

Superconformal models with non-abelian dual fields in 6d

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Based on Samtleben, Sezgin & Wimmer, arXiv:1108.4060
and **work in progress with Igor Bandos and Henning Samtleben**

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- Understanding theory of multiple M5

- 6d (2,0) superconformal theory with a non-abelian chiral tensor supermultiplet

$$(B_{\mu\nu}, \phi^A, \chi^i)$$

- How to endow this chiral multiplet with non-abelian structure?
- No free (dimensionless) parameter to make the theory weakly coupled. Does an action exist?
- Can, at least, equations of motion be constructed?

Motivation

- Rewrite and reinterpret 6d theory (compactified on a circle) in terms of a 5d SYM theory
 - *Lambert & Papageorgakis + Schmidt-Sommerfeld '10; Douglas '10*
 - *Singh '11*
 - *Ho, Huang & Matsuo '11*
 - *Chu & Ko '12*
 - *Bonetti, Grimm & Hohenegger '12*
 - *Hee-Cheol Kim and Kimyeong Lee '12*
 - ...
- Higher gauge theories, twistor space, gerbs...
 - *Kotov & Strobl '10; Saemann & Wolf '11, '12; Palmer & Saemann '12, ...*
- Construction of superconformal non-abelian tensor field theories directly in 6d
 - *Samtleben, Sezgin & Wimmer '11, + Wulff '12;*
 - *Chu '11*
 - *Akyol & Papadopoulos '12*

Each of the approaches has its own issues (limitations)

Ways of approaching the problem

- To show that a non-abelian deformation of 6d chiral tensor fields is possible
- Supersymmetrization and on-shell closure of susy algebra produces equations of motion
- Superconformal actions can be constructed for a subclass of these models

Issues:

- restrictions on possible gauge groups
- non-maximal $(1,0)$ 6d susy
- presence of ghosts in the action
- vector gauge fields are dynamical

Resemble issues
in BLG and ABJM

Goals and results

- Tensor hierarchy (*de Wit & Samtleben '05*) $(A_1, B_2, C_3, C_4, \dots)$

- to gauge 6d (1,0) chiral multiplets $(B_{\mu\nu}^I, \phi^I, \chi^{li})$

- use 6d non-abelian vector multiplet $(A_\mu^r, \lambda^{ir}, Y^{ijr})$

$\partial_\mu \rightarrow D_\mu = \partial_\mu - A_\mu^r T_r$ ← generators of a gauge group **G**

$$F_{\mu\nu}^r = 2\partial_{[\mu} A_{\nu]}^r - f_{st}^r A_\mu^s A_\nu^t + h_I^r B_{\mu\nu}^I$$

$$H_{\mu\nu\rho}^I = 3D_{[\mu} B_{\nu\rho]}^I + 6d_{rs}^I (A_{[\mu}^r \partial_{\nu} A_{\rho]}^s - \frac{1}{3} f_{pq}^s A_{[\mu}^r A_{\nu}^p A_{\rho]}^q) + g^{Ir} C_{\mu\nu\rho r}$$
 additional 3-form field

$h_I^r, g^{Ir}, d_{rs}^I, b_{Irs}$ - constant tensors

Gauge transformations:

extended:

naïve:

$$\delta A_\mu^r = D_\mu \Lambda^r(x)$$

$$\delta B_{\mu\nu}^I = 2D_{[\mu} \Lambda_{\nu]}^I(x) - T_{rJ}^I \Lambda^r B_{\mu\nu}^J$$

$$\delta A_\mu^r = D_\mu \Lambda^r(x) - h_I^r \Lambda_\mu^I$$

$$\Delta B_{\mu\nu}^I = \delta B_{\mu\nu}^I - 2d_{rs}^I A_{[\mu}^r \delta A_{\nu]}^s = 2D_{[\mu} \Lambda_{\nu]}^I(x) - 2d_{rs}^I \Lambda^r F_{\mu\nu}^s - g^{Ir} \Lambda_{\mu\nu r}$$

$$g^{Ir} \Delta C_{\mu\nu\rho r} = 3g^{Ir} D_{[\mu} \Lambda_{\nu\rho]r} + g^{Ir} b_{Irs} (3F_{[\mu\nu}^s \Lambda_{\rho]}^I + H_{\mu\nu\rho}^I \Lambda^s)$$

$$\delta H_{\mu\nu\rho}^I = F_{[\mu\nu}^r \Lambda_{\rho]}^J T_{rJ}^I \leftrightarrow D_{[\lambda} H_{\mu\nu\rho]}^I = F_{[\lambda\mu}^r B_{\nu\rho]}^J T_{rJ}^I$$

inhomogeneous gauge transformations
non-covariant Bianchi identities

Construction

- Covariant gauge transformations of field strengths

$$\delta F_{\mu\nu}^r = \Lambda^p T_{pq}^r F_{\mu\nu}^q, \quad \delta H_{\mu\nu\rho}^I = \Lambda^p T_{pJ}^I H_{\mu\nu\rho}^J$$

- Bianchi identities

$$D_{[\mu} F_{\nu\rho]}^r = \frac{1}{4} h_I^r H_{\mu\nu\rho}^I, \quad D_{[\lambda} H_{\mu\nu\rho]}^I = \frac{2}{3} d_{rs}^I F_{[\lambda\mu}^r F_{\nu\rho]}^s + \frac{1}{4} g^{Ir} H_{\lambda\mu\nu\rho}^{(4)}, \quad g^{Ir} D_{[\sigma} H_{\lambda\mu\nu\rho]}^{(4)} = -2g^{Ir} b_{Irs} F_{[\sigma\lambda}^r H_{\mu\nu\rho]}^I$$

- Gauge group generators and generalized Jacobi identities

$$T_{pq}^r = -f_{[pq]}^r + d_{(pq)}^I h_I^r, \quad T_{pJ}^I = 2h_J^s d_{sp}^I - g^{Is} b_{Jsp}$$

$$[T_p, T_q] = -T_{[pq]}^s T_s$$

$$h_I^r g^{Is} = 0$$

$$f_{[pq]}^s f_{r]s}^t - \frac{1}{3} h_I^t d_{s[p}^I f_{qr]}^s = 0$$

...

...

Constraints on tensors $\mathbf{b}, \mathbf{d}, \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{T}$

- (1,0) supersymmetry transformations are defined by closure of the susy algebra

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] = \bar{\varepsilon}_1 \gamma^\mu \varepsilon_2 \partial_\mu + \delta_\Lambda + \delta_{\Lambda_\mu} + \delta_{\Lambda_{\mu\nu}} + \delta(\text{eoms})$$

- susy transformations of the *off-shell* vector multiplet $(A_\mu^r, \lambda^{ir}, Y^{ijr})$

$$\delta A_\mu^r = -\bar{\varepsilon} \gamma_\mu \lambda^r$$

$$\delta \lambda^{ri} = \frac{1}{8} \gamma^{\mu\nu} F_{\mu\nu}^r \varepsilon^i - \frac{1}{2} Y^{ijr} \varepsilon_j + \frac{1}{4} h_I^r \phi^I \varepsilon^i$$

$$\delta Y^{ijr} = -\bar{\varepsilon}^{(i} \gamma^\mu D_\mu \lambda^{j)r} + 2h_I^r \bar{\varepsilon}^{(i} \chi^{j)I}$$

- susy transformations of the tensor multiplet $(B_{\mu\nu}^I, \phi^I, \chi^{li})$ and $C_{\mu\nu\rho r}$

$$\delta \phi^I = -\bar{\varepsilon} \chi^I$$

$$\delta \chi^{il} = \frac{1}{48} \gamma^{\mu\nu\rho} H_{\mu\nu\rho}^I \varepsilon^i + \frac{1}{4} \gamma^\mu D_\mu \phi^I \varepsilon^i - \frac{1}{2} d_{rs}^I \gamma^\mu \lambda^{ir} \bar{\varepsilon} \gamma_\mu \lambda^s$$

$$\Delta B_{\mu\nu}^I = -\bar{\varepsilon} \gamma_{\mu\nu} \chi^I$$

$$\Delta C_{\mu\nu\rho r} = -b_{Irs} \bar{\varepsilon} \gamma_{\mu\nu\rho} \lambda^s \phi^I \quad C_3 \text{ does not have its own superpartners}$$

it is expected to be dual to the vector field

Susy and superconformal field equations

- Tensor multiplet eom

$$H_{\mu\nu\rho}^I - *H_{\mu\nu\rho}^I = -d_{rs}^I \bar{\lambda}^r \gamma_{\mu\nu\rho} \lambda^s \quad - \text{self-duality condition}$$

$$\gamma^\mu D_\mu \chi^I = \frac{1}{4} d_{rs}^I F_{\mu\nu}^r \gamma^{\mu\nu} \lambda^s + \dots$$

$$D^\mu D_\mu \phi^I = -\frac{1}{2} d_{rs}^I F_{\mu\nu}^r F^{\mu\nu s} + 3d_{rs}^I h_J^r h_K^s \phi^J \phi^K + \dots$$

- Susy variations of these produce vector multiplet eom

$$g^{Irs} b_{Jrs} (Y_{ij}^s \phi^J - 2\bar{\lambda}_{(i}^s \chi_{j)}) = 0$$

$$g^{Irs} b_{Jrs} (\phi^J F_{\mu\nu}^s - 2\bar{\lambda}^s \gamma_{\mu\nu} \chi^J) = \frac{1}{4!} \varepsilon_{\mu\nu\lambda\rho\sigma\tau} g^{Irs} H_r^{(4)\lambda\rho\sigma\tau} \quad - \text{duality between } A_1 \text{ and } C_3$$

$$g^{Irs} b_{Jrs} \phi^J \gamma^\mu D_\mu \lambda^s = \dots$$

- Superconformal invariance: $\Phi = (\phi, Y_{ij}, A_\mu, B_{\mu\nu}, C_{\mu\nu\rho}, \chi, \lambda)$

$$\Delta = (2, 2, 1, 2, 3, 5/2, 3/2) \quad \text{conformal weights}$$

$$\partial_{(\mu} K_{\nu)} = \Omega \eta_{\mu\nu} \quad - \text{conformal Killing vectors}$$

$$\delta_C \Phi_p = \mathcal{L}_K \Phi_p + (\Delta - p) \Omega \Phi_p$$

Susy and superconformal field equations

- Metric in the representation space of the tensor multiplet is required

$$\frac{1}{2} \eta_{IJ} D_\mu \phi^I D^\mu \phi^J$$

- Additional constraints on the coupling tensors are required

$$b_{Irs} = 2\eta_{IJ} d_{rs}^J, \quad \eta_{IJ} d_{p(q}^I d_{rs)}^J = 0, \quad \left. \begin{array}{l} h_I^r = \eta_{IJ} g^{Jr} \\ h_I^r g^{Is} = 0 \end{array} \right\} \begin{array}{l} g^{Ir} \eta_{IJ} g^{Is} = 0 \\ \text{metric is indefinite} \end{array}$$

- Lagrangian for $\phi, Y_{ij}, A_\mu, C_{\mu\nu\rho}$

$$L = -\frac{1}{2} D^\mu \phi^I D_\mu \phi^J \eta_{IJ} + \frac{1}{4} b_{rsI} \phi^I (F_{\mu\nu}^r F^{s\mu\nu} - 4Y_{ij}^r Y^{sij}) + \frac{1}{2} b_{rsI} g_J^r g_K^s \phi^I \phi^J \phi^K + L_{top}$$

$$\int_{\partial M_7} L_{top} = \int_{M_7} (b_{rsI} F^r \wedge F^s \wedge H^I - H^I \wedge DH^J \eta_{IJ})$$

$$F^r = dA^r - f_{st}^r A^s A^t + g_I^r B_2^I, \quad H_3^I = DB_2^I + d_{rs}^I (A^r dA^s - \frac{1}{3} f_{pq}^r A^s A^p A^q) + g^{Ir} C_{3r}$$

Action

- Action for the self-dual field B_2

$$H_3 = *H_3$$

- Abelian case (*Henneaux & Teitelboim '87, Perry & Schwarz '96, Pasti, D.S. & Tonin '96*)

$$H_3 = dB_3$$

$$L_{HH} = -\frac{1}{6} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{2} v^\mu (H - *H)_{\mu\nu\lambda} (H - *H)^{\nu\lambda\rho} v_\rho,$$

$$v_\mu(x): v_\mu v^\mu = -1, \quad v_\mu = \frac{\partial_\mu a(x)}{\sqrt{-\partial_\mu a \partial^\mu a}}$$

- Local gauge symmetries:

$$\delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}(x)$$

$$\delta B_{\mu\nu} = v_{[\mu} \Phi_{\nu]}(x) \rightarrow v^\mu B_{\mu\nu} \text{ is pure gauge d.o.f. (enters Lagrangian under a total derivative)}$$

$$\delta a(x) = \varphi(x), \quad \delta B_{\mu\nu} = \frac{\varphi(x)}{\sqrt{-(\partial a)^2}} (H - *H)_{\mu\nu\lambda} v^\lambda \rightarrow \text{gauge fixing: } a(x) = x^0, \quad v_\mu = \delta_\mu^0$$

- Alternative form of the Lagrangian and equations of motion:

$$L_{HH} = -\frac{1}{8} v^\mu * H_{\mu\nu\lambda} (H - *H)^{\nu\lambda\rho} v_\rho$$

$$\frac{\delta L_{HH}}{\delta B_{\mu\nu}} = \varepsilon^{\mu\nu\rho\sigma\lambda\tau} \partial_\rho [v_\sigma (H - *H)_{\lambda\tau\kappa} v^\kappa] = 0 = d \wedge v \wedge i_v (H - *H) \Rightarrow v \wedge i_v (H - *H) = d(v \wedge \Phi_1)$$

Action

$$H_3 - *H_3 = 0$$

- Non-Abelian action (*Bandos, Samtleben & D.S. in preparation*)

$$L = -\frac{1}{8} v^\mu * H_{\mu\nu\lambda} (H - *H)^{\nu\lambda\rho} v_\rho + \frac{1}{4} b_{rsI} \phi^I (F_{\mu\nu}^r F^{s\mu\nu} - 4Y_{ij}^r Y^{sij}) + L_\phi + L_{top}$$

$$L_\phi = -\frac{1}{2} D^\mu \phi^I D_\mu \phi^J \eta_{IJ} + \frac{1}{2} b_{rsI} g_J^r g_K^s \phi^I \phi^J \phi^K$$

no kinetic term for C_3

$$\int_{\partial M_7} L_{top} = \int_{M_7} (b_{rsI} F^r \wedge F^s \wedge H^I - H^I \wedge DH^J \eta_{IJ})$$

- Symmetries

$$\delta a(x) = \varphi(x), \quad \delta B_{\mu\nu} = \frac{\varphi(x)}{\sqrt{-(\partial a)^2}} (H - *H)_{\mu\nu\lambda} v^\lambda, \quad \delta C_{\mu\nu\rho r} = -\frac{\varphi(x)}{\sqrt{-(\partial a)^2}} (H_{\mu\nu\rho\sigma}^{(4)} - *F_{\mu\nu\rho\sigma}^s b_{Isr} \phi^I) v^\sigma$$

$$\delta B_{\mu\nu}^I \neq v_{[\mu} \Phi_{\nu]}^I(x) \quad \text{but} \quad \delta B_{\mu\nu}^I = g^{Ir} v_{[\mu} \Phi_{\nu]r}(x), \quad \delta C_{\mu\nu\rho r} = v_{[\mu} \Phi_{\nu\rho]r}(x)$$

Action

- Variation with respect to C_3, B_2, A_1 and $a(x)$

$$\begin{aligned}
\delta L = & \Delta C_{3r} \wedge g_I^r i_\nu (H^I - *H^I) \wedge \nu \\
& + \Delta B_2^I \wedge D [i_\nu (H^I - *H^I) \wedge \nu] + \Delta B_2^I \wedge g_I^r (H_{4r} - *F^s b_{srJ} \phi^J) \\
& + \delta A_1^r \wedge F^s \wedge i_\nu (H^I - *H^I) \wedge \nu b_{srI} \\
& + \delta A_1^r \wedge [b_{srI} H^I \wedge F^s + h_{[I}^s b_{J]sr} \phi^I * D \phi^J - D(*F^s b_{srJ} \phi^J)] \\
& + \frac{1}{2} \delta a d \left[i_\nu (H^I - *H^I) \wedge i_\nu (H_I - *H_I) \wedge \frac{da}{\partial_\mu a \partial^\mu a} \right]
\end{aligned}$$

demonstrates how tensor hierarchy works

Action and equations of motion

Modified off-shell susy transformations

- susy transformations of the off-shell vector multiplet $(A_\mu^r, \lambda^{ir}, Y^{ijr})$

$$\delta A_\mu^r = -\bar{\varepsilon} \gamma_\mu \lambda^r$$

$$\delta \lambda^{ri} = \frac{1}{8} \gamma^{\mu\nu} F_{\mu\nu}^r \varepsilon^i - \frac{1}{2} Y^{ijr} \varepsilon_j + \frac{1}{4} h_I^r \phi^I \varepsilon^i$$

$$\delta Y^{ijr} = -\bar{\varepsilon}^{(i} \gamma^\mu D_\mu \lambda^{j)r} + 2h_I^r \bar{\varepsilon}^{(i} \chi^{j)I}$$

- susy transformations of the tensor multiplet $(B_{\mu\nu}^I, \phi^I, \chi^{Ii})$ and $C_{\mu\nu\rho r}$

$$\delta \phi^I = -\bar{\varepsilon} \chi^I$$

$$\delta \chi^{Ii} = \frac{1}{48} \gamma^{\mu\nu\rho} H_{\mu\nu\rho}^I \varepsilon^i + \frac{1}{4} \gamma^\mu D_\mu \phi^I \varepsilon^i - \frac{1}{2} d_{rs}^I \gamma^\mu \lambda^{ir} \bar{\varepsilon} \gamma_\mu \lambda^s + \frac{1}{48} \gamma^{\mu\nu\lambda} \nu_\mu (H^I - *H^I)_{\nu\lambda\rho} \nu^\rho \varepsilon^i$$

$$\Delta B_{\mu\nu}^I = -\bar{\varepsilon} \gamma_{\mu\nu} \chi^I$$

$$\Delta C_{\mu\nu\rho r} = -b_{Irs} \bar{\varepsilon} \gamma_{\mu\nu\rho} \lambda^s \phi^I$$

susy deformation is due to the structure of self-dual action:

$$L_{\text{HH}} = -\frac{1}{6} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{2} \nu^\mu (H - *H)_{\mu\nu\lambda} (H - *H)^{\nu\lambda\rho} \nu_\rho, \quad H_3 = DB_2 + AdA + AAA + C_3$$

Supersymmetry of the action

- *Samtleben et. al. '11, '12; Chong-Sung Chu '11*

- Minimal non-Lagrangian model: $g^{lr} = 0 = b_{lrs}$

$$F^r = dA^r - f_{st}^r A^s A^t + h_l^r B_2^l, \quad H_3^l = DB_2^l + d_{rs}^l (A^r dA^s - \frac{1}{3} f_{pq}^r A^s A^p A^q) + g^{lr} C_{3r}$$

$$A^r = (A^\alpha, \mathcal{A}^I), \quad B_2^l$$

$$f_{st}^r \rightarrow f_{\alpha\beta}^\gamma, (T_\alpha)_I^J \quad f_{\alpha\beta}^\gamma - \text{structure constants (generators) of } \text{adj } G$$

$$d_{rs}^l \rightarrow (T_\alpha)_I^J - \text{generators of some representation of } G$$

$$h_l^r \rightarrow \delta_l^r$$

$$F^r : F^\alpha = dA^\alpha - f_{\beta\gamma}^\alpha A^\beta A^\gamma, \quad \mathcal{B}_2^I = d\mathcal{A}^I - A^\alpha T_{\alpha J}^I \mathcal{A}^J + B_2^I = D\mathcal{A}^I + B_2^I$$

$$H_3^l = D\mathcal{B}_2^l$$

- Equations of motion imposed by susy

$$(H^I - *H^I)_{\mu\nu\rho} = T_{\alpha J}^I \bar{\lambda}^\alpha \gamma_{\mu\nu\rho} \lambda^J, \quad F_{\mu\nu}^\alpha - \text{remains on-shell}$$

Examples of gauge field structure

- Minimal Lagrangian model

$$A^r = (A^\alpha, \mathcal{A}^I), \quad B_2^{\hat{I}} = (B_2^I, B_{2J}), \quad C_{3J} \quad \text{upper and lower I, J label inequivalent reps.}$$

$$f_{st}^r \rightarrow f_{\alpha\beta}^\gamma, \quad (T_\alpha)_I^J \quad f_{\alpha\beta}^\gamma \quad - \text{structure constants (generators) of } adj G$$

$$d_{rs}^{\hat{I}} = \eta^{\hat{I}\hat{J}} b_{\hat{J}rs} \rightarrow (T_\alpha)_I^J \quad - \text{generators of some representation of } G$$

- field strengths

$$F^r : \quad F^\alpha = dA^\alpha - f_{\beta\gamma}^\alpha A^\beta A^\gamma, \quad \mathcal{B}_2^I = d\mathcal{A}^I - A^\alpha T_{\alpha J}^I \mathcal{A}^J + B_2^I = D\mathcal{A}^I + B_2^I$$

$$H_3^{\hat{I}} : \quad H_3^I = D\mathcal{B}_2^I, \quad \mathcal{C}_{3I} = DB_{2I} + C_{3I}, \quad H_{4I} = D\mathcal{C}_{3I}$$

- Lagrangian (of BF-type)

$$L = -\frac{1}{2} D^\mu \phi_I D_\mu \phi^I + T_{\alpha I}^J \phi_J \mathcal{B}_{\mu\nu}^I F^{\mu\nu\alpha} + \frac{1}{2} (\mathcal{C} + *\mathcal{C})^{\mu\nu\rho}{}_I D_\mu \mathcal{B}_{\nu\rho}^I$$

- Equations of motion: $H_3^I = *H_3^I, \quad T_{\alpha I}^J \phi_J F^{\mu\nu\alpha} = \frac{1}{2} D_\mu (\mathcal{C} + *\mathcal{C})^{\mu\nu\rho}{}_I = *H_{(4)I}^{\mu\nu+}$

$$T_{\alpha I}^J D_\mu (\phi_J \mathcal{B}^{\mu\nu I}) = \frac{1}{2} T_{\alpha I}^J \left((\mathcal{C} + *\mathcal{C})^{\mu\nu\rho}{}_J \mathcal{B}_{\nu\rho}^I + \phi_J D^\nu \phi^I + \phi^I D^\nu \phi_J \right), \quad D^\mu D_\mu \phi_I = 0, \quad D^\mu D_\mu \phi^I = \dots$$

Examples of gauge field structure

- A wide class of 6d $(1,0)$ superconformal models of non-abelian tensor multiplets has been constructed (*with highly restrictive constraints on possible gauge structure*)
 - *Akyel & Papadopoulos '12* studied BPS solutions and their string/brane interpretation
 - recently *Samtleben, Sezgin & Wimmer '12* coupled these models to $(1,0)$ hypermultiplets which together with the $(1,0)$ tensor multiplets form the field content of a non-abelian $(2,0)$ tensor multiplet (which one might be tempted to associate with the dofs of multiple M5).
- **Key question:** may some of these models have something to do with the $(2,0)$ theory of multiple M5-branes?
- Issues to be resolved
 - *presence of redundant dofs - vector gauge fields are dynamical*
 - presence of ghosts in the action
- To study relation to other proposals of non-abelian 6d chiral tensor models and 5d SYM. Can this give us a further hint at which direction to move?

Conclusion