5d Supersymmetric Yang-Mills-Chern-Simons Theories on RxCP^2 as the theory on M5 Branes

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Supersymmetric M5 Brane Theories on R×CP^2
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Symmetry and Geometry of Branes
in String/M Theory

6d (2,0) Superconformal Theories

- A, D, E type: type IIB on $R^{1+5} \times C^2/\Gamma$
- A_{N-1}, D_N type: N M5 branes, N M5 +OM5
- superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times Sp(2)_R$
- fields: B, Φ_I, Ψ
- selfdual strength $H=dB={}^*H$, purely quantum $\hbar=1$

- We do not know how to write down the theory for nonabelian case.
- covariant derivative?

- N^3 degrees of freedom

- Can you calculate something of (2,0) theories?

5d $\mathcal{N}$=2 SYM as the M5 brane theory

- compactification on R^{1+4} x S^1 with radius R
- the lowest KK modes ⇒ 5d SYM
- coupling constant $1/g_{YM}^2 = 4\pi^2/R$
- instanton = quantum of KK modes of unit momentum
- drop KK modes and keep instantons
- 5d SYM + instantons = ? 6d (2,0) theory

5d SYM on 6d (2,0) theories

* dyonic instanton index, DLCQ of 6d (2,0) theory

  Hee-Cheol Kim, Seok Ki, E. Koh, KL, Sungjay Lee

* 6-loop divergence in large N_c planar limit for four-point amplitude

  Z. Bern, J. J. Carrasco, L. J. Dixon, M. R. Douglas, H. Johansson, M. von Hippel

More Lessons from 5d SYM

- Instantons are magnetic objects.
  - Further compactification to 4d: D0 becomes D1 which is S-dual to F1=KK modes for 5 to 4 d compactification.
- 1/4 BPS selfdual string junction in the Coulomb phase of 6d (2,0) theory
  - possible solution for N^3 degrees of freedom.
  - counting works,
  - The entropy calculation in the Coulomb phase seems to work.

the index function on $S^1 \times S^5$

- 5d SYM on S^5 <Hee-Cheol Kim, Seok Kim> , < Lockhart, Vafa>, Imamura
- S-dual version of the index

- S^5 is S^1 fiber over CP^2: $ds^2_{S5} = ds^2_{CP2} + (dy + V)^2$, $dV = 2J$, $J = {*J}$
- 6d (2,0) Theories on R x S^5/Z_k with large k and twist with R-symmetry
- Question:
  - any supersymmetry preserved?
  - The large k limit leads to 5d limit.
  - Does it teach us anything about (2,0) theories?

6d Abelian Theory (Fermion+ Scalar)

- on R x S^5

$$-\frac{i}{2}\bar{\lambda}\Gamma^M\hat{\nabla}_M\lambda-\frac{1}{2}\partial_M\phi_I\partial^M\phi_I-\frac{2}{r^2}\phi_I\phi_I$$

- gamma matrices Γ^M, ρ^a
- Symplectic Majorana $\lambda = -BC\,\lambda^*$, $\varepsilon = BC\,\varepsilon^*$
- Weyl: $\Gamma^7\lambda=\lambda$, $\Gamma^7\varepsilon = -\varepsilon$
- 32 supersymmetry

$$\begin{aligned}\delta\phi_I &= -\bar{\lambda}\rho_I\epsilon=+\bar{\epsilon}\rho_I\lambda,\\ \delta\lambda &= +\frac{i}{6}H_{MNP}\Gamma^{MNP}\epsilon+i\partial_M\phi_I\Gamma^M\rho_I\epsilon-2\phi_I\rho_I\tilde{\epsilon},\\ \delta\bar{\lambda} &= -\frac{i}{6}H_{MNP}\bar{\epsilon}\Gamma^{MNP}+i\partial_M\phi_I\bar{\epsilon}\Gamma^M\rho_I-2\bar{\tilde{\epsilon}}\rho_I\phi_I.\end{aligned}$$

- additional condition on Killing spinor:

$$\hat{\nabla}_M\epsilon=\frac{i}{2r}\Gamma_M\tilde{\epsilon},\quad \Gamma^M\hat{\nabla}_M\tilde{\epsilon}=2i\epsilon,\qquad \tilde{\epsilon}=\pm\Gamma_0\epsilon.$$

Killing spinors

- The first 16 Killing spinors case: $\tilde{\epsilon} = +\Gamma_0 \epsilon.$
- two classes depending on the isometry SU(3) of CP^2
- $J = e^1 \wedge e^2 + e^3 \wedge e^4$
- SU(3) singlet: 4 $\gamma_{12}\varepsilon = \gamma_{34}\varepsilon = i\,\varepsilon$

$$\epsilon_+ \sim e^{-\frac{i}{2}t + \frac{3i}{2}y} \epsilon_0^{++},$$

- SU(3) triplet: 12

$$\epsilon_+ \sim e^{-\frac{i}{2}t - \frac{i}{2}y} (\epsilon_1^{+-}, \epsilon_2^{-+}, \epsilon_3^{--}),$$

Dimensional Reduction to CP^2

- Remove the y-dependence in the Killing spinor by twisting

- SU(3) singlet

$$\epsilon_{old} = e^{-\frac{3\rho_{45}}{2}y}\epsilon_{new}, \;\; \lambda_{old} = e^{-\frac{3\rho_{45}}{2}y}\lambda_{new}, \;\; (\phi_4 + i\phi_5)_{old} = e^{+3iy}(\phi_4 + i\phi_5)_{new}.$$

$$\rho_{45}\epsilon_+ \;=\; -i\epsilon_+. \qquad \partial_y \to \partial_y + 3iR_2$$

  - 4=2+2 ϵ_{new} is independent of y

- SU(3) triplet

$$\epsilon_{old} = e^{+\frac{\rho_{45}}{2}y}\epsilon_{new}, \;\; \lambda_{old} = e^{+\frac{\rho_{45}}{2}y}\lambda_{new}, \;\; (\phi_4 + i\phi_5)_{old} = e^{-iy}(\phi_4 + i\phi_5)_{new}.$$

$$\rho_{45}\epsilon_+ \;=\; -i\epsilon_+. \quad \partial_y \to \partial_y - iR_2.$$

  - 12=6+6 ϵ_{new} is independent of y

Z_k modding

- keep invariant under y ∾ y + 2 π/k

$$(\mathbf{I})\ \lambda(y)_{old} \sim e^{-\frac{3\pi\rho_{45}}{k}}\lambda(y+\frac{2\pi}{k})_{old},\quad (\phi_4+i\phi_5)(y)_{old} \sim e^{+\frac{6\pi i}{k}}(\phi_4+i\phi_5)(y+\frac{2\pi}{k})_{old},$$
$$(\mathbf{II})\ \lambda(y)_{old} \sim e^{+\frac{\pi\rho_{45}}{k}}\lambda(y+\frac{2\pi}{k})_{old},\quad (\phi_4+i\phi_5)(y)_{old} \sim e^{-\frac{2\pi i}{k}}(\phi_4+i\phi_5)(y+\frac{2\pi}{k})_{old}.$$

- removes some KK modes, reduces the degrees of freedom.

- dimensional reduction to R x CP^2

- expect instantons to represent KK modes.

Dimensional Reduction to R x CP²

- 5d Lagrangian for fermions and scalar
- 4-supersymmetric case

$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I - 2\sum_{a=1,2,3}\phi_a^2 - \frac{13}{2}(\phi_4^2+\phi_5^2)$$
$$-\frac{i}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda - \frac{1}{8}\bar{\lambda}\gamma^{\mu\nu}\lambda J_{mn} + \frac{3}{4}\bar{\lambda}\rho_{45}\lambda$$

- covariant derivative

$$D_\mu\phi_a = \partial_\mu\phi_a, (a=1,2,3)$$
$$D_\mu(\phi_4+i\phi_5) = (\partial_\mu - 3iV_\mu)(\phi_4+i\phi_5)$$

- complete supersymmetry
- abelian gauge $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}A_\rho F_{\sigma\eta}$

4 SUSY YMCS on R x CP^2

- lagrangian

$$
\begin{aligned}
&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta-\frac{i}{3}A_\rho A_\sigma A_\eta\Big)\\
&-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I+\frac{1}{4}[\phi_I,\phi_J]^2-i\epsilon_{abc}\phi_a[\phi_b,\phi_c]-2\phi_a^2-\frac{13}{2}\phi_i^2\\
&-\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda-\frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda]-\frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn}+\frac{3}{4}\bar\lambda\rho_{45}\lambda\Big], \quad (2
\end{aligned}
$$

- supersymmetric Transformation

$$
\begin{aligned}
\delta A_\mu &= +i\bar\lambda\gamma_\mu\epsilon=-i\bar\epsilon\gamma_\mu\lambda,\\
\delta\phi_I &= -\bar\lambda\rho_I\epsilon=\bar\epsilon\rho_I\lambda,\\
\delta\lambda &= +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon+iD_\mu\phi_I\rho_I\gamma^\mu\epsilon-\frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon+3i\epsilon_{ij}\phi_i\rho_j\epsilon-2\phi_I\rho_I\tilde\epsilon.
\end{aligned}
$$

- killing spinor

$$\gamma^{12}\epsilon=\gamma^{34}\epsilon=-\rho^{45}\epsilon,\ \partial_m\epsilon=0,\ D_m\epsilon=0$$

12 SUSY SYMCS Theory

* Lagrangian

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\left(A_\rho\partial_\sigma A_\eta-\frac{i}{3}A_\rho A_\sigma A_\eta\right)$$
$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I+\frac{1}{4}[\phi_I,\phi_J]^2+\frac{i}{3}\epsilon_{abc}\phi_a[\phi_b,\phi_c]-2\phi_a^2-\frac{5}{2}\phi_i^2$$
$$-\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda-\frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda]-\frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn}-\frac{1}{4}\bar\lambda\rho_{45}\lambda\Big], \quad (2.25)$$

* Supersymmetric Transformation

$$\delta A_\mu=i\bar\lambda\gamma_\mu\epsilon=-i\bar\epsilon\gamma_\mu\lambda,$$
$$\delta\phi_I=-\bar\lambda\rho_I\epsilon=\bar\epsilon\rho_I\lambda,$$
$$\delta\lambda=+\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon+iD_\mu\phi_I\rho_I\gamma^\mu\epsilon-\frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon+\epsilon_{ij}\phi_i\rho_j\epsilon-2\phi_I\rho_I\tilde\epsilon.$$

* Killing Spinor

$$\partial_m\epsilon\neq 0,\ \ D_m\epsilon=0$$

Comments on 4 Susy Case

- 6d Hamiltonian D = conformal dimension
- 5d Hamiltonian D= 6d D
- 5d harmonic analysis
  - scalar mass 2 …
  - fermion mass 5/2 …
  - vector mass 3+….
- KK modes = instantons=mass k
  - coupling constant, R=rad of S^5
  - quantization of CS: F=J case, k A_0

$$\frac{1}{g_{YM}^2} = \frac{k}{4\pi^2 R}$$

- anti-self dual instantons are fully BPS: F-*F = 0
- Myers' term: Fuzzy 2 sphere vacua

$$-i[\phi_1, \phi_2] - 2\phi_3 = 0, \cdots$$

5-d Chern-Simons term: Linander and Ohlsson

- 6d 2 form in terms of 5d 2 form and 1 form
- $B_6 = B_5 + A \wedge dy$
- $H_6 = dB_6 = dB_5 + F \wedge dy$
- $H_6 = dB_5 - F \wedge V + F \wedge (dy + V)$

$$ds^2_{R\times S^5} = ds^2_{R\times CP^2} + (dy + V)^2$$

- $H_6 = {}^*H_6$
- $dB_5 - F \wedge V = {}^*F$ on $R \times CP^2$
- $d(dB_5 - F \wedge V) = -2F \wedge J = d\,{}^*F$
- $d^*F + 2F \wedge J = 0$

Harmonic Analysis on S^5 & CP^2

Pope,Hosomich at.al.,Kim&Kim

- Scalar harmonics on R x S^5: $-\partial_t^2\Phi = (-\Delta_{S5} + 4)\Phi$

$$(-\nabla^2_{S^5}+4)Y^{\ell_1,\ell_2} = (\ell_1+\ell_2+2)^2 Y^{\ell_1,\ell_2}, \quad -i\partial_y Y^{\ell_1,\ell_2} = (\ell_1-\ell_2)Y^{\ell_1,\ell_2}.$$

  - highest weight vector of SU(3): $\ell_1 w_1 + \ell_2 w_2$ degeneracy: $(\ell_1+1)(\ell_2+1)(\ell_1+\ell_2+2)/2$
- On CP^2: y-independent mode for $\Phi_{1,2,3}$: $(-\nabla^2_{CP^2}+4)Y^{\ell,\ell} = 4(\ell+1)^2 Y^{\ell,\ell},$
  - conformal dimension: $\varepsilon=2\ell+2$ $\quad 2(\ell+1)^3$.
  - first KK mode: $Y^{0,k}$ $Y^{k,0}$: $\varepsilon=k+2$, $(k+1)(k+2)/2$
  - higher KK modes: $\ell_1 - \ell_2 = kn$, n=1, -1,2, -2,...
- $\Phi_{4,5}$: $(-\nabla^2_{S^5}+4)Y^{\ell,\ell+3} = (-D^2_{CP^2}+13)Y^{\ell,\ell+3} = (\ell+5)^2 Y^{\ell,\ell+3}$
- Fermions: 5/2+….

$$\Psi_1 = Y^{l,l+3}\epsilon_+, \quad \Psi_2 = \gamma^\tau\gamma^m D_m Y^{l,l+3}\epsilon_+, \quad \Psi_3 = Y^{l,l}\epsilon_-, \quad \Psi_4 = \gamma^\tau\gamma^m D_m Y^{l,l}\epsilon_-,$$

- Vector bosons: 4+...

$$\mathcal{A}_\tau = Y^{l,l}, \quad \mathcal{A}^1_m = D_m Y^{l,l}, \quad \mathcal{A}^2_m = J_{mn}D^n Y^{l,l}, \quad \mathcal{A}^3_m = \epsilon_-^\dagger \gamma_m \gamma^n D_n Y^{l,l+3}\epsilon_+.$$

Superconformal Index of 6d Theory on $S^1 \times S^5$

* choose Q & S to be one of four supercharges: SU(3) singlet

$$\{Q,S\} = \varepsilon - j_1 - j_2 - j_3 + 2R_1 + 2R_2 \equiv \Delta,$$

$$Q^{++}_{---}$$

* (2,0) superconformal index $\Delta=0$

$$I(x,y_1,y_2,q) = \mathrm{tr}\left[(-1)^F x^{\varepsilon+R_1} y_1^{j_1-j_2} y_2^{j_2-j_3} q^j\right],$$

$$x = e^{-\beta}, y_1 = e^{-i\gamma_1}, y_2 = e^{-i\gamma_2}$$

* partition function of 5d SYM on S^5: S-dual version: instanton action : $4\pi^2/\beta$

Lockhart, Vafa
H. Kim, S.Kim

* U(1) index: J. Bhattacharya, S. Bhattacharyya, S. Minwalla, S. Raju

$$I = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f(x^n, y_i^n, q^n)\right],$$

$$f(x, y_1, y_2, q) = \frac{x + x^2q^3 - x^2q^2(1/y_1 + y_1/y_2 + y_2) + x^3q^3}{(1 - xqy_1)(1 - xqy_2/y_1)(1 - xq/y_2)}.$$

* U(N) index in q=0 limit= half index (16 susy): S. Bhattacharyya, S. Minwalla

$$I_{1/2-\text{BPS}} = \prod_{m=1}^{N} \frac{1}{1 - x^m}.$$

Path Integral

- path integral

$$I(x, y_i, q) = \int_{\mathrm{S}^1 \times \mathrm{CP}^2} \mathcal{D}\Psi e^{-S_{\mathrm{I}}^E[\Psi]}.$$

- twisted boundary condition

$$\partial_\tau \to \partial_\tau + \frac{\beta}{\beta r} R_1 + \frac{i\gamma_1}{\beta r}(j_1 - j_2) + \frac{i\gamma_2}{\beta r}(j_2 - j_3),$$

- perturbative contribution: split to hyper and vector multiplets
  - $\rho_{12}\, \varepsilon = -i\, \varepsilon$, $\rho_{12}\, \psi = -i\, \psi$, $\rho_{12}\, \chi = i\chi$
  - hyper: $\varphi_1 + i\, \varphi_2$, $\varphi_4 - i\varphi_5$, ψ
  - vector: A_μ, χ, φ_3

* hyper and vector contributions

$$\frac{\det_{H,f}}{\det_{H,b}} = \prod_{\alpha\in root} \frac{1}{\sin\left(\frac{\alpha - i\beta}{2}\right)} \sim \exp\left[\sum_{n=1}^{\infty}\sum_{i,j}\frac{1}{n}x^n e^{ni\alpha_{ij}}\right]. \qquad \frac{\det_{V,f}}{\det_{V,b}} = 1.$$

* perturbative index

$$I = \frac{1}{N!}\int\prod_{i=1}^{N}[\frac{d\alpha_i}{2\pi}]\prod_{i<j}^{N}\left[2\sin\left(\frac{\alpha_i-\alpha_j}{2}\right)\right]^2 \times I_{1-loop}.$$

$$\begin{aligned} I(x,y_1,y_2)_{k\to\infty} &= \frac{1}{N!}\int\prod_{i=1}^{N}[\frac{d\alpha_i}{2\pi}]\prod_{i<j}^{N}\left[2\sin\left(\frac{\alpha_i-\alpha_j}{2}\right)\right]^2 \exp\left[\sum_{n=1}^{\infty}\sum_{i,j}\frac{1}{n}x^n e^{ni\alpha_{ij}}\right] \\ &= \prod_{m=1}^{N}\frac{1}{1-x^m}. \qquad (4. \end{aligned}$$

Supergravity

* AdS$_7$ x S^4

$$ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_5^2) + \frac{1}{4}R^2 d\Omega_4^2,$$
$$F_4 \sim N\epsilon_4,\ R/\ell_p = 2(\pi N)^{1/3}.$$

* Z$_k$ modding

$$y' = \frac{y}{k},\ \chi' = \chi + \frac{3y}{k},$$
$$ds^2_{S^5} = ds^2_{CP^2} + (dy' + V)^2,$$
$$ds^2_{S^4} = d\vartheta^2 + \sin^2\vartheta d\chi'^2 + \cos^2\vartheta ds_{S^2}.$$

* type IIA

$$ds^2_{11} = e^{-2\sigma/3} ds^2_{10} + e^{4\sigma/3}(dy + \mathcal{A})^2,$$
$$F^4_{11} = e^{4\sigma/3} F^4_{10} + e^{\sigma/3} F^3_{10} \wedge dy.$$

* 10-d metric

$$ds^2_{10} = \frac{R^3}{2k}\Big[(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho ds^2_{CP^2}) + \frac{1}{4}(d\vartheta^2 + \cos^2\vartheta ds^2_{S^2})\Big]. \tag{5.15}$$

The curvature scale of the type IIA theory is of order $\sqrt{R^3/2k} \sim \sqrt{N/k}$ which is large when 't Hooft coupling $\lambda = N/k$ is large.

* fiber radius

$$e^{2\sigma/3} \sim \frac{N^{1/3}}{k}\sinh\rho$$

* M-region: k < N$^{1/3}$

Conclusion

- New 5d supersymmetric theories for M5 are found.

- UV finite?
- Enhanced supersymmetry to 16 at k=2 and 32 at k=1
- Fuzzy sphere vacua, D6 branes?
- We are working on the full Index calculation including instantons.

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