When is Coalescing as fast as Meeting?

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Outline

Introduction

Interlude: Complete Graph

Relating Coalescing-Time to the Mixing and Meeting Time

Conclusion
Random Walk Notation

- **$P$ transition matrix** of a lazy walk on an undirected, connected graph $G$

  
  $p_{u,v} = \begin{cases} 
  \frac{1}{2} & \text{if } u = v, \\
  \frac{1}{2\deg(u)} & \text{if } \{u, v\} \in E(G), \\
  0 & \text{otherwise.} 
  \end{cases}$

- **$\pi$** with $\pi_v = \frac{\deg(v)}{2|E|}$ is the **stationary distribution**
Random Walk Notation

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**Fundamental Quantities**

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Random Walk Notation

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**Fundamental Quantities**

- **mixing time**: $t_{\text{mix}}(\frac{1}{e}) = \min\{ t \in \mathbb{N}: \forall u \in V: \frac{1}{2} \sum_{v \in V} |p^t_{u,v} - \pi_v| \leq \frac{1}{e} \}$
### Random Walk Notation

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**Fundamental Quantities**

- **mixing time**: $t_{\text{mix}}(\frac{1}{\epsilon}) = \min\{t \in \mathbb{N} : \forall u \in V : \frac{1}{2} \sum_{v \in V} |p_{u,v}^t - \pi_v| \leq \frac{1}{\epsilon}\}$

- **(maximum) hitting time**: $t_{\text{hit}} = \max_{u, v \in V} \mathbb{E}_u \left[ \min\{t : X_t = v\} \right]$
Random Walk Notation

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**Fundamental Quantities**

- **mixing time**: $t_{\text{mix}}\left(\frac{1}{e}\right) = \min\left\{ t \in \mathbb{N} : \forall u \in V: \frac{1}{2} \sum_{v \in V} \left| p_t^{u,v} - \pi_v \right| \leq \frac{1}{e} \right\}$

- (maximum) hitting time: $t_{\text{hit}} = \max_{u,v \in V} E_u \left[ \min \left\{ t : X_t = v \right\} \right]$  

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**Focus of this talk**

- **meeting time**: $t_{\text{meet}} = \max_{u,v \in V} E_{u,v} \left[ \min \left\{ t : X_t = Y_t \right\} \right]$  

- **coalescing time**: $t_{\text{coal}} = E_{1,2,\ldots,n} \left[ \ldots \right]$
Coalescing Random Walks (Example)

Time: 0
Particles: 16
Coalescing Random Walks (Example)

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Particles: 10
Coalescing Random Walks (Example)

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Particles: 10
Coalescing Random Walks (Example)

Time: 0
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Particles: 12

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Particles: 12

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Particles: 10

Time: 2.75
Particles: 10

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Particles: 7

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Particles: 7

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Particles: 7

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Particles: 6

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

Time: 0
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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Particles: 4

Time: 7.5
Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

Time: 9.5
Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

Time: 12
Particles: 2

Time: 12.5
Particles: 2

Time: 13
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Particles: 2

Time: 14
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Time: 15
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Time: 16
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Time: 21
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Time: 22
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Particles: 2

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Particles: 2

Time: 24
Particles: 2

Time: 24.5
Particles: 2

Time: 25
Particles: 1

Time: 25.5
Particles: 1

Time: 26
Particles: 1

Time: 26.5
Particles: 1

Time: 27
Particles: 1

Time: 27.5
Particles: 1

Time: 28
Particles: 1

Time: 10
Particles: 3
Coalescing Random Walks (Example)

Time: 10.5
Particles: 3
Coalescing Random Walks (Example)

Time: 0
Particles: 16

Time: 0.25
Particles: 16

Time: 0.5
Particles: 16

Time: 0.75
Particles: 16

Time: 1
Particles: 12

Time: 1.25
Particles: 12

Time: 1.5
Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

Time: 2.25
Particles: 10

Time: 2.5
Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

Time: 3.25
Particles: 7

Time: 3.5
Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

Time: 4.25
Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

Time: 5.5
Particles: 6

Time: 5.75
Particles: 6

Time: 6
Particles: 5

Time: 6.5
Particles: 5

Time: 7
Particles: 4

Time: 7.5
Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

Time: 9.5
Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

Time: 12
Particles: 2

Time: 12.5
Particles: 2

Time: 13
Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

Time: 24
Particles: 2

Time: 24.5
Particles: 2

Time: 25
Particles: 1

Time: 25.5
Particles: 1

Time: 26
Particles: 1

Time: 26.5
Particles: 1

Time: 27
Particles: 1

Time: 27.5
Particles: 1

Time: 28
Particles: 1
Coalescing Random Walks (Example)

Time: 0
Particles: 16

Time: 0.25
Particles: 16

Time: 0.5
Particles: 16

Time: 0.75
Particles: 16

Time: 1
Particles: 12

Time: 1.25
Particles: 12

Time: 1.5
Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

Time: 2.25
Particles: 10

Time: 2.5
Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

Time: 3.25
Particles: 7

Time: 3.5
Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

Time: 4.25
Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

Time: 5.5
Particles: 6

Time: 5.75
Particles: 6

Time: 6
Particles: 5

Time: 6.5
Particles: 5

Time: 7
Particles: 4

Time: 7.5
Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

Time: 9.5
Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

Time: 12
Particles: 2

Time: 12.5
Particles: 2

Time: 13
Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

Time: 24.5
Particles: 2

Time: 25
Particles: 1

Time: 25
Particles: 1

Time: 25
Particles: 1

Time: 25.5
Particles: 1

Time: 26
Particles: 1

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Particles: 1

Time: 28
Particles: 1
Coalescing Random Walks (Example)

Time: 0
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Particles: 16

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Particles: 16

Time: 0.75
Particles: 16

Time: 1
Particles: 12

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Particles: 12

Time: 1.5
Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

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Particles: 10

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Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

Time: 3.25
Particles: 7

Time: 3.5
Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

Time: 4.25
Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

Time: 5.5
Particles: 6

Time: 5.75
Particles: 6

Time: 6
Particles: 5

Time: 6.5
Particles: 5

Time: 7
Particles: 4

Time: 7.5
Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

Time: 9.5
Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

Time: 12
Particles: 2

Time: 12.5
Particles: 2

Time: 13
Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

Time: 24.5
Particles: 2

Time: 25
Particles: 1

Time: 25.5
Particles: 1

Time: 26
Particles: 1

Time: 26.5
Particles: 1

Time: 27
Particles: 1

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Particles: 1

Time: 28
Particles: 1

Particles: 2
Coalescing Random Walks (Example)

Time: 0
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Time: 0.25
Particles: 16

Time: 0.5
Particles: 16

Time: 0.75
Particles: 16

Time: 1
Particles: 12

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Particles: 12

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Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

Time: 2.25
Particles: 10

Time: 2.5
Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

Time: 3.25
Particles: 7

Time: 3.5
Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

Time: 4.25
Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

Time: 5.5
Particles: 6

Time: 5.75
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Particles: 5

Time: 7
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Particles: 4

Time: 8
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Particles: 3

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Particles: 3

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Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

Time: 12
Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 2

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Particles: 1

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Particles: 1

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Particles: 1
Coalescing Random Walks (Example)

Time: 0
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Particles: 16

Time: 1
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Particles: 12

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Particles: 12

Time: 2
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Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

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Particles: 7

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Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

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Particles: 6

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Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

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Particles: 6

Time: 5.75
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Time: 6
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Time: 6.5
Particles: 5

Time: 7
Particles: 4

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Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

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Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

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Particles: 3

Time: 12
Particles: 2

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Particles: 2

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Particles: 2

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Coalescing Random Walks (Example)

Time: 0
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Particles: 16

Time: 0.75
Particles: 16

Time: 1
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Particles: 12

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Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

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Particles: 10

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Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

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Particles: 7

Time: 3.5
Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

Time: 4.25
Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

Time: 5
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Time: 6.5
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Time: 7
Particles: 4

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Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

Time: 9.5
Particles: 3

Time: 10
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Particles: 3

Time: 11
Particles: 3

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Particles: 3

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Particles: 2

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Time: 24.5
Particles: 2

Time: 25
Particles: 1

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Particles: 1

Time: 26.5
Particles: 1

Time: 27
Particles: 1
Coalescing Random Walks (Example)

Time: 0
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Particles: 6

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Particles: 4

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Time: 11
Particles: 3

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Particles: 3

Time: 12
Particles: 2

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Particles: 2

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Coalescing Random Walks (Example)

Time: 0
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Time: 1
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Particles: 10

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Particles: 7

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Particles: 7

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Particles: 7

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Particles: 7

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Particles: 6

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Particles: 6

Time: 4.75
Particles: 6

Time: 5
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Particles: 3

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Particles: 3

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Time: 24
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Time: 24.5
Particles: 2

Time: 25
Particles: 1

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Coalescing Random Walks (Example)
Coalescing Random Walks (Example)

Time: 0
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Particles: 12

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Particles: 10

Time: 2.75
Particles: 10

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Particles: 7

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Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

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Particles: 6

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Particles: 6

Time: 4.75
Particles: 6

Time: 5
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Particles: 4

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Particles: 3

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Particles: 3

Time: 9
Particles: 3

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Particles: 3

Time: 10
Particles: 3

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Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

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Particles: 2

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Particles: 2

Time: 23
Particles: 2

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Particles: 2

Time: 24
Particles: 2

Time: 24.5
Particles: 2

Time: 25
Particles: 1

Time: 25
Particles: 1

Time: 25
Particles: 1

Time: 25.5
Particles: 1

Time: 26
Particles: 1

Time: 26.5
Particles: 1

Time: 27
Particles: 1

Time: 27.5
Particles: 1

Time: 28
Particles: 1

Particles: 2
Coalescing Random Walks (Example)

Time: 0
Particles: 16

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Particles: 16

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Particles: 16

Time: 0.75
Particles: 16

Time: 1
Particles: 12

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Particles: 12

Time: 1.5
Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

Time: 2.25
Particles: 10

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Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

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Particles: 7

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Particles: 7

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Particles: 7

Time: 4
Particles: 6

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Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

Time: 5.5
Particles: 6

Time: 5.75
Particles: 6

Time: 6
Particles: 5

Time: 6.5
Particles: 5

Time: 7
Particles: 4

Time: 7.5
Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

Time: 9.5
Particles: 3

Time: 10
Particles: 3

Time: 10.5
Particles: 3

Time: 11
Particles: 3

Time: 11.5
Particles: 3

Time: 12
Particles: 2

Time: 12.5
Particles: 2

Time: 13
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Time: 14
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Particles: 2

Time: 15
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Particles: 2

Time: 16
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Particles: 2

Time: 25
Particles: 1

Time: 25.5
Particles: 1

Time: 26
Particles: 1

Time: 26.5
Particles: 1

Time: 27
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Time: 27.5
Particles: 1

Time: 28
Particles: 1

Particles: 2
Coalescing Random Walks (Example)

Time: 0
Particles: 16

Time: 0.25
Particles: 16

Time: 0.5
Particles: 16

Time: 0.75
Particles: 16

Time: 1
Particles: 12

Time: 1.25
Particles: 12

Time: 1.5
Particles: 12

Time: 1.75
Particles: 12

Time: 2
Particles: 10

Time: 2.25
Particles: 10

Time: 2.5
Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

Time: 3.25
Particles: 7

Time: 3.5
Particles: 7

Time: 3.75
Particles: 7

Time: 4
Particles: 6

Time: 4.25
Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

Time: 5
Particles: 6

Time: 5.25
Particles: 6

Time: 5.5
Particles: 6

Time: 5.75
Particles: 6

Time: 6
Particles: 5

Time: 6.5
Particles: 5

Time: 7
Particles: 4

Time: 7.5
Particles: 4

Time: 8
Particles: 3

Time: 8.5
Particles: 3

Time: 9
Particles: 3

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Time: 10
Particles: 3

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Particles: 3

Time: 11
Particles: 3

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Particles: 3

Time: 12
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Particles: 2
Coalescing Random Walks (Example)

Time: 0
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Particles: 16

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Time: 0.75
Particles: 16

Time: 1
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Particles: 12

Time: 1.75
Particles: 12

Time: 2
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Particles: 10

Time: 2.75
Particles: 10

Time: 3
Particles: 7

Time: 3.25
Particles: 7

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Particles: 7

Time: 3.75
Particles: 7

Time: 4
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Particles: 6

Time: 4.5
Particles: 6

Time: 4.75
Particles: 6

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Particles: 3

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Coalescing Random Walks (Example)
Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

Time: 0
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Particles: 2
Coalescing Random Walks (Example)

Time: 0
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Particles: 7

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Particles: 7

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Particles: 6

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Particles: 3

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Particles: 3

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Time: 25
Particles: 1

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Particles: 1

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Particles: 1

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Particles: 1

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Particles: 2
Coalescing Random Walks (Example)

Time: 0
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Particles: 16

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Particles: 16

Time: 1
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Particles: 12

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Particles: 10

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Particles: 7

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Particles: 7

Time: 3.75
Particles: 7

Time: 4
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Time: 4.25
Particles: 6

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Particles: 6

Time: 4.75
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Particles: 1

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Particles: 2
Coalescing Random Walks (Example)

Time: 0
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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Time: 3.75
Particles: 7

Time: 4
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Time: 4.25
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Particles: 6

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Time: 21.5
Coalescing Random Walks (Example)
Coalescing Random Walks (Example)

Time: 0
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Particles: 10

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Particles: 7

Time: 3.75
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Time: 4.75
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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

Time: 0
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Coalescing Random Walks (Example)
Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

Time: 0
Particles: 16

Time: 0.25
Particles: 16

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Time: 0.75
Particles: 16

Time: 1
Particles: 12

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Particles: 12

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Particles: 12

Time: 2
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Coalescing Random Walks (Example)

Time: 0
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Coalescing Random Walks (Example)

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Coalescing Random Walks (Example)

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Time: 28
Particles: 1
Motivation: Voter Model

- Given a graph $G = (V, E)$ with $n$ nodes, each with a different opinion.
- At each round, each node "pulls" w.p. 1/2 the opinion of a random neighbor, otherwise keeps his current opinion.
Motivation: Voter Model

Voter Model

- Given a graph $G = (V, E)$ with $n$ nodes, each with a different opinion
- At each round, each node "pulls" w.p. 1/2 the opinion of a random neighbor, otherwise keeps his current opinion.

Duality

Time to reach consensus = Time for $n$ coalescing particles to merge.
Motivation: Voter Model

Voter Model

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Duality

Time to reach consensus = Time for $n$ coalescing particles to merge.
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**Voter Model**

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**Duality**

Time to reach consensus = Time for $n$ coalescing particles to merge.
Some Related Work and the Agenda of this Talk

For the discrete-time variant:

\[\frac{\text{coal}}{\text{uni}} \leq t \cdot \log n\] [Hassin, Peleg, DIST'01]

\[\frac{1}{1 - \lambda^2} \cdot \frac{1}{\log 4n + 1} \] [Cooper, Elsässer, Ono and Radzik, SIAM J. Discrete Math.'13]

\[\frac{\Phi}{\delta}, \text{ where } \delta \text{ is the minimum degree}\] [Berenbrink, Giakkoupis, Kermarrec and Mallmann-Trenn, ICALP'16]

For the continuous-time variant:

\[\frac{\text{coal}}{\text{uni}} \leq t \cdot \text{hit}\] [Oliveira, TAMS'12]

(simplified) For graphs with \(\frac{\text{mix}}{\text{uni}} n\), \(\frac{\text{coal}}{\text{uni}} \) behaves like on a clique [Oliveira, Ann. Prob.'12]

For many graphs, \(\frac{\text{coal}}{\text{uni}} \) can be \(t \cdot \text{meet}\) or even \(\frac{\text{coal}}{\text{uni}} n\) (if \(G\) is regular)

Under the premise that \(\text{mix}\) and \(\text{meet}\) are “simpler” quantities, when does \(\frac{\text{coal}}{\text{uni}}\) hold?
Some Related Work and the Agenda of this Talk

For the discrete-time variant:

- For any graph, $t_{\text{coal}} \preceq t_{\text{meet}} \cdot \log n$  

  [Hassin, Peleg, DIST'01]
Some Related Work and the Agenda of this Talk

For the discrete-time variant:

- For any graph, \( t_{\text{coal}} \leq t_{\text{meet}} \cdot \log n \) \[\text{[Hassin, Peleg, DIST'01]}\]
- For any graph, \( t_{\text{coal}} \leq \frac{1}{1-\lambda_2} \cdot \left( \log^4 n + \frac{1}{\|\pi\|^2} \right) \) \[\text{[Cooper, Elsässer, Ono and Radzik, SIAM J. Discrete Math.'13]}\]
- For any graph \( t_{\text{coal}} \leq \frac{1}{\Phi} \cdot \frac{|E|}{\delta} \), where \( \delta \) is the minimum degree \[\text{[Berenbrink, Giakkoupis, Kermarrec and Mallmann-Trenn, ICALP’16]}\]

For the continuous-time variant:

\( t_{\text{coal}} \) behaves like on a clique \[\text{[Oliveira, TAMS'12]}\]

For many graphs, \( t_{\text{coal}} \) behaves like on a clique or even \( t_{\text{coal}} \approx n \) (if \( G \) is regular)
Some Related Work and the Agenda of this Talk

For the discrete-time variant:

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  \[ \text{[Hassin, Peleg, DIST'01]} \]

- For any graph, \( t_{\text{coal}} \preceq \frac{1}{1-\lambda_2} \cdot \left( \log^4 n + \frac{1}{\| \pi \|^2} \right) \)  
  \[ \text{[Cooper, Elsässer, Ono and Radzik, SIAM J. Discrete Math.'13]} \]

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- (simplified) For graphs with \( t_{\text{mix}} \ll n \), \( t_{\text{coal}} \) behaves like on a clique  
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For the discrete-time variant:

- For any graph, $t_{\text{coal}} \preceq t_{\text{meet}} \cdot \log n$ \hfill [Hassin, Peleg, DIST'01]
- For any graph, $t_{\text{coal}} \preceq \frac{1}{1-\lambda_2} \cdot \left( \log^4 n + \frac{1}{\|\pi\|^2_2} \right)$ \hfill [Cooper, Elsässer, Ono and Radzik, SIAM J. Discrete Math.'13]
- For any graph $t_{\text{coal}} \preceq \frac{1}{\phi} \cdot \frac{|E|}{\delta}$, where $\delta$ is the minimum degree \hfill [Berenbrink, Giakkoupis, Kermarrec and Mallmann-Trenn, ICALP’16]

For the continuous-time variant:

- For any graph, $t_{\text{coal}} \preceq t_{\text{hit}}$ \hfill [Oliveira, TAMS’12]
- (simplified) For graphs with $t_{\text{mix}} \ll n$, $t_{\text{coal}}$ behaves like on a clique \hfill [Oliveira, Ann. Prob.’12]

- For many graphs, $t_{\text{coal}} \asymp t_{\text{meet}}$ or even $t_{\text{coal}} \asymp n$ (if $G$ is regular)
Some Related Work and the Agenda of this Talk

For the discrete-time variant:

- For any graph, $t_{coal} \preceq t_{meet} \cdot \log n$  
  \[ \text{[Hassin, Peleg, DIST'01]} \]
- For any graph, $t_{coal} \preceq \frac{1}{1-\lambda_2} \cdot \left( \log^4 n + \frac{1}{\|\pi\|_2^2} \right)$  
  \[ \text{[Cooper, Elsässer, Ono and Radzik, SIAM J. Discrete Math.'13]} \]
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- For any graph, $t_{coal} \preceq t_{hit}$  
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- (simplified) For graphs with $t_{mix} \ll n$, $t_{coal}$ behaves like on a clique  
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- For many graphs, $t_{coal} \asymp t_{meet}$ or even $t_{coal} \asymp n$ (if $G$ is regular)
- Under the premise that $t_{mix}$ and $t_{meet}$ are “simpler” quantities, when does $t_{coal} \asymp t_{meet}$ hold?
Outline

Introduction

Interlude: Complete Graph

Relating Coalescing-Time to the Mixing and Meeting Time

Conclusion
For the continuous-time variant:

Waiting times are i.i.d. exponentials with mean 1.

Suppose we are left with $k$ random walks. The waiting time until the next walk moves

$$\sim \text{Exp}(k)$$

and then walk hits one of the others with probability

$$\left(1 - \frac{1}{n}\right)\frac{k - 1}{n - 1}$$

Time until $k - 1$ walks left is an exponential with mean:

$$\frac{1}{k} \cdot \frac{n - 1}{k - 1} = \frac{1}{2} \cdot \frac{n - 1}{k - 1}.$$

Since

$$\sum_{k=2}^{\infty} \frac{1}{k^2} = 2,$$

the expected coalescence time is

$$\frac{n}{\sum_{k=2}^{\infty} \frac{1}{k^2}} = \frac{n}{2} \cdot \frac{n - 1}{k - 1} = \frac{1}{2} \cdot \frac{n - 1}{k - 1} = \frac{1}{2} \cdot \frac{n - 1}{k - 1}.$$

For the discrete-time variant:

Answer "should be"

$$8 + o(1) \cdot n$$

for lazy random walks (loop probability $1/2$).
Warm-Up: Complete Graph

Waiting times are i.i.d. exponentials with mean 1.

For the continuous-time variant:

Suppose we are left with $k$ random walks. The waiting time until the next walk moves $\sim \text{Exp}(k)$, and then the walk hits one of the others with probability $\frac{k-1}{n-1}$. The time until $k-1$ walks are left is an exponential with mean:

$$\frac{1}{k} \cdot \frac{n-1}{k-1} = \frac{1}{2} \cdot \frac{n-1}{k^2}.$$ 

Since $\sum_{k=2}^{\infty} \frac{1}{k^2} = 2$, the expected coalescence time is $\frac{n}{\sum_{k=2}^{\infty} \frac{1}{k^2}} = \frac{1}{2} \cdot \frac{n-1}{k^2}. \quad \text{(1)}$

For the discrete-time variant:

Answer "should be" \(8 + o(1)\) for lazy random walks (loop probability $\frac{1}{2}$).
Warm-Up: Complete Graph

Waiting times are i.i.d. exponentials with mean 1.

For the continuous-time variant:

- Suppose we are left with \( k \) random walks

For the discrete-time variant:

Answer “should be” \( \frac{8}{3} \) + \( o(1) \) \( \cdot \) \( n \) for lazy random walks (loop probability \( \frac{1}{2} \)).
For the continuous-time variant:

- Suppose we are left with $k$ random walks
- Waiting time until the next walk moves $\sim \text{Exp}(k)$, and then walk hits one of the others with probability $(k - 1)/(n - 1)$
Waiting times are i.i.d. exponentials with mean 1.

For the continuous-time variant:

- Suppose we are left with \( k \) random walks
- Waiting time until the next walk moves \( \sim \) Exp\((k)\), and then walk hits one of the others with probability \((k - 1)/(n - 1)\)
- Time until \( k - 1 \) walks left is an exponential with mean:

\[
\frac{1}{k} \cdot \frac{n - 1}{k - 1} = \frac{1}{2} \cdot \frac{n - 1}{\binom{k}{2}}.
\]
Waiting times are i.i.d. exponentials with mean 1.

For the continuous-time variant:

- Suppose we are left with $k$ random walks
- Waiting time until the next walk moves $\sim \text{Exp}(k)$, and then walk hits one of the others with probability $(k - 1)/(n - 1)$
- Time until $k - 1$ walks left is an exponential with mean:

$$
\frac{1}{k} \cdot \frac{n - 1}{k - 1} = \frac{1}{2} \cdot \frac{n - 1}{\binom{k}{2}}.
$$

- Since $\sum_{k=2}^{\infty} \frac{1}{\binom{k}{2}} = 2$, expected coalescence time is

$$
\sum_{k=2}^{n} \frac{1}{2} \cdot \frac{n - 1}{\binom{k}{2}} = \frac{1}{2} (n - 1) \cdot \sum_{k=2}^{n} \frac{1}{\binom{k}{2}} = (1 + o(1)) \cdot n.
$$

For the discrete-time variant:

Answer "should be" $(\frac{1}{2} + o(1)) \cdot n$ for lazy random walks (loop probability $\frac{1}{2}$).
Warm-Up: Complete Graph

Waiting times are i.i.d. exponentials with mean 1.

For the continuous-time variant:

- Suppose we are left with $k$ random walks
- Waiting time until the next walk moves $\sim \text{Exp}(k)$, and then walk hits one of the others with probability $(k - 1)/(n - 1)$
- Time until $k - 1$ walks left is an exponential with mean:

$$\frac{1}{k} \cdot \frac{n - 1}{k - 1} = \frac{1}{2} \cdot \frac{n - 1}{\binom{k}{2}}.$$ 

- Since $\sum_{k=2}^{\infty} \frac{1}{\binom{k}{2}} = 2$, expected coalescence time is

$$\sum_{k=2}^{n} \frac{1}{2} \cdot \frac{n - 1}{\binom{k}{2}} = \frac{1}{2} (n - 1) \cdot \sum_{k=2}^{n} \frac{1}{\binom{k}{2}} = (1 + o(1)) \cdot n.$$

For the discrete-time variant:

Answer “should be” $(\frac{8}{3} + o(1)) \cdot n$ for lazy random walks (loop probability $1/2$)
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The Upper Bound and some Consequences

Theorem (Upper Bound)

For any graph $G = (V,E)$,

$$t_{coal} \preceq t_{meet} \cdot \left(1 + \sqrt{\frac{t_{mix}}{t_{meet}} \cdot \log n}\right)$$
Theorem (Upper Bound)

For any graph $G = (V, E)$,

$$t_{coal} \preceq t_{meet} \cdot \left(1 + \sqrt{\frac{t_{mix}}{t_{meet}} \cdot \log n}\right)$$

- Whenever $\frac{t_{meet}}{t_{mix}} \geq (\log n)^2$, we have $t_{coal} \preceq t_{meet}$
The Upper Bound and some Consequences

Theorem (Upper Bound)

For any graph $G = (V, E)$,

$$t_{coal} \preceq t_{meet} \cdot \left(1 + \sqrt{\frac{t_{mix}}{t_{meet}}} \cdot \log n\right)$$

- Whenever $\frac{t_{meet}}{t_{mix}} \gtrsim (\log n)^2$, we have $t_{coal} \preceq t_{meet}$
- If $\frac{t_{meet}}{t_{mix}} \asymp 1$, our bound states $t_{coal} \preceq t_{meet} \cdot \log n$

$\Rightarrow$ bound can be viewed as a refinement of the basic $t_{coal} \preceq t_{meet} \cdot \log n$
The Upper Bound and some Consequences

Theorem (Upper Bound)

For any graph $G = (V, E)$,

$$t_{\text{coal}} \preceq t_{\text{meet}} \cdot \left( 1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n} \right)$$

- Whenever $\frac{t_{\text{meet}}}{t_{\text{mix}}} \gtrsim (\log n)^2$, we have $t_{\text{coal}} \sim t_{\text{meet}}$
- If $\frac{t_{\text{meet}}}{t_{\text{mix}}} \asymp 1$, our bound states $t_{\text{coal}} \preceq t_{\text{meet}} \cdot \log n$

$\Rightarrow$ bound can be viewed as a refinement of the basic $t_{\text{coal}} \preceq t_{\text{meet}} \cdot \log n$

Application to “Real World” Graph Models

If the max-degree satisfies $\Delta \preceq n/\log^3 n$ and $t_{\text{mix}} \preceq \log n$, then $t_{\text{coal}} \sim t_{\text{meet}}$. 
The Upper Bound and some Consequences

Theorem (Upper Bound)

For any graph $G = (V, E)$,

$$t_{coal} \preceq t_{meet} \cdot \left(1 + \sqrt{\frac{t_{mix}}{t_{meet}}} \cdot \log n\right)$$

- Whenever $\frac{t_{meet}}{t_{mix}} \gtrsim (\log n)^2$, we have $t_{coal} \preceq t_{meet}$
- If $\frac{t_{meet}}{t_{mix}} \asymp 1$, our bound states $t_{coal} \preceq t_{meet} \cdot \log n$

$\Rightarrow$ bound can be viewed as a refinement of the basic $t_{coal} \preceq t_{meet} \cdot \log n$

Application to “Real World” Graph Models

If the max-degree satisfies $\Delta \preceq n/\log^3 n$ and $t_{mix} \preceq \log n$, then $t_{coal} \preceq t_{meet}$.

Unfortunately we are not able to determine $t_{meet}$
(it is conceivable though that $t_{meet} \asymp 1/\|\pi\|_2^2$)
A Glimpse at the Proof of the Upper Bound

Proof is quite technical, and we will only glance over one challenging part.

Consider two random walks $(X_t)_{t \geq 0}$, $(Y_t)_{t \geq 0}$ starting from stationarity. By a scaling argument,

$$\Pr\left[\int (X, Y, t_{\text{mix}}) \geq t_{\text{mix}}^{1/16} t_{\text{meet}} = : p, \right]$$

If we have $j$ random walks $Y_1, Y_2, \ldots, Y_j$, do we have

$$\Pr[\bigcap_{\ell=1}^j \int (X, Y_\ell, \tau) \geq 1 - (1 - p)^j] ?$$

Define $C_1 : = \{ (x_0, \ldots, x_\tau) \in T_\tau : \Pr[\int (x, Y, \tau) \geq p^3] \}$

$C_2 : = \{ (x_0, \ldots, x_\tau) \in T_\tau : \Pr[\int (x, Y, \tau) \geq \sqrt{p}] \}$.

Then,

$$\Pr[ (X_t)_{t=0}^{\tau} \in C_1 ] \geq \sqrt{p^3} \quad \text{or} \quad \Pr[ (X_t)_{t=0}^{\tau} \in C_2 ] \geq p^3.$$  

Suppose $\Pr[ (X_t)_{t=0}^{\tau} \in C_2 ] \geq p^3$. Then a $p$-fraction of all walks have a "good" trajectory that is hit by a stationary walk with probability at least $\sqrt{p}$. 

(Issue: Random walks coalesce and could therefore have terminated earlier!) This is of course wrong, since the events are not independent!
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Proof is quite technical, and we will only glance over one challenging part.

- Consider two random walks $\{X_t\}_{t \geq 0}$, $\{Y_t\}_{t \geq 0}$ starting from stationarity.
A Glimpse at the Proof of the Upper Bound

Proof is quite technical, and we will only glance over one challenging part.

- Consider two random walks $\left( X_t \right)_{t \geq 0}$, $\left( Y_t \right)_{t \geq 0}$ starting from stationarity.
- By a scaling argument,

$$\Pr \left[ \text{int}(X, Y, t_{\text{mix}}) \right] \geq \frac{t_{\text{mix}}}{16t_{\text{meet}}} =: p,$$

... (Issue: Random walks coalesce and could therefore have terminated earlier!)

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  \[
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  \]

- If we have \(j\) random walks \(Y^1, Y^2, \ldots, Y^j\), do we have
  \[
  \Pr\left[ \bigcup_{\ell=1}^{j} \text{int}(X, Y^\ell, \tau) \right] \geq 1 - (1 - p)^j
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- Define

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C_1 := \{(x_0, \ldots, x_\tau) \in \mathcal{T}_\tau: \Pr[\text{int}(x, Y, \tau)] \geq \frac{p}{3}\}
\]

\[
C_2 := \{(x_0, \ldots, x_\tau) \in \mathcal{T}_\tau: \Pr[\text{int}(x, Y, \tau)] \geq \sqrt{p}\}.
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- Then, \(\Pr[(X_t)_{t=0}^\tau \in C_1] \geq \frac{\sqrt{p}}{3}\) or \(\Pr[(X_t)_{t=0}^\tau \in C_2] \geq \frac{p}{3}\).
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- Then,

\[
\Pr\left[ (X_t)_{t=0}^{\tau} \in C_1 \right] \geq \frac{\sqrt{p}}{3} \quad \text{or} \quad \Pr\left[ (X_t)_{t=0}^{\tau} \in C_2 \right] \geq \frac{p}{3}.
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- Suppose \(\Pr[(X_t)_{t=0}^\tau \in C_2] \geq \frac{p}{3}\). Then a \(p\)-fraction of all walks have a “good” trajectory that is hit by a stationary walk with probability at least \(\sqrt{p} \ldots\).
A Glimpse at the Proof of the Upper Bound

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- (Issue: Random walks coalesce and could therefore have terminated earlier!)
A Graph Demonstrating Tightness

\[ z_1, z_2, z_3, \ldots, z_\sqrt{n} \]

\[ G_1, G_2, G_3, \ldots, G_\sqrt{n} \]

Node \( z_\ast \) is connected to one designated node in each \( G_i \) and to \( \sqrt{n} / \alpha \) distinct nodes in \( G_2 \).

\[ G_2 \] is a \( \sqrt{n} \)-regular Ramanujan graph on \( n / \sqrt{\alpha} \) nodes.
A Graph Demonstrating Tightness

$z^*$ is connected to one designated node in each $G_i^1$ and to $\sqrt{n}/\alpha$ distinct nodes in $G_2$.
A Graph Demonstrating Tightness

- $G_1^i$, $1 \leq i \leq \sqrt{n}$ are cliques over $\sqrt{n}$ nodes, where $\alpha = t_{\text{meet}}/t_{\text{mix}}$
- $G_2$ is a $\sqrt{n}$-regular Ramanujan graph on $n/\sqrt{\alpha}$ nodes
- Node $z^*$ is connected to one designated node in each $G_1^i$ and to $\sqrt{n/\alpha}$ distinct nodes in $G_2$
Intuition of the Construction

- $G^i_1$, $1 \leq i \leq \sqrt{n}$ are cliques over $\sqrt{n}$ nodes, where $\alpha = \frac{t_{\text{meet}}}{t_{\text{mix}}}$.
- $G^*_2$ is a $\sqrt{n}$-regular Ramanujan graph on $\frac{n}{\sqrt{\alpha}}$ nodes.
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Random Walk Quantities
Intuition of the Construction

- $G^i_1, 1 \leq i \leq \sqrt{n}$ are cliques over $\sqrt{n}$ nodes, where $\alpha = t_{\text{meet}} / t_{\text{mix}}$
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Random Walk Quantities

- $t_{\text{mix}} \approx n$
Intuition of the Construction

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Random Walk Quantities

- $t_{\text{mix}} \asymp n$
  - “$\geq$”: Cheeger's Inequality
  - “$\leq$”: use principle of “Mixing-Time equal to Hitting-Time of Large Sets”

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- $t_{\text{meet}} \asymp \alpha n$
  - very unlikely to meet outside $G_2$
  - After $t_{\text{mix}}$ steps, w.p. $(1/\sqrt{\alpha})^2$ both walks on $G_2 \Rightarrow$ meet w.c.p.
Intuition of the Construction

- $G^i_1, 1 \leq i \leq \sqrt{n}$ are cliques over $\sqrt{n}$ nodes, where $\alpha = \frac{t_{\text{meet}}}{t_{\text{mix}}}$
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    \[\text{[Peres, Sousi, J. of. Theor. Prob.’15]}\]
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  - After $t_{\text{mix}}$ steps, w.p. $(1/\sqrt{\alpha})^2$ both walks on $G_2 \Rightarrow$ meet w.c.p.
- $t_{\text{coal}} \gtrsim \sqrt{\alpha n \log n}$
Intuition of the Construction

- $G_i^i, 1 \leq i \leq \sqrt{n}$ are cliques over $\sqrt{n}$ nodes, where $\alpha = t_{\text{meet}}/t_{\text{mix}}$
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  - After $t_{\text{mix}}$ steps, w.p. $(1/\sqrt{\alpha})^2$ both walks on $G_2 \Rightarrow$ meet w.c.p.

- $t_{\text{coal}} \geq \sqrt{\alpha n \log n}$
  - $\exists$ one walk starting from $G_i^i$ that doesn’t reach $G_2$ in $\sqrt{\alpha n \log n}$ steps
Contrasting the Example with the Upper Bound

For the example $t_{\text{mix}} \asymp \sqrt{n}$, $t_{\text{meet}} \asymp \alpha \sqrt{n}$ and $t_{\text{coal}} \gtrsim \sqrt{\alpha \cdot n \log n}$:
Contrasting the Example with the Upper Bound

For the example \( t_{\text{mix}} \asymp \sqrt{n} \), \( t_{\text{meet}} \asymp \alpha \sqrt{n} \) and \( t_{\text{coal}} \gtrapprox \sqrt{\alpha \cdot n \log n} \):

**Theorem (Lower Bound)**

For any \( \alpha = \frac{t_{\text{meet}}}{t_{\text{mix}}} \in [1, \log^2 n] \) there exists a family of almost-regular graphs such that:

\[
    t_{\text{coal}} \gtrapprox t_{\text{meet}} \cdot \left( 1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n} \right)
\]
Contrasting the Example with the Upper Bound

For the example $t_{\text{mix}} \asymp \sqrt{n}$, $t_{\text{meet}} \asymp \alpha \sqrt{n}$ and $t_{\text{coal}} \asymp \sqrt{\alpha \cdot n \log n}$:

**Theorem (Lower Bound)**

For any $\alpha = \frac{t_{\text{meet}}}{t_{\text{mix}}} \in [1, \log^2 n]$ there exists a family of almost-regular graphs such that:

$$t_{\text{coal}} \asymp t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n}\right)$$

**Theorem (Upper Bound)**

For any graph $G = (V, E)$,

$$t_{\text{coal}} \preceq t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n}\right)$$
Contrasting the Example with the Upper Bound

For the example $t_{\text{mix}} \gtrsim \sqrt{n}$, $t_{\text{meet}} \gtrsim \alpha \sqrt{n}$ and $t_{\text{coal}} \gtrsim \sqrt{\alpha \cdot n \log n}$:

**Theorem (Lower Bound)**

For any $\alpha = \frac{t_{\text{meet}}}{t_{\text{mix}}} \in [1, \log^2 n]$ there exists a family of almost-regular graphs such that:

$$t_{\text{coal}} \gtrsim t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n}\right)$$

**Theorem (Upper Bound)**

For any graph $G = (V, E)$,

$$t_{\text{coal}} \lesssim t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n}\right)$$

- For almost-regular graphs, $t_{\text{coal}}$ might be as large as $t_{\text{meet}} \cdot \log n$
Contrasting the Example with the Upper Bound

For the example $t_{\text{mix}} \asymp \sqrt{n}$, $t_{\text{meet}} \asymp \alpha \sqrt{n}$ and $t_{\text{coal}} \asymp \sqrt{\alpha \cdot n \log n}$:

**Theorem (Lower Bound)**

For any $\alpha = \frac{t_{\text{meet}}}{t_{\text{mix}}} \in [1, \log^2 n]$ there exists a family of almost-regular graphs such that:

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For any graph $G = (V, E)$,

$$t_{\text{coal}} \preceq t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}}} \cdot \log n\right)$$

- For almost-regular graphs, $t_{\text{coal}}$ might be as large as $t_{\text{meet}} \cdot \log n$
- However, for any vertex-transitive graph, $t_{\text{coal}} \asymp t_{\text{meet}} (\asymp t_{\text{hit}})$
Outline

Introduction

Interlude: Complete Graph

Relating Coalescing-Time to the Mixing and Meeting Time

Conclusion
Application to Concrete Networks
Application to Concrete Networks

1D Grid

- $t_{\text{mix}} \asymp n^2$
- $t_{\text{hit}} \asymp t_{\text{meet}} \asymp n^2$
- $t_{\text{coal}} \asymp n^2$ (✓)

2D Grid

- $t_{\text{mix}} \asymp n$
- $t_{\text{hit}} \asymp t_{\text{meet}} \asymp n \log n$
- $t_{\text{coal}} \asymp n \log n$ (✓)

3D Grid

- $t_{\text{mix}} \asymp n^{2/3}$
- $t_{\text{hit}} \asymp t_{\text{meet}} \asymp n$
- $t_{\text{coal}} \asymp n$ ✓
Application to Concrete Networks

1D Grid

1D Grid

1D Grid

2D Grid

2D Grid

2D Grid

3D Grid

3D Grid

3D Grid

16
1. For arbitrary graphs, $t_{coal} \lesssim t_{meet} \cdot \left(1 + \sqrt{\frac{t_{mix}}{t_{meet}}} \cdot \log n\right)$
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2. For any $\frac{t_{\text{meet}}}{t_{\text{mix}}} \in [0, \log^2 n]$, there is an almost-regular matching graph
Summary and Open Questions (1/2)

Results

1. For arbitrary graphs, \( t_{\text{coal}} \lesssim t_{\text{meet}} \cdot \left( 1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n} \right) \)
2. For any \( \frac{t_{\text{meet}}}{t_{\text{mix}}} \in [0, \log^2 n] \), there is an almost-regular matching graph
3. For graphs with constant \( \Delta/d \), \( t_{\text{mix}} \lesssim t_{\text{meet}} \lesssim t_{\text{coal}} \lesssim t_{\text{hit}} \lesssim t_{\text{cov}} \)
Summary and Open Questions (1/2)

Results

1. For arbitrary graphs, $t_{\text{coal}} \preceq t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n}\right)$
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Open Questions

Can we prove $t_{\text{coal}} \preceq t_{\text{hit}}$ for all graphs?
Is it true that $t_{\text{coal}}(\text{disc}) \preceq t_{\text{coal}}(\text{cont})$ for any graph?
Reduce the number of walks to some threshold $\kappa \in [1, n]$.
Conjecture: For any (regular) graph, no. walks can be reduced to $\sqrt{n}$ in $O(n)$ time.
More generally, it takes $O\left(\left(\frac{n}{\kappa}\right)^2\right)$ time to go from $n$ to $\kappa$. 

Summary and Open Questions (1/2)

Results

1. For arbitrary graphs, \( t_{coal} \preceq t_{meet} \cdot \left(1 + \sqrt{\frac{t_{mix}}{t_{meet}} \cdot \log n}\right) \)
2. For any \( \frac{t_{meet}}{t_{mix}} \in [0, \log^2 n] \), there is an almost-regular matching graph
3. For graphs with constant \( \Delta/d \), \( t_{mix} \preceq t_{meet} \preceq t_{coal} \preceq t_{hit} \preceq t_{cov} \)

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1. For arbitrary graphs, $t_{\text{coal}} \lesssim t_{\text{meet}} \cdot \left(1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n}\right)$
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- Can we prove $t_{\text{coal}} \lesssim t_{\text{hit}}$ for all graphs?
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Summary and Open Questions (1/2)

**Results**

1. For arbitrary graphs,  \( t_{\text{coal}} \lesssim t_{\text{meet}} \cdot \left( 1 + \sqrt{\frac{t_{\text{mix}}}{t_{\text{meet}}} \cdot \log n} \right) \)
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- Is it true that \( t_{\text{coal}}^{(\text{disc})} \lesssim t_{\text{coal}}^{(\text{cont})} \) for any graph?
- Reduce the number of walks to some threshold \( \kappa \in [1, n] \).

**Conjecture:**

- For any (regular) graph, no. walks can be reduced to \( \sqrt{n} \) in \( O(n) \) time.
- More generally, it takes \( O((n/\kappa)^2) \) time to go from \( n \) to \( \kappa \).
The End

THANK YOU
The End

THANK YOU
Another Direction: Cat-and-Mouse Game

Definition

- The mouse picks a deterministic walk \((v_0, v_1, v_2, \ldots)\), unaware of the transitions of the cat.

Comments on the Cat-and-Mouse Game:
Easier to deal with in the sense there is only one random object (the cat!). Clearly, 

\[ t_{\text{meet}} / t_{\text{hit}} \] 

But do we have 

\[ t_{\text{cat}} - t_{\text{mouse}} \] 

\[ t_{\text{hit}} \]
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- The **cat** performs lazy random walk \((Y_t)_{t \geq 0}\) from \(u\)
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- The **expected duration** of the game is

\[
\max_u \left\{ \min_{t \geq 0} \left\{ Y_t = v_t \right\} \right\}
\]
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t_{\text{cat-mouse}} := \max \mathbb{E}_u \left[ \min \{ t \geq 0 : Y_t = v_t \} \right].
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\[
t_{\text{cat-mouse}} := \max_{u,(v_0,v_1,\ldots)} \mathbb{E}_u \left[ \min\{t \geq 0 : Y_t = v_t\} \right].
\]

- very similar version in Aldous and Fill (Section 4.3)
- we may assume w.l.o.g. that the cat starts from stationarity by simply letting the cat perform \(t_{\text{mix}}\) steps
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- The expected duration of the game is

\[
t_{\text{cat-mouse}} := \max_{u, (v_0, v_1, \ldots)} E_u \left[ \min\{t \geq 0 : Y_t = v_t\} \right].
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Comments on the Cat-and-Mouse Game:

- Easier to deal with in the sense there is only one random object (the cat!)
- Clearly, \(t_{\text{meet}} \leq t_{\text{cat-mouse}}\) and \(t_{\text{hit}} \leq t_{\text{cat-mouse}}\).
  But do we have \(t_{\text{cat-mouse}} \sim t_{\text{hit}}\)?