joining gives rise to local interactions. IF we can understand this, it would go a long way to understanding of 10-D emergent geometry from N=4 SYM.
- What about giants growing into AdS? Needs a nice gadget like determinants to make our collective coordinates.