Making Up for Lost Time:
A 5D Euclidean View of the M5-Brane

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Symmetry and Geometry of Branes in String/M Theory

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C. Hull and NL., to appear

Outline

- Introduction
- A Little Review:
  - A non-Abelian (2,0) algebra
  - Spacelike and Null Cases
- Timelike Reduction
- Hidden $SO(5.1)$
- Comments

Introduction

Our beloved M5-brane remains very mysterious. Low-energy, decoupled, dynamics governed by a 6D theory with:

- $(2,0)$ supersymmetry
- conformal invariance
- $\mathrm{SO}(5)$ R-symmetry

Multiplet contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions

Very rich and novel 6D CFT dual to $AdS_7 \times S^4$

Strong Coupling, UV completion of 5D SYM

Various conjectures to define/describe/compute/cope with

- DLCQ of QM on instanton moduli space [Aharony, Berkooz, Kachru, Seiberg, Silverstein]
- Deconstruction from D=4 SCFT [Arkani-Hamed,Cohen,Karch,Motl]
- Strong coupling limit of 5D SYM [Douglas],[NL,Papageorgakis,Schmodt-Sommerfeld]
- 5D SYM on $\mathbb{R} \times \mathbb{C}P^2$ [Kim,Lee]

all based on lower dimensional theories and strongly related to each other. More recently there are 6D approaches

- [Chu],[Ho, Huang, Matsuo],[Saemann, Wolf],[Samtleben, Sezgin, Wulf][Bonetti, Grimm, Hohenegger],[Bandos, Here we will continue looking at the M5 from lower-dimensional field theories. This time consider 5D Euclidean SYM

Can be obtained as dimensional reduction of 5+5-dimensional SYM

- 16 supersymmetries
- $SO(5)$ rotational symmetry (and translations)
- $SO(5)$ R-symmetry

But we will construct it from an explicit relalization of the $(2,0)$ system given by [NL, Papageogakis]

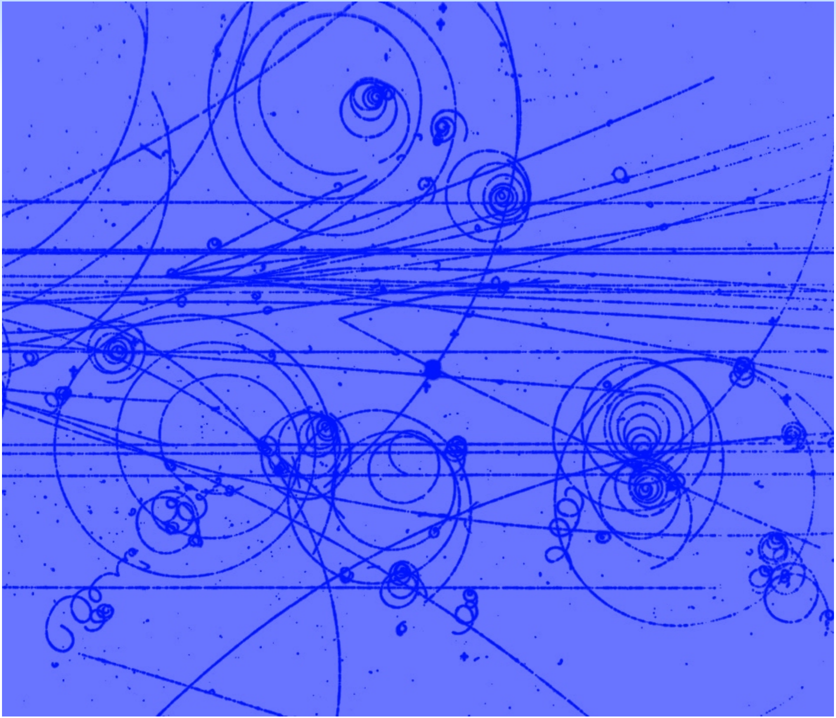
Conjectured by [Hull] to arise as the theory of E5-branes arising from timelike reduction of M-theory.

We will show that it has a hidden $SO(5,1)$ symmetry acting on it string soliton states.

So this Euclidean theory knows about time and dynamics.

- Sees entire worldline/worldsheet
- can define energy and momentum which respect $SO(5,1)$.
- Analogous to time-independent Schrödinger equation

We should view it like one views bubble chamber tracks.



A Little Review

At linearized level the free susy variations of the (2,0) theory are

$$\begin{aligned}\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma^{\mu\nu\lambda}H_{\mu\nu\lambda}\epsilon \\ \delta H_{\mu\nu\lambda} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}\partial_{\lambda]}\Psi\ ,\end{aligned}$$

and the equations of motion are those of free fields with $dH = 0$ (and hence $dH = d\star H = 0$).

Reduction to the D4-brane theory sets $\partial_5 = 0$ and

$$F_{\mu\nu} = H_{\mu\nu5}$$

We wish to generalise this algebra to nonabelian fields with

$$D_\mu X^I_a = \partial_\mu X^I_a - A^b_{\mu a} X^I_b$$

Thus we need a term in $\delta\Psi$ that is quadratic in X^I and which has a single Γ_μ:

- need to invent a field C^μ

N.B. In the original formulation $C^\mu \to C^\mu_a$

- Lie algebra structure constants $f^{ab}{}_c \to f^{abc}{}_d$ - 3-algebra structure constants.

After starting with a suitably general anstaz we find closure of the susy algebra implies [NL, Papageorgakis]

$$
\begin{aligned}
\delta X^I_a &= i\bar{\epsilon}\Gamma^I\psi_a \\
\delta\psi_a &= \Gamma^\mu\Gamma^I\epsilon D_\mu X^I_a + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}\epsilon H^{\mu\nu\lambda}_a - \frac{1}{2}\Gamma_\lambda\Gamma^{IJ}\epsilon C^\lambda X^I_c X^J_d f^{cd}{}_a \\
\delta H_{\mu\nu\lambda\ a} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\psi_a + i\bar{\epsilon}\Gamma^I\Gamma_{\mu\nu\lambda\kappa}C^\kappa X^I_c\psi_d f^{cd}{}_a \\
\delta A_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_{\mu\lambda}C^\lambda\psi_d f^{db}{}_a \\
\delta C^\mu &= 0\,,
\end{aligned}
$$

N.B. 3-algebra version can be rephrased in terms of

- Loop Groups [Papageorgakis, Saemann]
- Lie-Crossed-Modules of Gerbes [Palmer, Saemann]

The algebra closes with the on-shell conditions [NL, Papageorgakis]

$$0 = \Gamma^\mu D_\mu \psi_a + X^I_c C^\nu \Gamma_\nu \Gamma^I \psi_d f^{cd}{}_a$$

$$0 = D^2 X^I_a - \frac{i}{2}\bar{\psi}_c C^\nu \Gamma_\nu \Gamma^I \psi_d f^{cd}{}_a + C^\nu C_\nu X^J_c X^J_e X^I_f f^{ef}{}_d f^{cd}{}_a$$

$$0 = D_{[\mu} H_{\nu\lambda\rho]\ a} + \frac{1}{4}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma X^I_c D^\tau X^I_d f^{cd}{}_a + \frac{i}{8}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma \bar{\psi}_c \Gamma^\tau \psi_d f^{cd}{}_a$$

$$0 = F_{\mu\nu}{}^b{}_a - C^\lambda H_{\mu\nu\lambda\ d} f^{db}{}_a$$

$$0 = D_\mu C^\nu$$

$$0 = C^\rho D_\rho X^I_d = C^\rho D_\rho \psi_d = C^\rho D_\rho H_{\mu\nu\lambda\ a}\ .$$

Thus C^μ picks out a fixed direction in space and in the 3-algebra and $C^\mu D_\mu = 0$.

So apparently we are simply pushed back to 5D. But not so

[NL, Richmond]

$$
\begin{aligned}
T_{\mu\nu} =& D_\mu X^I_a D_\nu X^{Ia} - \frac{1}{2}\eta_{\mu\nu} D_\lambda X^I_a D^\lambda X^{Ia} \\
&+ \frac{1}{4}\eta_{\mu\nu} C^\lambda X^I_a X^J_c C_\lambda X^I_f X^J_e f^{cda} f^{ef}{}_d + \frac{1}{4} H_{\mu\lambda\rho\,a} H_\nu{}^{\lambda\rho\,a} \\
&- \frac{i}{2}\bar{\psi}_a \Gamma_\mu D_\nu \psi^a + \frac{i}{2}\eta_{\mu\nu}\bar{\psi}_a \Gamma^\lambda D_\lambda \psi^a + \frac{i}{2}\eta_{\mu\nu}\bar{\psi}_a C^\lambda X^I_c \Gamma_\lambda \Gamma^I \psi_d f^{acd} \\
J^\mu =& \frac{1}{2}\frac{1}{3!} H_{\nu\lambda\rho\,a}\Gamma^{\nu\lambda\rho}\Gamma^\mu \psi^a - D_\nu X^I_a \Gamma^\nu \Gamma^I \Gamma^\mu \psi^a \\
&- \frac{1}{2} C^\nu X^I_c X^J_d \Gamma_\nu \Gamma^{IJ} \Gamma^\mu \psi^a f^{cd}{}_a
\end{aligned}
$$

And we also obtain 6D expressions for the central charges.

Thus the system is 6D, with a compact direction:

$$
C^\mu P_\mu = \int d^5x C^\mu T_{0\mu} \sim \mathrm{Tr}\int F \wedge H \in \mathbb{Z}
$$

There are three cases for C^μ of interest:

- Spacelike: $C^\mu = g^2\delta^\mu_5$ and the previous system reduces to 4+1D SYM [NL, Papageorgakis]

$$P_5 = -\frac{1}{8g^2_{YM}}\int d^4x\ \mathrm{tr}(F_{ij}F_{kl}\varepsilon_{ijkl}) = \frac{k}{R_5}$$

- Null: $C^\mu = g^2\delta^\mu_+$ and the previous system reduces to DLCQ quantum mechanics on instanton moduli space with x^- acting as time [NL, Richmond]

$$P_+ = -\frac{1}{8g^2_{YM}}\int d^4x\ \mathrm{tr}(F_{ij}F_{kl}\varepsilon_{ijkl}) = \frac{k}{R_+}$$

  as in [Aharony, Berkooz, Kachru, Seiberg, Silverstein]

- Timelike: $C^\mu = g^2\delta^\mu_0$ which we now consider

Timelike reduction

The constraint $C^\mu D_\mu = 0$ means that there is no explicit time dependence. The equations of motion all follow from the euclidean action

$$S = -\mathrm{tr}\int d^5x - \frac{1}{4g^4}F_{ij}F^{ij} + \frac{1}{2}D_iX^ID^iX^I + \frac{g^4}{4}[X^I, X^J]^2$$
$$- \frac{i}{2}\psi^T\Gamma_0\Gamma^iD_i\psi + \frac{1}{2}g^2\psi^T\Gamma^I[X^I, \psi]\,.$$

where

$$F_{ij} = g^2 H_{0ij}$$

This has an $ISO(5)$ euclidean symmetry and $SO(5)$ R-symmetry

Also arises from reduction of 5+5D super-Yang-Mills.

This action is invariant under the supersymmetry

$$\delta X^I = i\bar{\epsilon}\Gamma^I\psi$$
$$\delta\psi = \Gamma^\mu\Gamma^I\epsilon D_\mu X^I + \frac{1}{2g^2}\Gamma^{ij}\Gamma^0\epsilon F_{ij} + \frac{i}{2}g^2\Gamma_0\Gamma^{IJ}[X^I, X^J]$$
$$\delta A_i = -ig^2\epsilon^T\Gamma_i\psi \; .$$

Conserved currents associated with the symmetries

$$T_{ij} = \mathrm{tr}\left(D_iX^ID_jX^I - \frac{1}{2}\delta_{ij}D_kX^ID^kX^I - \frac{1}{4}\delta_{ij}g^4[X^I, X^J]^2 \right.$$
$$\left. -\frac{1}{g^4}F_{ik}F_j{}^k + \frac{1}{4g^4}\delta_{ij}F_{kl}F^{kl} + fermions\right) \; ,$$
$$J^i = \mathrm{tr}\left(-\frac{1}{2g^2}F_{jk}\Gamma^{jk}\Gamma_0\Gamma^i\psi - D_jX^I\Gamma^j\Gamma^I\Gamma^i\psi + \frac{i}{2}g^2[X^I, X^J]\Gamma_0\Gamma^{IJ}\Gamma^i\psi\right)$$

In addition there is a topologically conserved current

$$K_i = \frac{1}{8g^4}\varepsilon_{ijklm}\mathrm{tr}(F^{jk}F^{lm}) \; .$$

In a theory with time conserved currents lead to dynamically conserved quantities

$$q = \int d^5 x j_0 \qquad \partial_0 q = 0$$

In a euclidean theory we still find Ward identities that impose the symmetry on correlation functions. But no 'conserved charges' such as energy and momentum.

Consider an alternative evolution where one picks a direction, say $y = x^5$, and thinks of y as a 'time'

$$\hat{q} = \int d^4 j_y \qquad \partial_y \hat{q} = 0$$

subject to certain boundary conditions. Can be physical in certain circumstances

$$\hat{P}_0 = \int d^4 x T_{0y} \propto \int d^5 x T_{0y} = P_y$$

We could examine the superalgebra of the associated $\hat{Q}$'s

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = \int d^5x \; (\delta_\epsilon S_y C^{-1})_{\alpha\beta}$$
$$= 2(\Gamma^\mu C^{-1})_{\alpha\beta}\hat{P}_\mu + (\Gamma^\mu \Gamma^I C^{-1})_{\alpha\beta}\hat{Z}^I_\mu + (\Gamma^{\mu\nu\lambda}\Gamma^{IJ}C^{-1})_{\alpha\beta}\hat{Z}^{IJ}_{\mu\nu\lambda}$$

If so one finds

$$\hat{P}_y = \int d^4x T_{yy}$$
$$\hat{P}_0 = \int d^4x K_y$$
$$\hat{P}_i = \int d^4x T_{iy} \; .$$

this suggests that we identify $T_{0y} = K_y$ and more generally

$$T_{0i} = K_i$$

Alternatively, from the results of the superalgebra for generic C^μ one finds (formally - there is no dynamical Poisson Bracket),

$$\begin{aligned}\{Q_\alpha, Q_\beta\} =& -\int d^5x\ (\delta_\epsilon J^0 C^{-1})_{\alpha\beta} \\ =& 2(\Gamma^\mu C^{-1})_{\alpha\beta} P_\mu + (\Gamma^\mu \Gamma^I C^{-1})_{\alpha\beta} Z^I_\mu + (\Gamma^{\mu\nu\lambda}\Gamma^{IJ} C^{-1})_{\alpha\beta} Z^{IJ}_{\mu\nu\lambda}\end{aligned}$$

where

$$E = \mathrm{tr}\int d^5x \frac{1}{4g^4} F_{ij}F^{ij} + \frac{1}{2} D_i X^I D^i X^I - \frac{g^4}{4}[X^I, X^J]^2$$

$$P_i = \mathrm{tr}\int d^5x \frac{1}{8g^4} \varepsilon_{0ijklm} F_{jk} F^{lm}\ .$$

and, for example,

$$Z^I_i = \frac{1}{g^2}\mathrm{tr}\int d^5x\ 2F_{ij} D^j X^I + D^j F_{ij} X^I - 2ig^4 [X^I, X^J] D_i X^J$$

This defines dynamical ’charges’ even in the euclidean case.

Hidden $SO(5.1)$

Let us look for simple solutions with energy and momentum. In the symmetric phase the simplest thing is a pure 1/2 BPS pp-wave along x^5

$$F_{ij} = \pm\frac{1}{2}\varepsilon_{ijkl}F^{kl} \qquad i, j \neq 5$$

This has

$$\mathcal{P}_5 = \frac{4\pi^2 n}{g^4} \qquad \mathcal{E} = \frac{4\pi^2 |n|}{g^4} \ .$$

Since this is a time-averaged image of the process we see the whole world line extended along x^5. As if one took a long exposure photograph of the particle as it traverses along x^5.

Next we look at the Coulomb phase with say $\langle X^6 \rangle \neq 0$. Here we expect string states, extended along say x^5 with $Z_5^6 \neq 0$.

Without momentum we just have non-solitonic states:

$$F_{i5} = \partial_i X^6$$

and extended along x^5

If we are looking at a 6D theory we should see boosted versions of these, say along x^4. What should these look like?

- still 1/2 BPS
- extended along x^5 and x^4

These are BPS Monopoles:

- $F_{ij} = \varepsilon_{ijk} D^k \Phi \; A_5 = \frac{1}{v}\Phi \; X^6 = \frac{\gamma}{vg^2}\Phi$
- 1/2 BPS if $\gamma^2 = 1 - v^2$
- Φ is the usual BPS scalar field:

$$\Phi = \frac{vg^2}{\gamma}\langle X^6 \rangle - \frac{Q_M}{2r} + \mathcal{O}(1/r^2) \; , \qquad e^{i\oint F} = e^{2\pi i Q_M} = 1 \; .$$

The 'charges' are

$$\mathcal{Z}_5^6 = -\frac{4\pi}{vg^2}\mathrm{tr}(\langle X^6 \rangle Q_M) \quad \mathcal{P}_4 = \frac{2\pi}{g^2\gamma}\mathrm{tr}(\langle X^6 \rangle Q_M)$$

$$\mathcal{E} = \frac{2\pi\alpha^2}{g^2 v\gamma}\mathrm{tr}(\langle X^6 \rangle Q_M)$$

and satisfy

$$\mathcal{E}^2 = \mathcal{P}_4^2 + \frac{1}{4}(\mathcal{Z}_5^6)^2 \; .$$

This looks like a boosted string whose rest tension is

$$\mathcal{T} = \frac{1}{2}|\mathcal{Z}_5^6| = \frac{2\pi}{g^2 v}\left|\mathrm{tr}(\langle X^6\rangle Q_M)\right| \ ,$$

But what is the factor of v doing in the denominator?

To see this note that the 'charges' have been evaluated as densities over x^4 and x^5

- x^5 is the length along the string
- but x^4 is the distance traveled in unit time
- so $x^4 = vt$, where t is time
- thus the physical tension (mass per unit length per unit time) is

$$T = \frac{2\pi}{g^2}\left|\mathrm{tr}(\langle X^6\rangle Q_M)\right|$$

Next we should look at 1/4 BPS excited states of strings extended along x^5 with momentum along x^5

In the rest frame these are 'dyonic instantons' [NL, Tong]

$$F_{ij} = \pm\frac{1}{2}\varepsilon_{ijkl}F^{kl} \qquad F_{i5} = D_i X^6 \qquad i,j \neq 5$$

with $D^2 X^6 = 0$

These have

$$P_5 = \frac{4\pi^2 n}{g^4} \qquad Z_5^6 = -\frac{4\pi^2}{g^2}\mathrm{tr}(\langle X^6\rangle Q_E) \qquad E = |\mathcal{P}_5| + \frac{1}{2}|Z_5^6|$$

N.B. $E = |P_5| + \frac{1}{2}|Z_5^6|$

So now we look for boosted versions of these:

- still 1/4 BPS
- extended along x^5 and x^4
- 1/4 BPS monopoles (for example see [Lee,Yi])

One finds the generic 1/4 BPS state has

$$F_{ij} = \varepsilon_{ijk} D^k \Phi \qquad X^6 = v g^{-2} A_4 + \gamma g^{-2} A_5 \ , \qquad \Phi = \gamma A_4 - v A_5$$

with $\gamma^2 = 1 - v^2$ and

$$D^2 \Phi = 0 \qquad D^2 X^6 = [\Phi, [\Phi, X^6]]$$

Solutions have the form

$$\Phi = \frac{1}{v\gamma}Y_0 - \frac{g^2 v}{\gamma}\langle X^6\rangle - \frac{Q_M}{2r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$
$$X^6 = \langle X^6\rangle - \frac{Q_E}{2r} + \mathcal{O}\left(\frac{1}{r^2}\right) ,$$

Requiring that $Z_4^6 = 0$ gives

$$0 = \gamma \mathrm{tr}(\langle X^6\rangle Q_M) + v g^2 \mathrm{tr}(\langle X^6\rangle Q_E)$$
$$Z_5^6 = \frac{4\pi}{g^2}\mathrm{tr}(\langle X^6\rangle Q_M)$$

as before.

The momentum and energy (per unit length per unit time)

$$P_4 = -\frac{v}{\sqrt{1-v^2}}\left(|P_5| + \frac{1}{2}|Z_5^6|\right)$$
$$P_5 = \frac{2\pi}{g^4}\mathrm{tr}(Y_0 Q_M)$$
$$E = \frac{1}{\sqrt{1-v^2}}\left(|P_5| + \frac{1}{2}|Z_5^6|\right)$$

which are just the boosted versions of the dyonic instanton expressions $E = |P_5| + \frac{1}{2}|Z_5^6|$

Thus the spectrum of 1/2 and 1/4 BPS string states is consistent with a lorentzian 5+1 D theory

- Strings have tension

$$T = \frac{2\pi}{g^2} \left| \mathrm{tr}(\langle X^6 \rangle Q_M) \right|$$

- Excited states carry quantized momentum along the string (at least in the rest frame)

$$P_5 = \frac{4\pi^2 n}{g^4}$$

Consistent with an emergent time and emergent $SO(5,1)$ symmetry as $g^2 \to \infty$.

Comments

We have shown how 5D euclidean super-Yang-Mills arises as the worldvolume theory of the M5.

Used supersymmetry algebra to define dynamical conserved quantities such as energy and momentum

- leads to a dynamical interpretation

Shown that it possess a hidden $SO(5,1)$ symmetry that acts on string states:

- spectrum is consistent with a 6D dynamical theory

Altogether the $(2,0)$ system of [NL, Papageorgakis] paints a consistent, interconnected picture of the M5-brane in terms of lower dimensional theories

Some important technical points remain

- In DLCQ picture instanton moduli space has singularities (but these are mild orbifold singularities)
- Is 5D SYM well-defined non-perturbatively? We have ignored this here. One approach is the conjecture of [Douglas],[NL, Papageorgakis, Schmidt-Sommerfeld]. But even without this we hope these results will stand

Rather pretty picture whereby a euclidean field theory knows about time from its topological sectors. As if one were looking at the time-independent Schrödinger equation, i.e. in a basis where E and P_i have been diagonalized.

Hopefully gives a new perspective on the emergence of role of lower-dimensional field theories to describe higher-dimensional ones.

Can be viewed as an emergent time or $SO(5,1)$ Lorentz symmetry at the strong coupling (UV) fixed point c.f. [Horava].