Automatic Model Order Reduction for Hydraulics modeling

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1. Introduction

Managed Pressure Drilling (MPD) is used for drilling operations to guarantee the control of the down-hole pressure. To support the design of MPD strategies and to assess their performance in operational conditions, a hydraulic model accurately predicting such effects is essential. On the other hand, such model should be simple enough to perform analyses supporting real time drilling decision making, real time estimation and control of the pressure.

2. Hydraulic Model for One-Phase Flow

Isothermal Euler equations are usually applied for one-phase flow simulation:

$$\left.\begin{array}{l}\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0\\ \frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho v^2+p)}{\partial x}=-\rho g\,\sin\theta-\frac{32\mu v}{d^2}\end{array}\right\}\text{ and }\boxed{p=p_0+a_l^2(\rho-\rho_0)}\qquad(1)$$

with ρ, v, p, a_l as density, velocity, pressure of the fluid and sound velocity in the fluid and ρ_0, p_0 as density and pressure reference. Moreover, θ, μ, d and g are pipe inclination, viscosity of the fluid and diameter of the pipe, acceleration due to gravity respectively.

A Finite Volume Method (**FVM**) is generally used to discretize the hyperbolic systems. Nevertheless, the computational effort required by this technique is beyond the limit for having a real time simulation and it does not lead to models simple enough for estimation and control.

3. Challenges for Model Order Reduction (MOR)

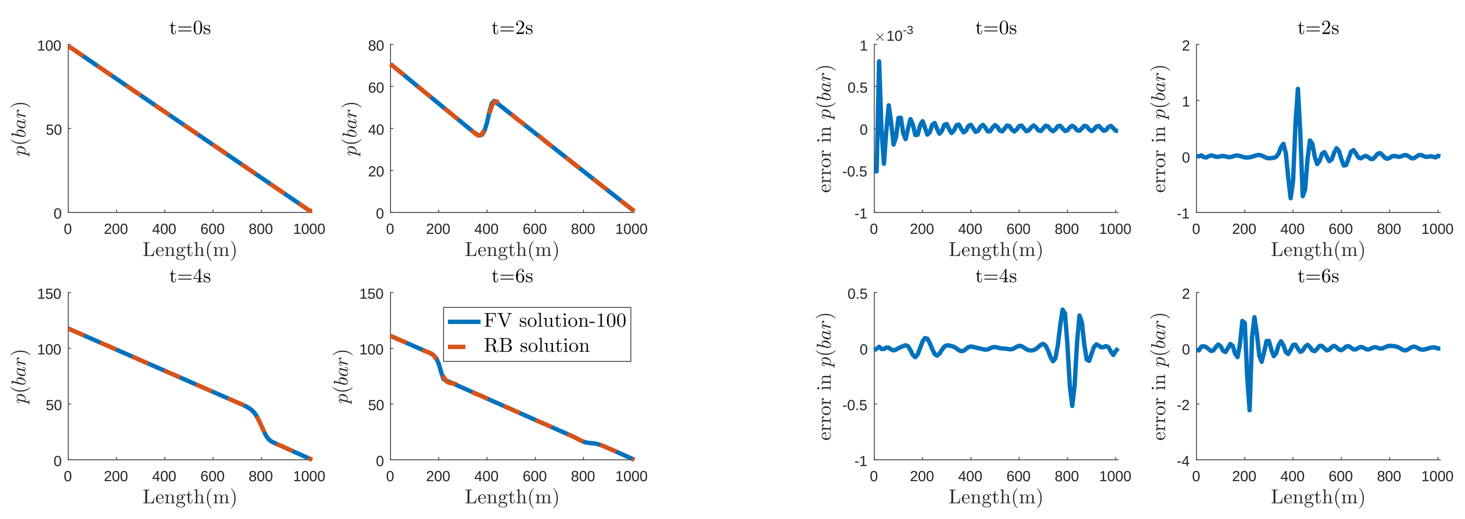
- Robust and accurate resolution of **multiple time scales**;
- Effective resolution of **sharp gradients** and **moving discontinuities**;
- **Large Kolmogorov N-width** (representative of convection dominated phenomena);
- Model-based distributed non-linearities (due to compressibility, equations of state, etc.) and discretization-based non-linearities (flux limiters, mixed sound velocity, etc.);
- High sensitivity on varying parameter domain, initial and boundary conditions;
- Solutions to class of hyperbolic PDEs that govern transport dominated phenomena are reflected by **diagonal structure in space-time**;
- **Implicit, highly nonlinear and non-Lipschitz boundary conditions**;
- Extremely conservative error bound for Reduced Basis method.

4. Undertaken Steps for Linear MOR with Localized Nonlinearities

Applying any classical FV scheme on Equation (1) leads to the following matrix equations followed by the Reduced Basis (RB) analysis [4]:

$$\left\{\begin{array}{l}\mathbf{U}^0(\bar\mu)=u_0(x;\bar\mu)\\ \mathbf{U}^{k+1}(\bar\mu)=L_E^k[\mathbf{U}^k(\bar\mu)]+b^k(\bar\mu)\end{array}\right.\xrightarrow{RB\ approach}\left\{\begin{array}{l}\mathbf{a}^0=[\int_\Omega u_0(x;\bar\mu)\varphi_1,\dots,\int_\Omega u_0(x;\bar\mu)\varphi_N]^T\\ \mathbf{a}^{k+1}=\mathbf{L}_E^k(\bar\mu)\mathbf{a}^k+\mathbf{b}^k\\ \mathbf{U}_N^k(x;\bar\mu)=\sum_{n=1}^{N}a_n^k(\bar\mu)\varphi_n(x)\end{array}\right.\qquad(2)$$

where $U=(\rho,\rho v)^T$ is the outcome of FV solution after imposing initial condition u_0, and $\mathbf{U}_N$ is the result of RB analysis,L_E, b are FVM-dependent operators with similar RB counterparts displayed by $\mathbf{L}_E, \mathbf{b}$; $\mathbf{a}, \varphi_n$ are RB coefficients and basis function, and finally $\bar\mu$ are the varying parameters which are sound velocity, viscosity and reference density of the fluid, pipe inclination, length and diameter, and pump flow rate. Comparison of FV with 100 control volumes and RB with 40 basis functions is presented in Figure 1.



(a) pressure evaluation comparison (b) pressure error along the pipe

Figure 1: *Finite Volume and Reduced Basis comparison.*

Decomposing the non-linear ODE in a large linear part and a local non-linear part [6] yields:

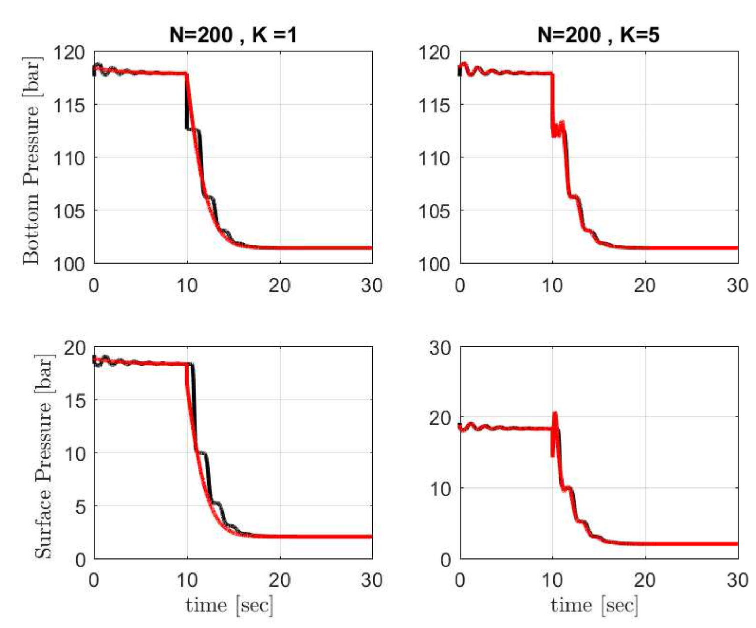


Figure 2: *Results after decomposing the model in a linear and nonlinear part. N is the order of the original system and K is the order of the reduced system.*

5. Envisaged Approaches for Non-Linear MOR

Spatial discretization of time dependent PDEs results in the system of the non-linear ODEs of the form:

$$\left\{\begin{array}{l}\frac{d}{dt}\mathbf{y}(t,\mu)=\mathbf{A}\mathbf{y}(t,\mu)+\mathbf{F}(\mathbf{y}(t,\mu));\mathbf{y}(0)=\mathbf{y}_0;\mathbf{y}(t,\mu)\in\mathbb{R}^n;\mathbf{A}\in\mathbb{R}^{n*n};\mathbf{F}:\mathbb{R}^n\to\mathbb{R}^n\\ \frac{d}{dt}\tilde{\mathbf{y}}(t,\mu)=\mathbf{V}^T\mathbf{A}\mathbf{V}\tilde{\mathbf{y}}(t,\mu)+\mathbf{V}^T\mathbf{F}(\mathbf{V}\tilde{\mathbf{y}}(t,\mu));\tilde{\mathbf{y}}(t)\in\mathbb{R}^r\\ \tilde{\mathbf{N}}(\tilde{\mathbf{y}}):=\underbrace{\mathbf{V}^T}_{r*n}\underbrace{\mathbf{F}(\mathbf{V}\tilde{\mathbf{y}}(t,\mu))}_{n*1}\end{array}\right.\qquad(3)$$

1. Proper Orthogonal Decomposition (**POD**); but it requires a prolongation of the reduced state variables back to the high dimensional state space to evaluate the non-linear terms;
2. Empirical Interpolation Method (**EIM**) [2] and POD - Discrete Empirical Interpolation Method (**DEIM**) [1] are envisioned for non-linear MOR.

Hybrid MOR Proposition :
The localized hybrid MOR relies on underlying hyperbolic dynamics to extract the **transport structure** [5]. This transport structure is used for **localized basis construction** [3] and also envisioned to furnish transformation operator to **transform diagonal structure to rectangular structure in space-time**. This would contribute to obtain lowest dimensional representation.

6. Numerical Results for non-linear MOR

Motivation to model-reduce the Burger's Equation:

$$\left\{\begin{array}{l}y_t+\left(\frac{1}{2}y^2-\mu y_x\right)_x=f,\ (x,t)\in(0,L)*(0,T),\\ y(t,0)=y(t,L)=0,\ t\in(0,T),\\ y(0,x)=y_0(x),\ x\in(0,L).\end{array}\right.\qquad(4)$$

- Simple analog of the Euler equations for fluid flow and mimics nonlinear wave equation where each point on the wave-front can propagate with different speed;
- Possess coalescence of characteristics and formation of discontinuous solutions similar to shock waves/ rarefaction waves in DFM/ general fluid mechanics problem.

Applying POD-DEIM approach to viscous Burger's equation yields:

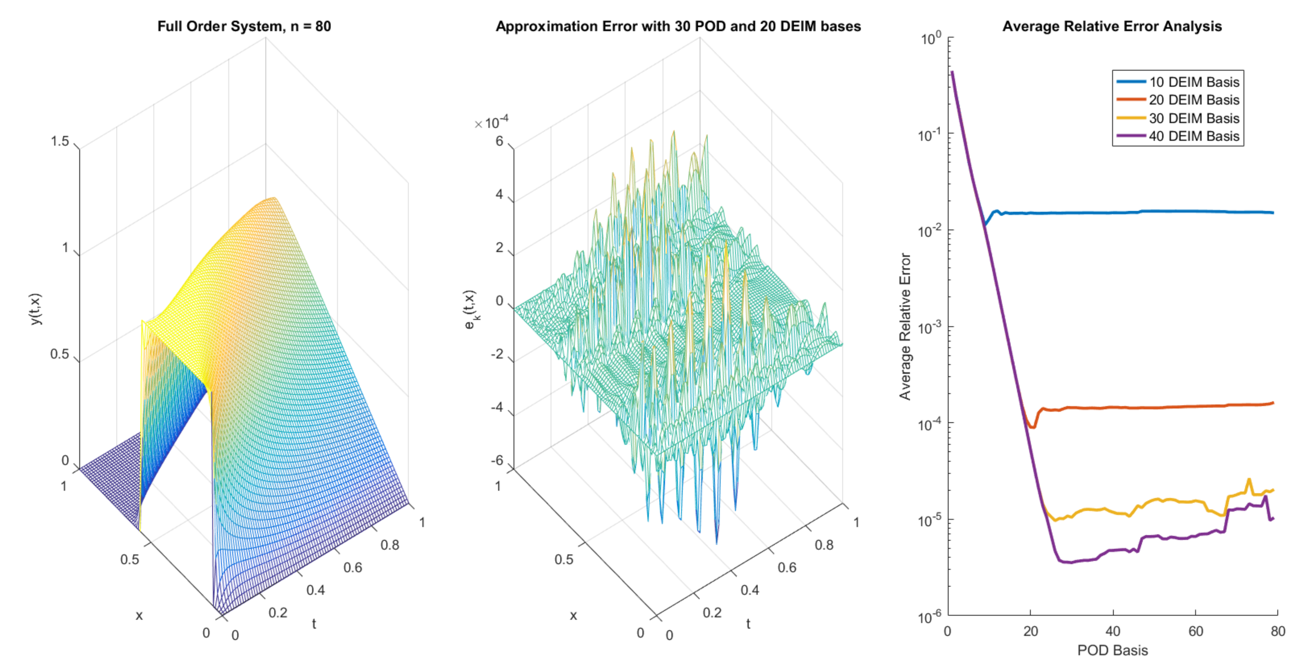


Figure 3: *POD-DEIM results for viscous Burger's Equation at μ = 0.01.*

7. Future Plan

Since most of complexities in the well flow occur in the presence of a mixture of gas and liquid, a model should be found to represent this phenomena. For multi-phase flow modelling, the Drift Flux Model (DFM) consisting of a system of non-linear Hyperbolic Partial Differential Equations (PDEs) has been employed.

$$\begin{pmatrix}\rho_l\alpha_l\\ \rho_g\alpha_g\\ \rho_l\alpha_l v_l+\rho_g\alpha_g v_g\end{pmatrix}_t+\begin{pmatrix}\rho_l\alpha_l v_l\\ \rho_g\alpha_g v_g\\ \rho_l\alpha_l v_l^2+\rho_g\alpha_g v_g^2+P\end{pmatrix}_x=\begin{pmatrix}0\\ 0\\ Q_g+Q_v\end{pmatrix}\text{ and }\left\{\begin{array}{l}\alpha_g+\alpha_l=1\\ \rho_g=p/a_g^2\\ \rho_l=\rho_0+(p-p_0)/a_l^2\\ v_g=\frac{(Kv_l\alpha_l+S)}{(1-K\alpha_g)}\end{array}\right.\qquad(5)$$

where $Q_g=-g(\alpha_l\rho_l+\alpha_g\rho_g)sin\theta$ and $Q_v=-\frac{32\mu(\alpha_l v_l+\alpha_g v_g)}{d^2}$, α is the phase volume fraction with subscripts l, g referring to liquid and gas phases and K and S are flow dependent parameters. The future plan is to perform MOR on DFM in the context of many query simulations and real time estimation and control.

8. References

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