1. Introduction

Managed Pressure Drilling (MPD) is used for drilling operations to guarantee the control of the downhole pressure. To support the design of MPD strategies and to assess their performance in operational conditions, a hydraulic model accurately predicting such effects is essential. On the other hand, such models should be simple enough to perform analyses supporting real-time drilling decision making, real-time estimation and control of the pressure.

2. Hydraulic Model for One-Phase Flow

Isothermal Euler equations are usually applied for one-phase flow simulation:

\[\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\nabla p + \frac{\partial \mathbf{g}}{\partial x} + \frac{1}{\rho} \nabla \cdot \mathbf{T}
\end{align*}\]

with \(\rho\) as density, \(p\) as pressure, \(\mathbf{v}\) as velocity, \(\mathbf{g}\) as gravity, \(\mathbf{T}\) as stress tensor, and \(\nabla\) as gradient operator.

3. Challenges for Model Order Reduction (MOR)

- Robust and accurate resolution of multiple time scales;
- Effective resolution of sharp gradients and moving discontinuities;
- Large Kolmogorov N-width (representative of convective dominated phenomena);
- Model-based distributed non-linearities (due to compressibility, equations of state, etc.) and discretization-based non-linearities (flux limiters, mixed sound velocity, etc.);
- High sensitivity on varying parameter domain, initial and boundary conditions; and discretization-based non-linearities (flux limiters, mixed sound velocity, etc.);
- Solutions to class of hyperbolic PDEs that govern transport dominated phenomena are reflected by diagonal structure in space-time;
- Implicit, highly nonlinear and non-Lipschitz boundary conditions;
- Extremely conservative error bound for Reduced Basis method.

4. Undertaken Steps for Linear MOR with Localized Nonlinearities

Applying any classical FV scheme on Equation (1) leads to the following matrix equations followed by the Reduced Basis (RB) analysis (4):

\[\begin{align*}
U^{(0)}(\bar{x},\bar{\mu}) &= u_0(\bar{x},\bar{\mu}) \\
U^{(1)}(\bar{x},\bar{\mu}) &= \bar{L}_R U^{(0)}(\bar{x},\bar{\mu}) + V(\bar{x},\bar{\mu})
\end{align*}\]

where \(U^{(0)}(\bar{x},\bar{\mu})\) is the outcome of FV solution after imposing initial condition \(u_0\) and \(U^{(1)}\) is the result of RB analysis, \(L_R\) and \(V\) are FVM-dependent operators with similar RB counterparts, \(u_0\) are RB coefficients and basis function, and finally \(\bar{\mu}\) are the varying parameters which are sound velocity, viscosity and reference density of the fluid, pipe inclination, length and diameter, and pump flow rate. Comparison of FV with 100 control volumes and RB with 40 basis functions is presented in Figure 1.

Figure 1: Finite Volume and Reduced Basis comparison.

Decomposing the non-linear ODE in a large linear part and a local non-linear part \(f\) yields:

\[\begin{align*}
\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) &= -\nabla p + \frac{\partial \mathbf{g}}{\partial x} + \frac{1}{\rho} \nabla \cdot \mathbf{T} \\
\partial_t \rho &= \rho_f \left( \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla \cdot \mathbf{T} \right)
\end{align*}\]

Spatial discretization of time dependent PDEs results in the system of the non-linear ODEs of the form:

\[\begin{align*}
\frac{d}{dt} \begin{pmatrix} \rho \mathbf{v} \end{pmatrix} &= \begin{pmatrix} F_1(\rho) \mathbf{v} \end{pmatrix} + \begin{pmatrix} F_2(\rho) \mathbf{v} \end{pmatrix} \\
\frac{d}{dt} \begin{pmatrix} \rho \end{pmatrix} &= \rho_f \left( \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla \cdot \mathbf{T} \right)
\end{align*}\]

5. Envisaged Approaches for Non-Linear MOR

6. Numerical Results for non-linear MOR

7. Future Plan

8. References