Memoryless Computation and Universal Simulation

Alonso Castillo-Ramirez

Joint work with Maximilien Gadouleau

Durham University

alonso.castillo-ramirez@durham.ac.uk

July 2015

1. Introduction

What is memoryless computation?

$$A^n = \{(a_1, a_2, \dots, a_n) \mid a_i \in A\}$$

$$f : A^n \to A^n$$

- Let A be a finite set of size $q \geq 2$ and let $n \geq 2$ be an integer.

What is memoryless computation?

$$A^n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A\}$$

$$f : A^n \to A^n$$

- Let A be a finite set of size $q \geq 2$ and let $n \geq 2$ be an integer.
- **Memoryless computation** (MC) is a new model for computing transformations of A^n with **instructions** that only update one coordinate at a time while using no memory.

The XOR swap algorithm

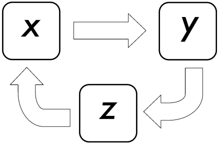
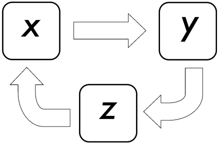


Fig.: Swap of x and y using a temporary variable z.

The XOR swap algorithm

Fig.: Swap of x and y using a temporary variable z.

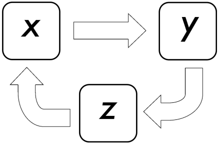


MC generalises the famous **XOR swap algorithm**:

$$
\begin{aligned}
&\text{Input: } (x, y) \in \mathbb{Z}^2;\\
&\quad x := x + y;\\
&\quad y := x - y;\\
&\quad x := x - y;\\
&\text{Output: } (x, y).
\end{aligned}
$$

The XOR swap algorithm

Fig.: Swap of x and y using a temporary variable z.



MC generalises the famous **XOR swap algorithm**:

$$\begin{aligned} \text{Input: } & (x, y) \in \mathbb{Z}^2; \\ x :=& x + y; \\ y :=& x - y; \\ x :=& x - y; \\ \text{Output: } & (x, y). \end{aligned}$$

$$\begin{aligned} \textit{Example}\text{: } & (x, y) := (3, 2); \\ x :=& 3 + 2 = 5; \\ y :=& 5 - 2 = 3; \\ x :=& 5 - 3 = 2; \\ \text{Output: } & (2, 3). \end{aligned}$$

Why is MC interesting?

Here are some reasons:

Why is MC interesting?

Here are some reasons:

1. Using memory consumes time and resources.

Why is MC interesting?

Here are some reasons:

1. Using memory consumes time and resources.
2. **Every** transformation of A^n may be computed without memory.

Why is MC interesting?

Here are some reasons:

1. Using memory consumes time and resources.
2. **Every** transformation of A^n may be computed without memory.
3. If we use all possible instructions, every transformation of A^n may be computed without memory in **linear time**.

Why is MC interesting?

Here are some reasons:

1. Using memory consumes time and resources.
2. **Every** transformation of A^n may be computed without memory.
3. If we use all possible instructions, every transformation of A^n may be computed without memory in **linear time**.
4. We only need $\mathbf{n+1}$ fixed instructions in order to compute without memory every transformation of A^n.

2. Memoryless Computation

Instructions

- Let $\mathrm{Tran}(A^n)$ be the full transformation monoid of A^n.

Instructions

- Let $\mathrm{Tran}(A^n)$ be the full transformation monoid of A^n.
- Denote any $f \in \mathrm{Tran}(A^n)$ by $f = (f_1, f_2, \dots, f_n)$, where $f_i : A^n \to A$ is the *i*-**th coordinate function** of f.

Instructions

- Let $\mathrm{Tran}(A^n)$ be the full transformation monoid of A^n.
- Denote any $f \in \mathrm{Tran}(A^n)$ by $f = (f_1, f_2, \dots, f_n)$, where $f_i : A^n \to A$ is the *i*-**th coordinate function** of f.
- An **instruction** of A^n is a transformation $f \in \mathrm{Tran}(A^n)$ with at most one nontrivial coordinate function f_i, i.e. $f_i \neq \mathrm{pr}_i$.

Instructions

- Let $\mathrm{Tran}(A^n)$ be the full transformation monoid of A^n.
- Denote any $f \in \mathrm{Tran}(A^n)$ by $f = (f_1, f_2, \dots, f_n)$, where $f_i : A^n \to A$ is the *i*-**th coordinate function** of f.
- An **instruction** of A^n is a transformation $f \in \mathrm{Tran}(A^n)$ with at most one nontrivial coordinate function f_i, i.e. $f_i \neq \mathrm{pr}_i$.
- For example, the following are instructions of $\mathbb{Z}_q^2$:

Instruction	**Update form**
$(x_1, x_2)f = (x_1 + 1, x_2)$	$x_1 \leftarrow x_1 + 1$
$(x_1, x_2)g = (x_1, x_1 + x_2)$	$x_2 \leftarrow x_1 + x_2$

Memoryless Complexity

- Let $\mathcal{H}$ be a set of instructions of A^n. Denote by $\langle \mathcal{H} \rangle$ the subsemigroup of $\mathrm{Tran}(A^n)$ generated by $\mathcal{H}$.

Memoryless Complexity

- Let $\mathcal{H}$ be a set of instructions of A^n. Denote by $\langle \mathcal{H} \rangle$ the subsemigroup of $\mathrm{Tran}(A^n)$ generated by $\mathcal{H}$.

- A **program** of length ℓ computing $g \in \langle \mathcal{H} \rangle$ is a sequence $f^{(1)}, \dots, f^{(\ell)} \in \mathcal{H}$ such that $g = f^{(1)} \circ \cdots \circ f^{(\ell)}$.

Memoryless Complexity

- Let $\mathcal{H}$ be a set of instructions of A^n. Denote by $\langle \mathcal{H} \rangle$ the subsemigroup of $\mathrm{Tran}(A^n)$ generated by $\mathcal{H}$.
- A **program** of length ℓ computing $g \in \langle \mathcal{H} \rangle$ is a sequence $f^{(1)}, \dots, f^{(\ell)} \in \mathcal{H}$ such that $g = f^{(1)} \circ \cdots \circ f^{(\ell)}$.
- For example, a program computing $(x_1, x_2)g = (x_2, x_1)$ is

$$f^{(1)} : x_1 \leftarrow x_1 + x_2,\ f^{(2)} : x_2 \leftarrow x_1 - x_2,\ f^{(3)} : x_1 \leftarrow x_1 - x_2.$$

Memoryless Complexity

- Let $\mathcal{H}$ be a set of instructions of A^n. Denote by $\langle \mathcal{H} \rangle$ the subsemigroup of $\mathrm{Tran}(A^n)$ generated by $\mathcal{H}$.
- A **program** of length ℓ computing $g \in \langle \mathcal{H} \rangle$ is a sequence $f^{(1)}, \dots, f^{(\ell)} \in \mathcal{H}$ such that $g = f^{(1)} \circ \cdots \circ f^{(\ell)}$.
- For example, a program computing $(x_1, x_2)g = (x_2, x_1)$ is

$$f^{(1)} : x_1 \leftarrow x_1 + x_2, \; f^{(2)} : x_2 \leftarrow x_1 - x_2, \; f^{(3)} : x_1 \leftarrow x_1 - x_2.$$

- The shortest length of a program computing $g \in \langle \mathcal{H} \rangle$ with instructions in $\mathcal{H}$ is called the **memoryless complexity of** g **with respect to** $\mathcal{H}$.

Main Results

Theorem (Burckel '96; Gadouleau-Riis '15)

Let A be a finite set and $n \geq 2$. Let $\mathcal{I}$ be the set of all instructions of A^n. Then, $\langle \mathcal{I} \rangle = \mathrm{Tran}(A^n)$.

Main Results

Theorem (Burckel '96; Gadouleau-Riis '15)

Let A be a finite set and $n \geq 2$. Let $\mathcal{I}$ be the set of all instructions of A^n. Then, $\langle \mathcal{I} \rangle = \mathrm{Tran}(A^n)$.

Proof.

Let $c^1, c^2, \dots, c^{q^n}$ be a **Gray code** for A^n, where $|A| = q$, so states c^i and c^{i+1} differ in exactly one coordinate.

Main Results

Theorem (Burckel '96; Gadouleau-Riis '15)

Let A be a finite set and $n \geq 2$. Let $\mathcal{I}$ be the set of all instructions of A^n. Then, $\langle \mathcal{I} \rangle = \mathrm{Tran}(A^n)$.

Proof.

Let $c^1, c^2, \ldots, c^{q^n}$ be a **Gray code** for A^n, where $|A| = q$, so states c^i and c^{i+1} differ in exactly one coordinate. Hence, the set of transpositions

$$\mathcal{H} := \{(c^i, c^{i+1}) : 1 \leq i \leq q^n - 1\}$$

is contained in $\mathcal{I}$ and coincides with the set of Coxeter generators for $\mathrm{Sym}(A^n)$.

Main Results

Theorem (Burckel '96; Gadouleau-Riis '15)

Let A be a finite set and $n \geq 2$. Let $\mathcal{I}$ be the set of all instructions of A^n. Then, $\langle \mathcal{I} \rangle = \mathrm{Tran}(A^n)$.

Proof.

Let $c^1, c^2, \dots, c^{q^n}$ be a **Gray code** for A^n, where $|A| = q$, so states c^i and c^{i+1} differ in exactly one coordinate. Hence, the set of transpositions

$$\mathcal{H} := \{(c^i, c^{i+1}) : 1 \leq i \leq q^n - 1\}$$

is contained in $\mathcal{I}$ and coincides with the set of Coxeter generators for $\mathrm{Sym}(A^n)$. Thus, $\mathcal{H}$ together with any instruction of defect 1 generates $\mathrm{Tran}(A^n)$. □

Main Results

Theorem (Gadouleau-Riis '15)

The memoryles complexity of any $g \in \mathrm{Tran}(A^n)$ with respect to the set of all instructions is at most $4n - 3$.

Main Results

Theorem (Gadouleau-Riis '15)

The memoryles complexity of any $g \in \mathrm{Tran}(A^n)$ *with respect to the set of all instructions is at most* $4n - 3$.

Theorem (Cameron-Fairbairn-Gadouleau '14)

1. *Unless* $|A| = n = 2$, $\mathrm{Sym}(A^n)$ *is generated by* n *instructions, and* $\mathrm{Tran}(A^n)$ *is generated by* $n + 1$ *instructions.*

Main Results

Theorem (Gadouleau-Riis '15)

The memoryles complexity of any $g \in \mathrm{Tran}(A^n)$ with respect to the set of all instructions is at most $4n - 3$.

Theorem (Cameron-Fairbairn-Gadouleau '14)

1. *Unless $|A| = n = 2$, $\mathrm{Sym}(A^n)$ is generated by n instructions, and $\mathrm{Tran}(A^n)$ is generated by $n + 1$ instructions.*
2. *If A is a finite field, the group $GL(A^n)$ is generated by n instructions.*

3. Universal Simulation

Motivation

Let A be a finite set of size $q \geq 2$, and let $m \geq 2$.

- We want to study sets $\{F^{(1)}, \dots, F^{(m)}\}$ of instructions of A^m such that $F^{(i)}$ updates the i-th coordinate.

Motivation

Let A be a finite set of size $q \geq 2$, and let $m \geq 2$.

- We want to study sets $\{F^{(1)}, \dots, F^{(m)}\}$ of instructions of A^m such that $F^{(i)}$ updates the i-th coordinate.
- The set $\{F^{(1)}, \dots, F^{(m)}\}$ never generates $\mathrm{Tran}(A^m)$, but is is possible that, for some $2 \leq n \leq m$, every transformation of A^n may be "**simulated**" by these instructions.

Motivation

Let A be a finite set of size $q \geq 2$, and let $m \geq 2$.

- We want to study sets $\{F^{(1)}, \dots, F^{(m)}\}$ of instructions of A^m such that $F^{(i)}$ updates the i-th coordinate.
- The set $\{F^{(1)}, \dots, F^{(m)}\}$ never generates $\mathrm{Tran}(A^m)$, but is is possible that, for some $2 \leq n \leq m$, every transformation of A^n may be "**simulated**" by these instructions.
- We shall formalize this idea and study different schemes of simulation such as **sequential**, **parallel**, and **quasi-parallel**.

Motivation

Let A be a finite set of size $q \geq 2$, and let $m \geq 2$.

- We want to study sets $\{F^{(1)}, \dots, F^{(m)}\}$ of instructions of A^m such that $F^{(i)}$ updates the i-th coordinate.
- The set $\{F^{(1)}, \dots, F^{(m)}\}$ never generates $\mathrm{Tran}(A^m)$, but is is possible that, for some $2 \leq n \leq m$, every transformation of A^n may be "**simulated**" by these instructions.
- We shall formalize this idea and study different schemes of simulation such as **sequential**, **parallel**, and **quasi-parallel**.

Notation

Let $m \geq n \geq 2$.

- For any $f = (f_1, \dots, f_m) \in \mathrm{Tran}(A^m)$, define

$$S_f := \langle F^{(1)}, \dots, F^{(m)} \rangle$$

where $F^{(i)}$ is the instruction of A^m defined by $x_i \leftarrow (x)f_i$.

Notation

Let $m \geq n \geq 2$.

- For any $f = (f_1, \ldots, f_m) \in \mathrm{Tran}(A^m)$, define

  $$S_f := \langle F^{(1)}, \ldots, F^{(m)} \rangle$$

  where $F^{(i)}$ is the instruction of A^m defined by $x_i \leftarrow (x)f_i$.

- Denote $[n] := \{1, \ldots, n\}$.

Notation

Let $m \geq n \geq 2$.

- For any $f = (f_1, \dots, f_m) \in \mathrm{Tran}(A^m)$, define

$$S_f := \langle F^{(1)}, \dots, F^{(m)} \rangle$$

where $F^{(i)}$ is the instruction of A^m defined by $x_i \leftarrow (x)f_i$.

- Denote $[n] := \{1, \dots, n\}$.

- Consider the $[n]$-projection $\mathrm{pr}_{[n]} : A^m \to A^n$, where

$$(x_1, \dots, x_m)\mathrm{pr}_{[n]} := (x_1, \dots, x_n).$$

Universal Transformations

Definition (CR-Gadouleau '15; cf. Dömösi-Nehaniv '05)

Let $m \geq n \geq 2$. A transformation $f \in \mathrm{Tran}(A^m)$ **simulates** $g \in \mathrm{Tran}(A^n)$ if there exists $h \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g = h \circ \mathrm{pr}_{[n]};$$

Universal Transformations

Definition (CR-Gadouleau '15; cf. Dömösi-Nehaniv '05)

Let $m \geq n \geq 2$. A transformation $f \in \mathrm{Tran}(A^m)$ **simulates** $g \in \mathrm{Tran}(A^n)$ if there exists $h \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g = h \circ \mathrm{pr}_{[n]};$$

$$(x_1, \dots, x_n)g = ((x)h_1, \dots, (x)h_n), \quad (\forall x \in A^m).$$

Universal Transformations

Definition (CR-Gadouleau '15; cf. Dömösi-Nehaniv '05)

Let $m \geq n \geq 2$. A transformation $f \in \mathrm{Tran}(A^m)$ **simulates** $g \in \mathrm{Tran}(A^n)$ if there exists $h \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g = h \circ \mathrm{pr}_{[n]};$$

$$(x_1, \dots, x_n)g = ((x)h_1, \dots, (x)h_n), \quad (\forall x \in A^m).$$

The **time of simulation** of g by f is the minimum possible memoryless complexity of h with respect to $\{F^{(1)}, \dots, F^{(m)}\}$.

Universal Transformations

Definition (CR-Gadouleau '15; cf. Dömösi-Nehaniv '05)

Let $m \geq n \geq 2$. A transformation $f \in \mathrm{Tran}(A^m)$ **simulates** $g \in \mathrm{Tran}(A^n)$ if there exists $h \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g = h \circ \mathrm{pr}_{[n]};$$

$$(x_1, \dots, x_n)g = ((x)h_1, \dots, (x)h_n), \quad (\forall x \in A^m).$$

The **time of simulation** of g by f is the minimum possible memoryless complexity of h with respect to $\{F^{(1)}, \dots, F^{(m)}\}$.

An n-**universal transformation of size** m is a transformation of A^m that may simulate any transformation of A^n.

Universal Transformations of Small Size

Theorem (CR-Gadouleau '15)

There is no n-universal transformation of size n, but there exists one of $\mathbf{size\ n+2}$ *and time of simulation* $\mathbf{3(q-1)nq^n + O(q^n)}$.

Universal Transformations of Small Size

Theorem (CR-Gadouleau '15)

There is no n-universal transformation of size n, but there exists one of $\mathbf{size\ n+2}$ *and time of simulation* $\mathbf{3(q-1)nq^n + O(q^n)}$.

Sketch of the Proof.

We find the required set $\{F^{(1)}, \dots, F^{(n+2)}\} \subseteq \mathrm{Tran}(A^{n+2})$:

1. Choose a generating set of instructions $\mathcal{H} \subseteq \mathrm{Tran}(A^n)$ such that for any $i \in [n]$, at most two instructions in $\mathcal{H}$ update i.

Universal Transformations of Small Size

Theorem (CR-Gadouleau '15)

There is no n-universal transformation of size n, but there exists one of **size** $\mathbf{n+2}$ *and time of simulation* $\mathbf{3(q-1)nq^n + O(q^n)}$.

Sketch of the Proof.

We find the required set $\{F^{(1)}, \dots, F^{(n+2)}\} \subseteq \mathrm{Tran}(A^{n+2})$:

1. Choose a generating set of instructions $\mathcal{H} \subseteq \mathrm{Tran}(A^n)$ such that for any $i \in [n]$, at most two instructions in $\mathcal{H}$ update i.
2. If there exist $A, B \in \mathcal{H}$, $A \neq B$, that update $i \in [n]$, let

$$F^{(i)} : x_i \leftarrow \begin{cases} (x)\mathrm{pr}_{[n]} \circ A_i & \text{if } x_{n+1} = x_{n+2} \\ (x)\mathrm{pr}_{[n]} \circ B_i & \text{if } x_{n+1} \neq x_{n+2}. \end{cases}$$

Universal Transformations of Small Size

Sketch of the Proof (continuation).

3. If there is a unique $C \in \mathcal{H}$ that update $i \in [n]$, let $F^{(i)} : x_i \leftarrow (x)\mathrm{pr}_{[n]}\circ_i$.

Universal Transformations of Small Size

Sketch of the Proof (continuation).

3. If there is a unique $C \in \mathcal{H}$ that update $i \in [n]$, let $F^{(i)} : x_i \leftarrow (x)\mathrm{pr}_{[n]}\circ_i$.

4. Let $F^{(n+1)} : x_{n+1} \leftarrow x_{n+2}$ and

$$F^{(n+2)} : x_{n+2} \leftarrow \begin{cases} x_{n+2} + 1 & \text{if } x_{n+1} = x_{n+2} \\ x_{n+2} & \text{if } x_{n+1} \neq x_{n+2}. \end{cases}$$

Universal Transformations of Small Size

Sketch of the Proof (continuation).

3. If there is a unique $C \in \mathcal{H}$ that update $i \in [n]$, let $F^{(i)} : x_i \leftarrow (x)\mathrm{pr}_{[n]}\circ_i$.

4. Let $F^{(n+1)} : x_{n+1} \leftarrow x_{n+2}$ and

$$F^{(n+2)} : x_{n+2} \leftarrow \begin{cases} x_{n+2} + 1 & \text{if } x_{n+1} = x_{n+2} \\ x_{n+2} & \text{if } x_{n+1} \neq x_{n+2}. \end{cases}$$

5. Any $g \in \mathrm{Tran}(A^n)$ has a program in $\mathcal{H}$, so we may use this program to define $h \in S_f$ such that $\mathrm{pr}_{[n]} \circ g = h \circ \mathrm{pr}_{[n]}$.

Universal Transformations of Small Size

Sketch of the Proof (continuation).

3. If there is a unique $C \in \mathcal{H}$ that update $i \in [n]$, let $F^{(i)} : x_i \leftarrow (x)\mathrm{pr}_{[n]} \circ_i$.

4. Let $F^{(n+1)} : x_{n+1} \leftarrow x_{n+2}$ and

$$F^{(n+2)} : x_{n+2} \leftarrow \begin{cases} x_{n+2} + 1 & \text{if } x_{n+1} = x_{n+2} \\ x_{n+2} & \text{if } x_{n+1} \neq x_{n+2}. \end{cases}$$

5. Any $g \in \mathrm{Tran}(A^n)$ has a program in $\mathcal{H}$, so we may use this program to define $h \in S_f$ such that $\mathrm{pr}_{[n]} \circ g = h \circ \mathrm{pr}_{[n]}$.

Question: Is there an n-universal transformation of size $n + 1$?

Fast Universal Transformations

Theorem (CR-Gadouleau '15)

There is an n-universal transformation of A^m with time of simulation at most $\mathbf{q^n + O(n)}$.

Fast Universal Transformations

Theorem (CR-Gadouleau '15)

There is an n-universal transformation of A^m with time of simulation at most $\mathbf{q^n + O(n)}$.

Fast Universal Transformations

Theorem (CR-Gadouleau '15)

There is an n-universal transformation of A^m with time of simulation at most $\mathbf{q^n} + \mathbf{O}(\mathbf{n})$.

Idea of the Proof.

The key idea of the proof is to enumerate all the coordinate functions $A^n \to A$ and use a one-error correcting code to decide which one of them shall be computed in each simulation.

Fast Universal Transformations

Theorem (CR-Gadouleau '15)

There is an n-universal transformation of A^m with time of simulation at most $\mathbf{q^n + O(n)}$.

Idea of the Proof.

The key idea of the proof is to enumerate all the coordinate functions $A^n \to A$ and use a one-error correcting code to decide which one of them shall be computed in each simulation.

Question: Is there an n-universal transformation with maximum time of simulation less than $q^n + O(n)$?

Sequential Simulation

Definition

A transformation $f \in \mathrm{Tran}(A^m)$ **sequentially simulates** a sequence of transformations $g^{(1)}, \dots, g^{(\ell)} \in \mathrm{Tran}(A^n)$ if there are $h^{(1)}, \dots, h^{(\ell)} \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g^{(i)} = h^{(1)} \circ \cdots \circ h^{(i)} \circ \mathrm{pr}_{[n]}, \qquad \forall (1 \leq i \leq \ell).$$

Sequential Simulation

Definition

A transformation $f \in \mathrm{Tran}(A^m)$ **sequentially simulates** a sequence of transformations $g^{(1)}, \ldots, g^{(\ell)} \in \mathrm{Tran}(A^n)$ if there are $h^{(1)}, \ldots, h^{(\ell)} \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g^{(i)} = h^{(1)} \circ \cdots \circ h^{(i)} \circ \mathrm{pr}_{[n]}, \qquad \forall (1 \leq i \leq \ell).$$

An n-universal transformation is **complete** if it may sequentially simulate any finite sequence of transformations of A^n.

Sequential Simulation

Definition

A transformation $f \in \mathrm{Tran}(A^m)$ **sequentially simulates** a sequence of transformations $g^{(1)}, \dots, g^{(\ell)} \in \mathrm{Tran}(A^n)$ if there are $h^{(1)}, \dots, h^{(\ell)} \in S_f \subseteq \mathrm{Tran}(A^m)$ such that

$$\mathrm{pr}_{[n]} \circ g^{(i)} = h^{(1)} \circ \dots \circ h^{(i)} \circ \mathrm{pr}_{[n]}, \qquad \forall (1 \leq i \leq \ell).$$

An n-universal transformation is **complete** if it may sequentially simulate any finite sequence of transformations of A^n.

Lemma

Any complete n-universal transformation has size $m \geq 2n$.

Other Schemes of Simulation

Theorem (CR-Gadouleau '15)

Let A be a finite set and $m \geq n \geq 2$.

1. *There is an n-universal complete transformation of size $2n+3$.*

Other Schemes of Simulation

Theorem (CR-Gadouleau '15)

Let A be a finite set and $m \geq n \geq 2$.

1. *There is an n-universal complete transformation of size $2n+3$.*
2. *There is no transformation $f \in \mathrm{Tran}(A^m)$ that may* **simulate in parallel** *(i.e. with $h \in \langle f \rangle$ instead of $h \in S_f$) every transformation of A^n.*

Other Schemes of Simulation

Theorem (CR-Gadouleau '15)

Let A be a finite set and $m \geq n \geq 2$.

1. *There is an n-universal complete transformation of size $2n+3$.*
2. *There is no transformation $f \in \mathrm{Tran}(A^m)$ that may* **simulate in parallel** *(i.e. with $h \in \langle f \rangle$ instead of $h \in S_f$) every transformation of A^n.*
3. *There is a transformation $f \in \mathrm{Tran}(A^m)$ that may* **simulate in quasi-parallel** *(i.e. with $h \in \left\langle (f_1, \dots, f_{m-1}, \mathrm{pr}_m), F^{(m)} \right\rangle$) every finite sequence of $\mathrm{Tran}(A^n)$.*

Thanks for listening!

Universal Simulation of Automata Networks,
joint with M. Gadouleau
arXiv:1504.00169.