# Practical challenges faced when using modern approaches to numerical PDEs to simulate petroleum reservoirs

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# Our groups work

- Which subject do we come from
  - Hyperbolic conservation laws
  - (Geometrical Integration, computational geometry, Physics)
- History of the research in reservoirs,
  - From: Complicated methods for simple problems like (incompressible 2phase flow)
    - Discretization: (Eliptic; mimetic, mpfa, Hyperbolic: fronttracking, reordering, operator splitting)
    - Multiscale (Mixed finite element,m Finite Volume .)
    - Streamlines (Fronttracking)
  - To: Simple Methods for complicated problems
    - fast prototyping, model reduction, optimization, EOR
- Software:
  - Matlab Reservoir Simulation Toolbox (MRST)
    - Collection of our research
    - Research tool
    - Fast prototyping
  - Open Porous Media (OPM) C++
    - Platform for implementing methods on Industry standard models

People (Current):

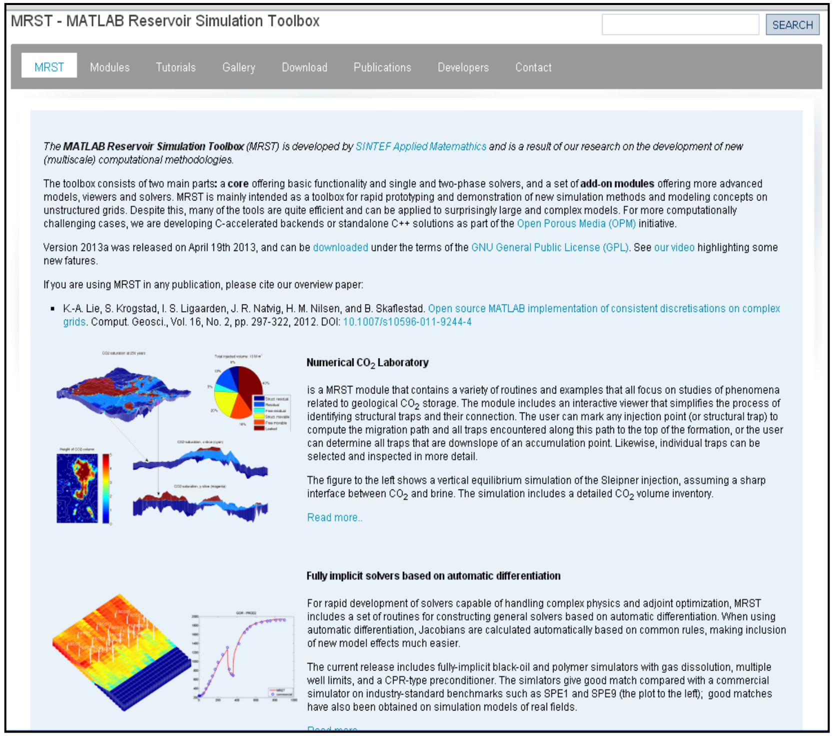
Knut Andreas Lie
Stein Krogstad
Atgeirr Rasmussen
Xavier Raynaud
Olav Møyner
Bård Skaflestad

# Matlab Reservoir Simulation Toolbox - MRST

- An *open source* comprehensive set of routines for reading, visualising and running numerical simulations on reservoir models.
- Developed at SINTEF Applied Mathematics.
- MRST core: grid + basic functionality
- Add-on modules: discretizations (TPFA, MPFA, mimetic), black oil, thermal, upscaling, coarsening, multiscale, flow diagnostics, CO2 laboratory,….

**Statistics**: (release 2013b)
- Number of downloads:  ~3000
- Number of countries:  ~120
- Number og institutions: ~1080

http://www.sintef.no/MRST/



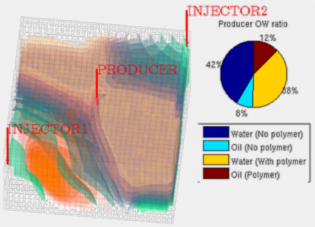

<u>Main idea:</u> flexibility and rapid prototyping

Light weight/
special purpose

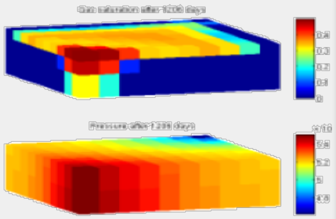
Black box/
general purpose

complexity/ computational complexity

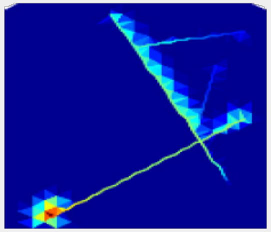
# MRST add-on modules



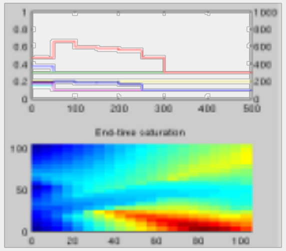

Fully implicit solvers (AD and gradients)



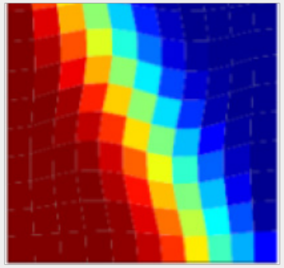
IMPES black-oil solvers



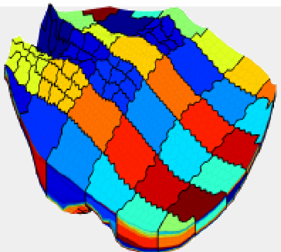
Discrete fracture models



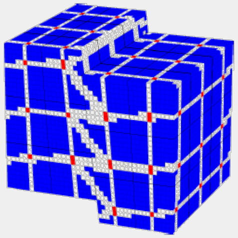

Adjoint methods



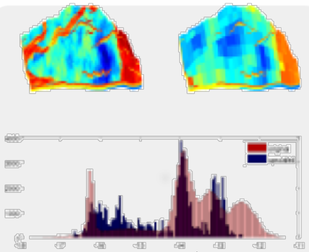
MPFA methods



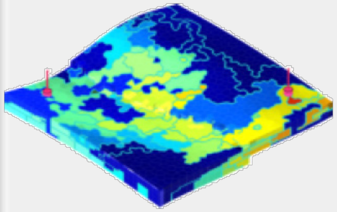
Multiscale mixed finite elements



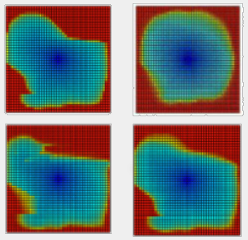
Multiscale finite volumes



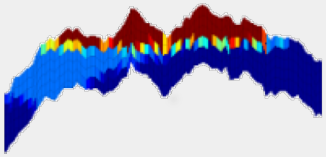
Single and two-phase upscaling



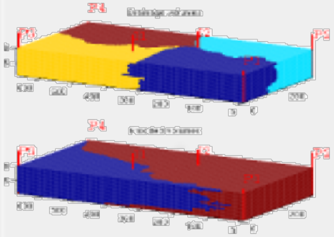
Grid coarsening



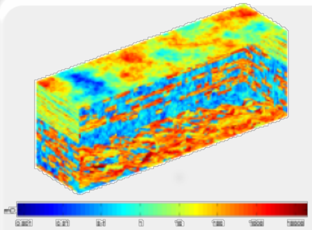
Ensamble Kalman filter



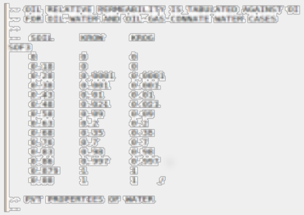
CO2 laboratory



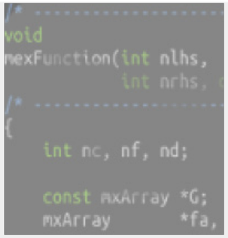
Flow diagnostics



Data sets (e.g. SPE 10)



Industry standard input formats




C-accelerated routines

# Question:

Why is almost all simulations of reservoirs  today using a fully implicit Two Point  Method with Mobility upwinding.

# Outline

- Reservoir simulation: model ,  challenges
- Fully implicit two point method's
  - Problems, (Advantages)
- Why not (?)
  - Higher order
  - Explicit saturation
  - Operator splitting based
  - MPFA,  MIMETIC ...
- Conclusion/Challenges

# Model: Black-oil model

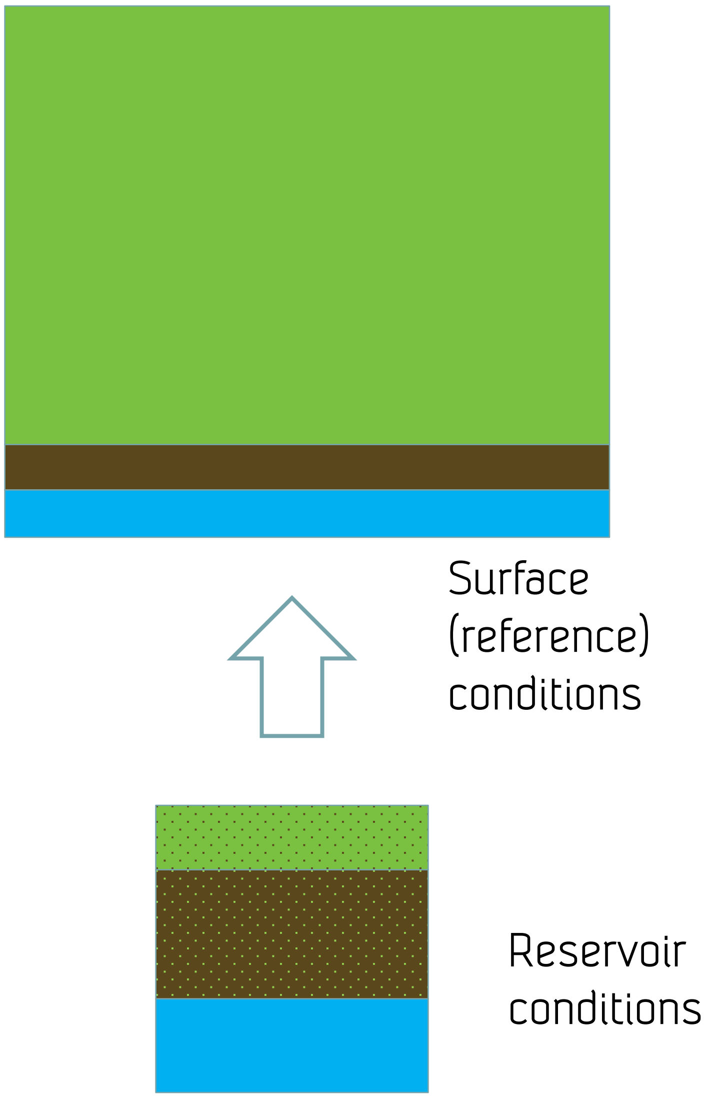
- 3 component – 3phase model

components

| phases | W | O | G |
|---|---|---|---|
| W | X | | |
| O | | X | X |
| G | | X | X |

**Unknowns**

- Phase pressures $p_\alpha$
- Phase saturations $s_\alpha$
- Gas comp. in oil phase $r_s$
- Oil comp. in gas phase $r_v$



# Black-oil model

$$\frac{d}{dt}(\phi b_w s_w) + \nabla \cdot (b_w v_w) - b_w q_w = 0$$

$$\frac{d}{dt}(\phi(b_g r_v s_g + b_o s_o)) + \nabla \cdot (b_g r_v v_g + b_o v_o) - (b_g r_v q_g + b_o q_o) = 0$$

$$\frac{d}{dt}(\phi(b_g s_g + b_o r_s s_o)) + \nabla \cdot (b_g v_g + b_o r_s v_o) - (b_g q_g + b_o r_s q_o) = 0$$

$$v_j = -\frac{k_{rj}}{\mu_j} K(\nabla p_j - \rho_j g \nabla z)$$

Primary variables:

- Oil pressure
- Water saturation , gas saturation(/dissolved gas/dissolved oil)

Two point flux mobility upwinding:

$$dp_{i,j} = (p_j - p_i - \rho_{i,j} g(z_j - z_i))$$

$$v_{i,j} = \begin{cases} -\frac{k_{r,j}}{\mu_j} T_{i,j} dp_{i,j} & \text{for} \quad dp_{i,j} \geq 0 \\ -\frac{k_{r,i}}{\mu_i} T_{i,j} dp_{i,j} & \text{for} \quad dp_{i,j} < 0 \end{cases}$$

# Black-oil model: wells

For each connection:

$$q_j = T m_{wj}(p_w - p + Hw)$$

- Well head computed explicitly based on phase distribution along well
- For producing connection:

$$m_{wj} = m_j = \frac{k_{rj}}{\mu_j}$$

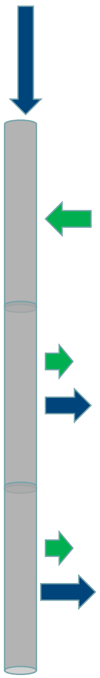
- For injecting connection:

$$m_{wj} = \alpha_j (m_w + m_o + m_g)$$

- $\alpha_j$ is the volume fraction of phase *j* in the injected mixture at connection conditions

**Handling of *cross-flow (implicit)*:**

1) Compute inflow from producing connections (at reference conditions)
2) Compute average wellbore mixture (at reference conditions)
3) Compute average volumetric mixture at injection connection conditions
4) Compute injection connection mobilities



# Black-oil model: Jacobian

Setting up the Jacobian:

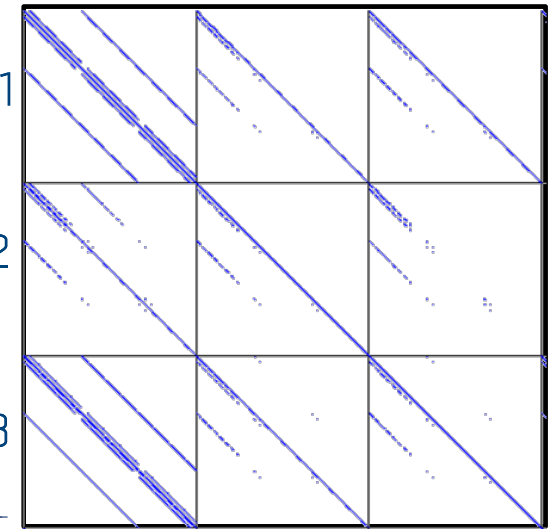
- Primary variables:

$$p_o, s_w, \underbrace{x}_{s_g, r_s, r_v}, q_o^s, q_w^s, q_g^s, p_w$$

- Equations:
  - 1-3 : reservoir equations
  - 4-6 : $q_w^s - \sum b_w^c q_w^c = 0, \; q_o^s = \ldots$
  - 7 : well control (phase rates, bhp, ...)

$$\overbrace{\frac{\partial}{\partial q_o^s}, \frac{\partial}{\partial q_w^s}, \frac{\partial}{\partial q_g^s}, \frac{\partial}{\partial p_w^s}}$$

$$\frac{\partial}{\partial p_o} \qquad \frac{\partial}{\partial s_w} \qquad \frac{\partial}{\partial x}$$




$$\frac{V}{\Delta t}(\phi^{n+1} b_w^{n+1} s_w^{n+1} - \phi^n b_w^n s_w^n) + \nabla \cdot (b_w^{n+1} v_w^{n+1}) - b_w^{n+1} q_w^{n+1} = 0$$

```
dpW     = s.grad(p-pcOW) - g*(rhoWf.*s.grad(z));upc = (double(dpW)>=0);
bWvW = s.faceUpstr(upc, bW.*mobW).*s.T.*dpW;
eqs{2}        = (pv/dt).*( pvMult.*bW.*sW - pvMult0.*f.bW(p0).*sW0 ) + s.div(bWvW);
```

# Black-oil model: linear system

**Solution procedure for linear equation**

$$\frac{\partial F(x^i)}{\partial x^i}\delta x^{i+1} = -F(x^i)$$

1. Eliminate $\delta q_o^s, \delta q_w^s, \delta q_g^s$
2. Eliminate $\delta p_w$

$$\begin{bmatrix} A_{op_o} & A_{os_w} & A_{om} \\ A_{wp_o} & A_{ws_w} & A_{wm} \\ A_{gp_o} & A_{gs_w} & A_{gm} \end{bmatrix} \begin{bmatrix} \delta p_o \\ \delta s_w \\ \delta m \end{bmatrix} = \begin{bmatrix} r_o \\ r_w \\ r_g \end{bmatrix}$$

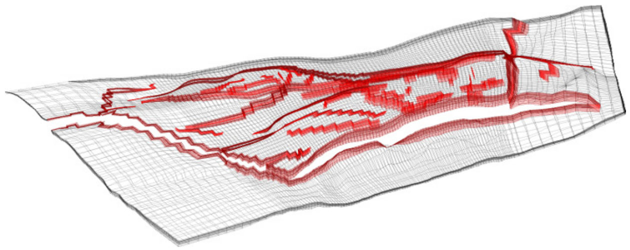
3. After approximate decoupling of pressure, we solve the resulting linear system using GMRES with CPR precontitioner,
4. Recover remaining variables

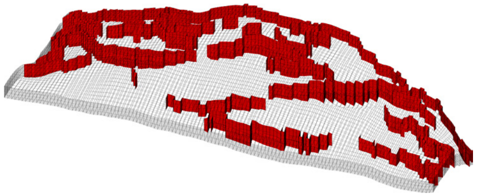
- Similar (transposed) approach implemented for adjoint equations
- *Appleyard chop* performed when updating saturations

- The CPR preconditioner consist of
  1. ILU on whole system
  2. Algebraic mulitgrid on pressure sub-system ,

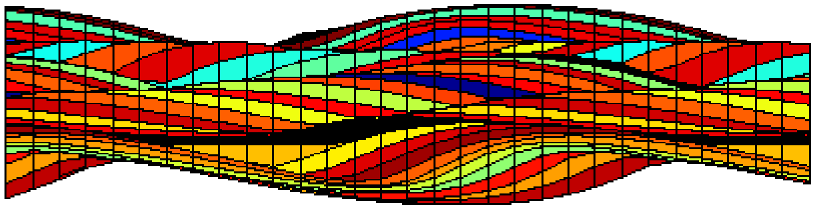
# Grid: model and data

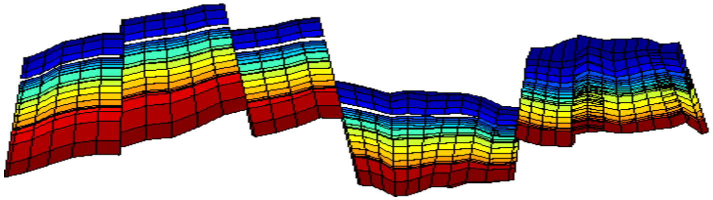
- The structure of the reservoir ( geological , surfaces, faults, etc)



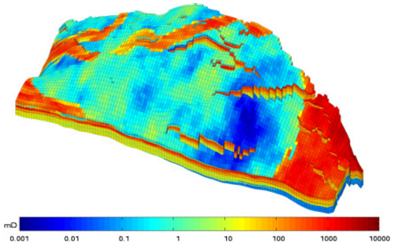


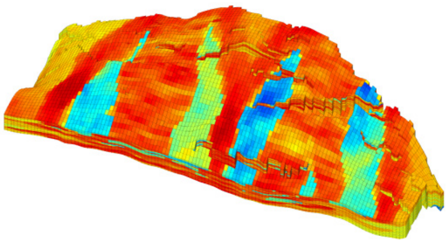
- The stratigraphy of the reservoir (sedimentary structure)

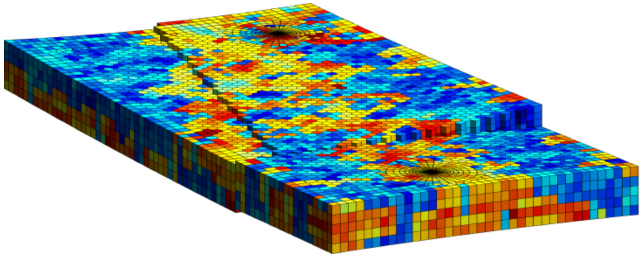




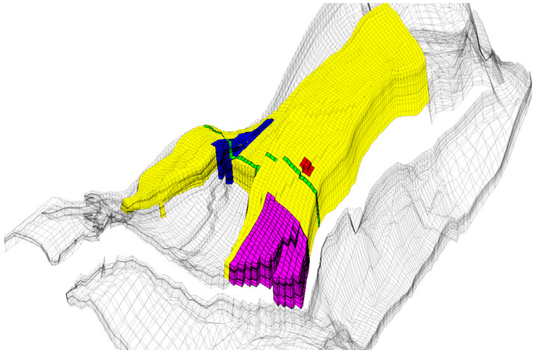
- Petrophysical parameters (permeability, porosity, net-to-gross, ….)

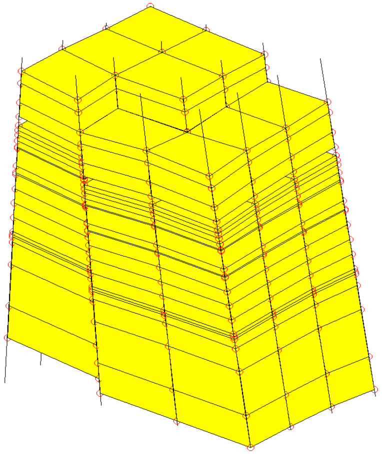


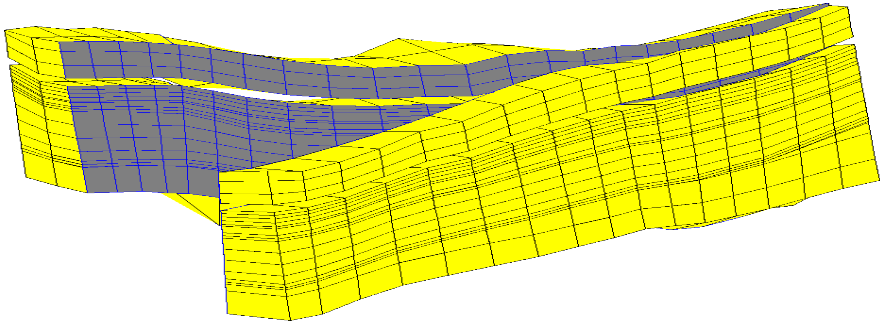



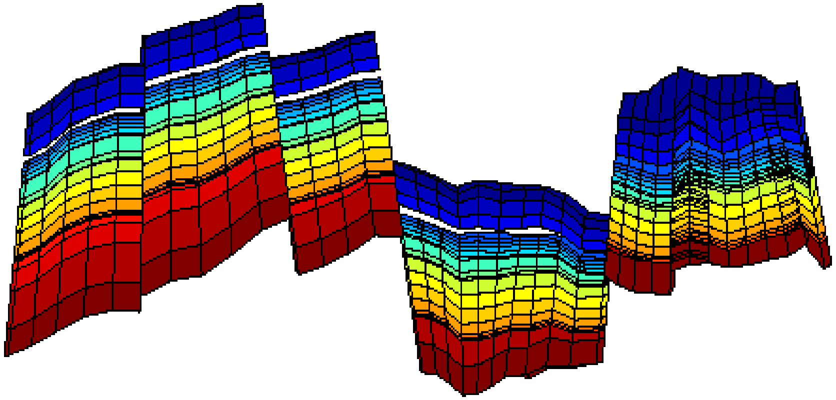


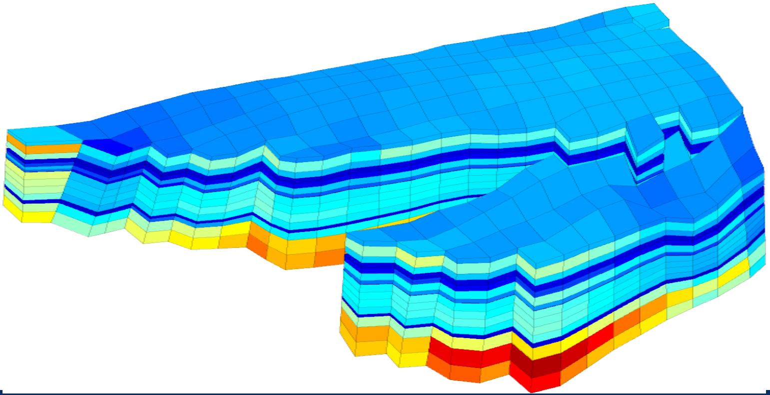
# Grid: North Sea Model



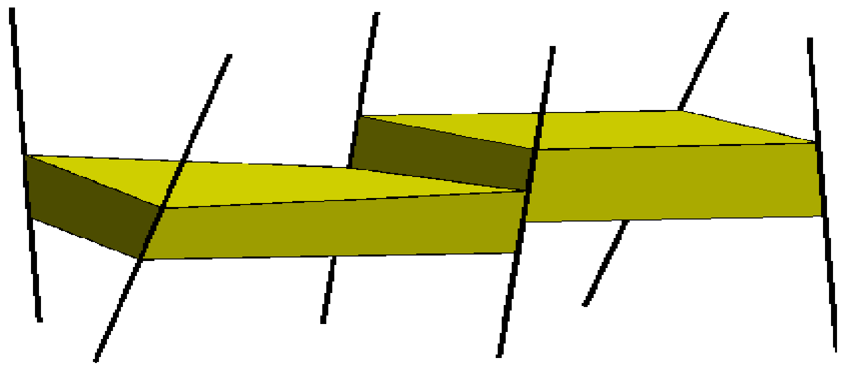


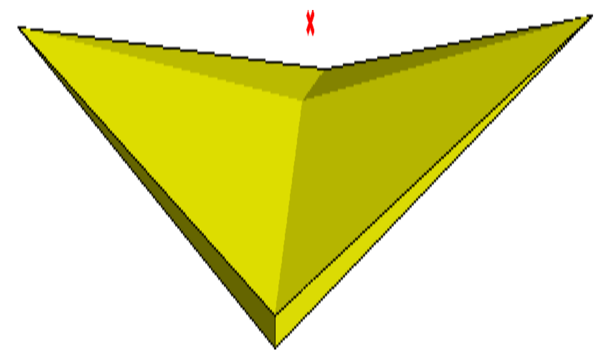


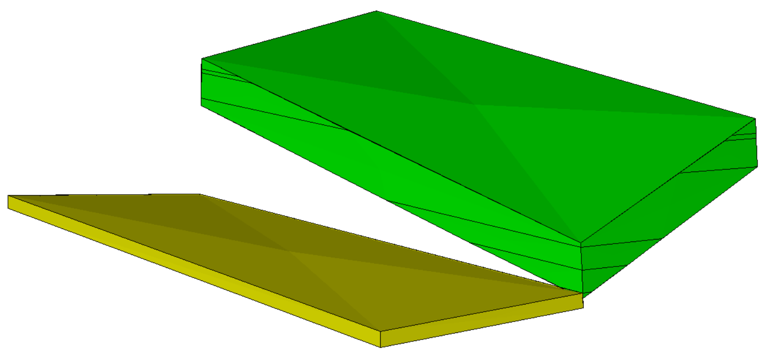


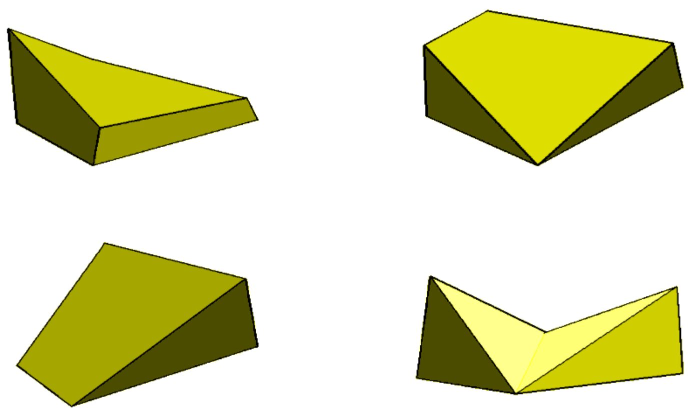


# Grid: strange cells



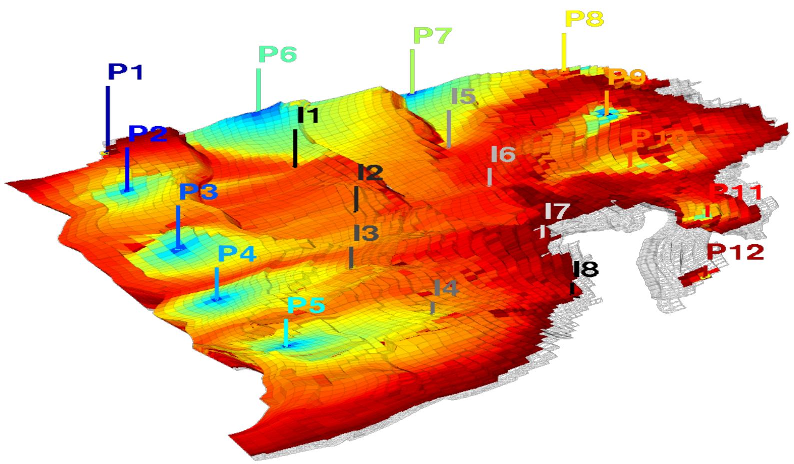






# Few observations, few data

- Wells are the observables




The incompressible single phase case have only n-1 degrees of freedom  for all possible  boundary conditions

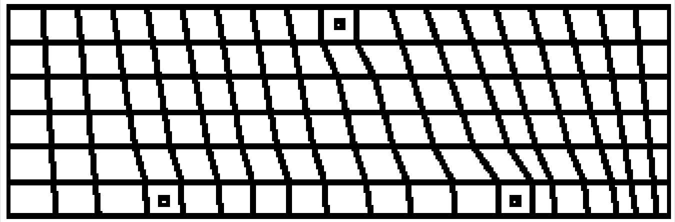
- Observables:
  - Well rates (oil, water, gas)
  - Bottom hole pressure
- Parameter knowledge
  - Horizons – seismic
  - Permeability , porosity, relative permeability from cores
  - 'Geological interpretation/knowleadge, interpolation, geostatistic
  - historymatching

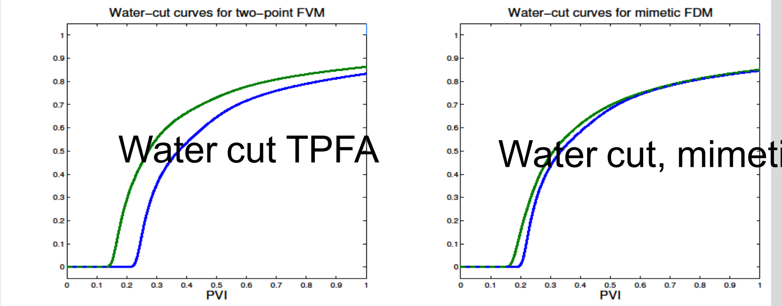
# Grid orientation effects/ tensor permeability

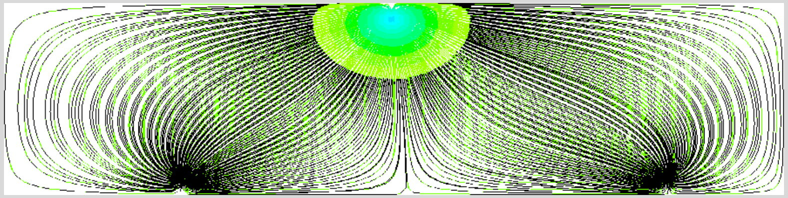
Standard method + skew grid = grid-orientation effects

MPFA/mimetic : Consistent discretization methods capable of handling general polyhedral grids

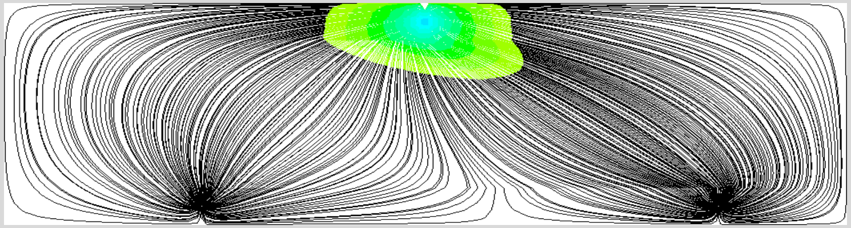
Example:
Homogenous and isotropic medium with a symmetric well pattern





Streamlines Mimetic

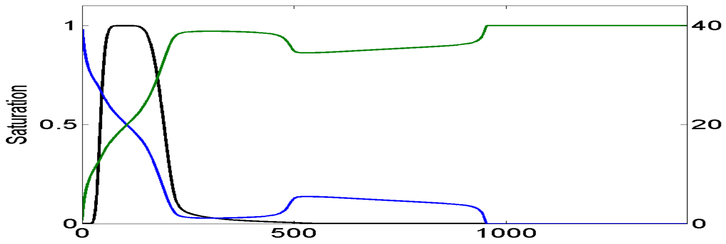


Streamlines TPFA

Upscaled models do have tensor permeability and relative permeability

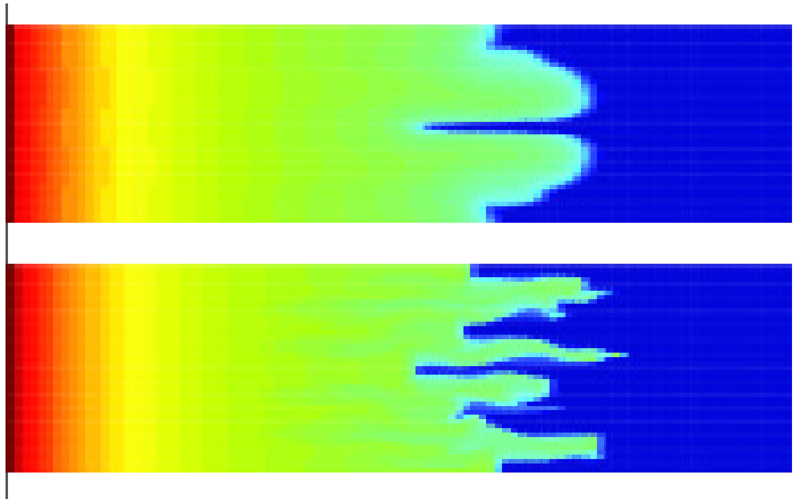
# Numerical diffusion

- Front capturing




Upwind need fine grid and small time steps to resolve a polymer slug

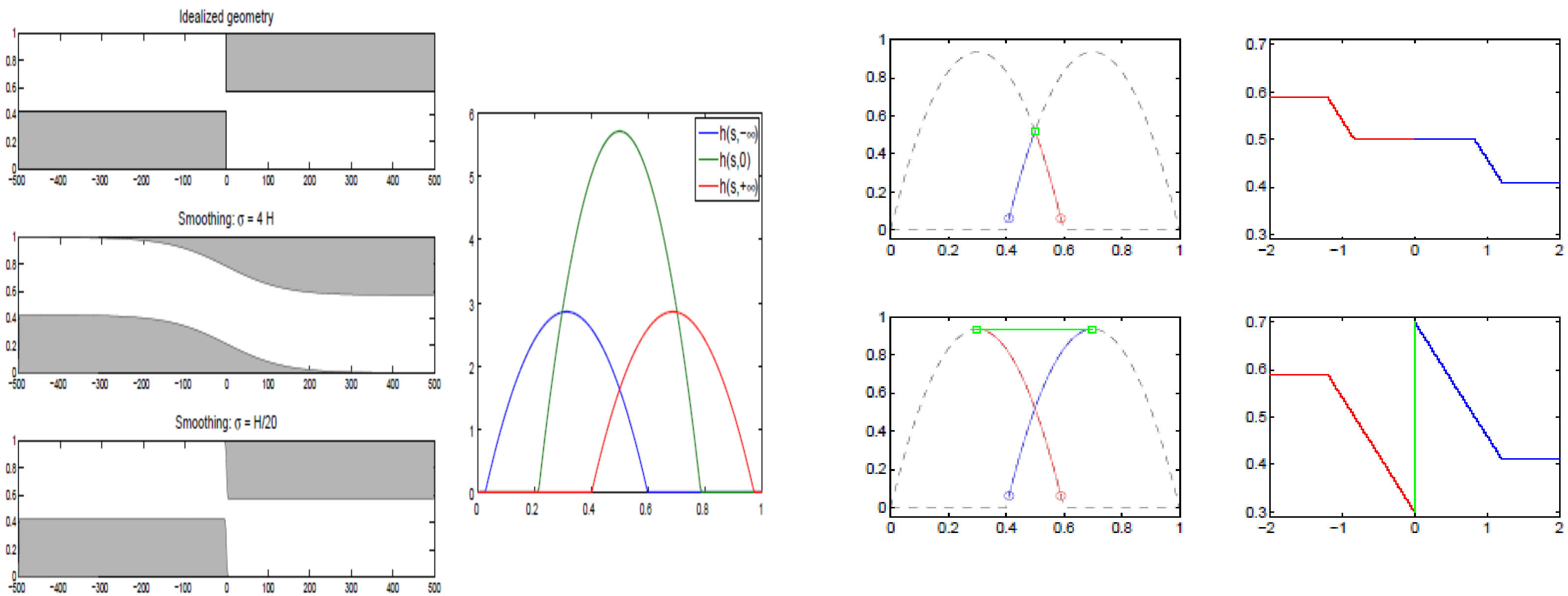
- Viscous fingering instabilities



Viscous fingering comparing a fully implicit single-point upwind and 'TVD-type' schemes

# Discontinuous Riemann problem

$$\frac{\partial s}{\partial t} + \frac{\partial}{\partial x} h(s,x) = q(x), \qquad h(s,x) = f_g(s,x) g(x),$$



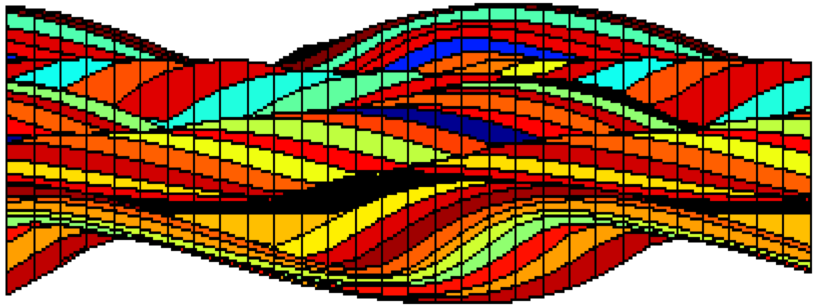

Upwind method do not always give the physical solution

# Proposed methods:

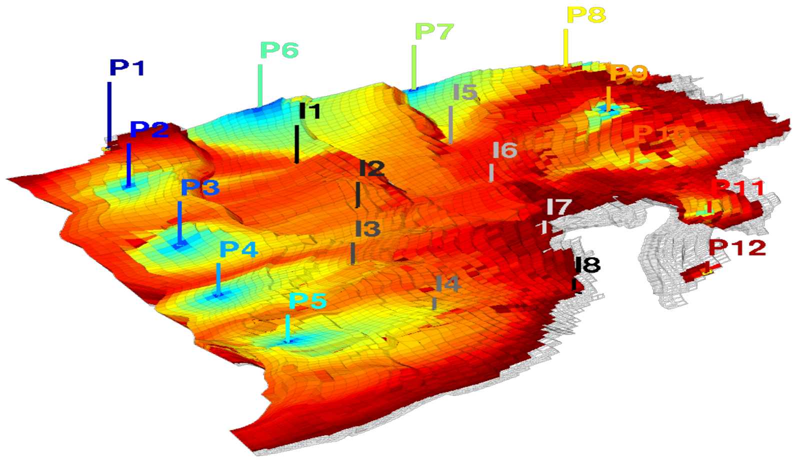
- Explicit
- Splitting:
  - Full system
    - Pressure and transport
  - Transport:
    - Advection, (convection)  diffusion
- High order:
- MPFA,  MIMETIC, Mixed finite element, DG
- Parallelization:

# Explicit methods

- Heterogeneity (grids):
  - small cells
  - high porosity



- Wells
  - Velocity $$v \sim \frac{1}{r}$$




High CFL numbers from localized features

# Splitting:
## Pressure ("elliptic") – transport ("hyperbolic")

- Incompressible two phase flow:

$$\nabla \cdot \vec{v} = q \tag{1}$$

$$\vec{v} + \lambda K \left[ \nabla p - \lambda_w \rho_w + \lambda_n \rho_n ) \vec{g} \right] = 0$$

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \left[ f_w ( \vec{v} + \lambda_n ( \rho_w - \rho_n ) K \vec{g} ) \right] = q_w . \tag{2}$$

- Equation 1) independent of saturation (and pressure)
- Equation 2) has solution if $\nabla \cdot \vec{v} = q$

# Splitting:
## Pressure ("elliptic") – transport splitting ("hyperbolic")

$$\phi(c_w S_w + (1 - S_w)c_o)\frac{\partial p}{\partial t} + (f_w c_w + (1 - f_w)c_o)\nabla p \cdot \vec{v} + \nabla \cdot \vec{v} = q \qquad (1)$$

$$\vec{v} + \lambda K\left[\nabla p - \lambda_w \rho_w + \lambda_n \rho_n)\vec{g}\right] = 0$$

$$\phi \frac{\partial \rho_w s_w}{\partial t} + \nabla \cdot \left[\rho_w f_w(\vec{v} + \lambda_n(\rho_w - \rho_n)K\vec{g})\right] = q_w. \qquad (2)$$

- Equation 1) not independent of saturation
- There may be no solution to 2) if 1) is not fulfilled
    - Saturation outside range (0,1)

# **Strong coupling:** Vertical equilibrium model

Integration in vertical direction $\longrightarrow$ pressure equation:

$$\nabla_{\|} \cdot \vec{v} = q_{\text{tot}}, \quad \vec{v} = -\lambda_t \Big[ \nabla_{\|} p_t - \Big( f_v \rho_{\mathrm{CO_2}} + [1 - f_v] \rho_w \Big) \vec{g}_{\|} + \frac{\lambda_w}{\lambda_t} \nabla_{\|} g_c \Big]$$

and transport equation:

$$\phi H(x) \frac{\partial s}{\partial t} + \nabla_{\|} \cdot \Big( f_v(s,x) \vec{v} + f_g(s,x) \big[ \vec{g}_{\|} + \boxed{\nabla_{\|}\, g_c(s,x)} \big] \Big) = q(x)$$

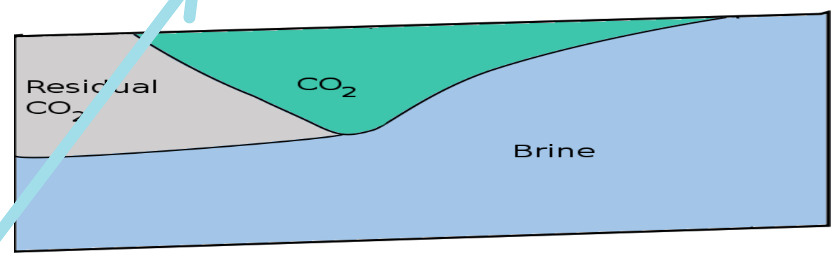
$$\lambda_c(s,x) = \frac{1}{\mu_c} \int_0^{sH} k_c(1) K_x(z,x) dz$$

$$\lambda_w(s,x) = \frac{1}{\mu_w} \int_{sH}^{H} k_w(1) K_x(z,x) dz$$

$$f_v(s,x) = \frac{\lambda_c(s,x)}{\lambda_c(s,x) + \lambda_w(s,x)}$$

$$f_g(s,x) = \lambda_w(s,x) f(s,x)$$

$$g_c(s,x) = H(x)(\rho_c - \rho_w) g_{\perp} s$$



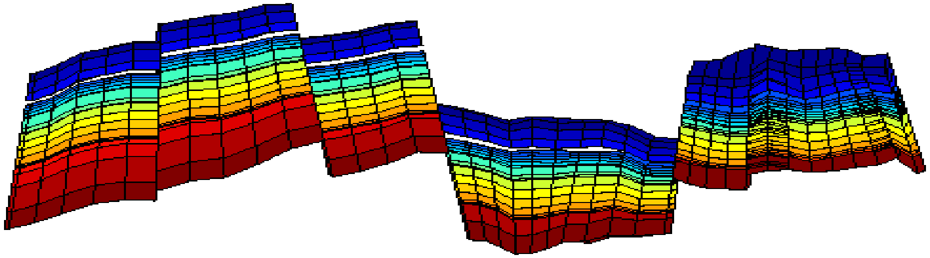

| | | | | | |
|---|---|---|---|---|---|
| $H$ | – | height of formation | $\lambda_\alpha$ | – | pseudo mobility |
| $h$ | – | height of CO2 | $p_t$ | – | pressure at top surface |
| $s$ | – | relative height, $h/H$ | $\|$ | – | parallel to top surface |

The "transport" equation have obtained a parabolic term, by strong gravity coupling to pressure equation.

# High order

- Pressure
  - Heterogeneity permeability
  - Large uncertainty

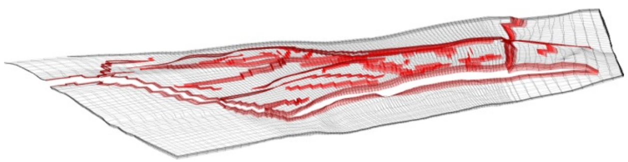
  - No gain?

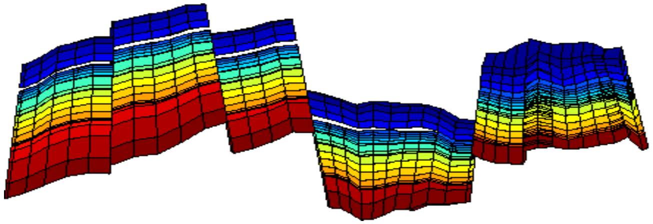


- Transport ( DG?)
  - Splitting to transport problem?
  - Explicit methods excluded, need to be implicit

# MIMETIC, MPFA, ..

- Pressure equation
  - Problematic for aspect ratio: anisotropy (MPFA/mimetic(?))
  - More expensive : (Mimic 3 times dof, 2 times bandwidth)



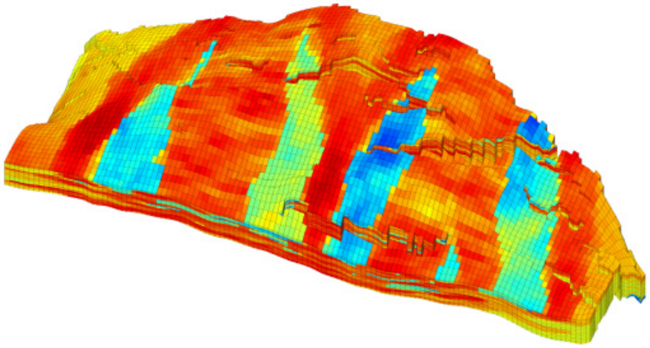


  - Limited experience: Nonlinear methods

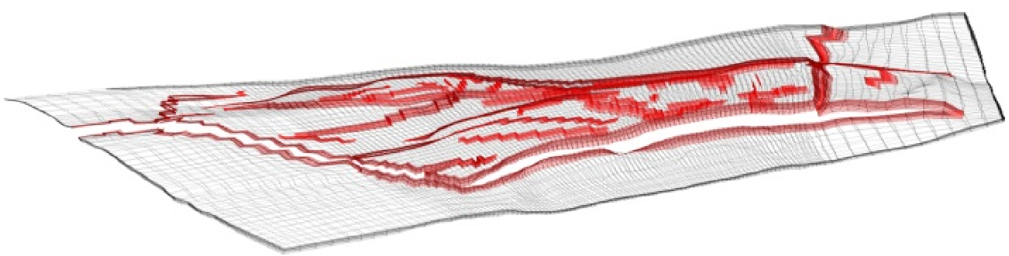
- Coupled system
  - Formulation ? (Mixed, mimetic,…)
  - Stability for hyperbolic part: Upwinding ?, numerical flux ?
  - Physical effects
    - Gravity, Capillary pressure, wells and dissolution

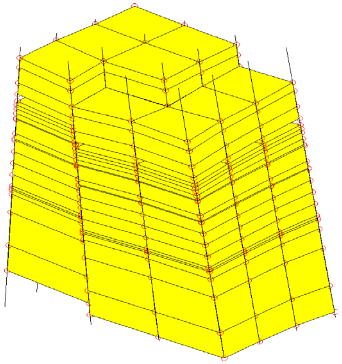
# Others

- Parallelization
  - Communication costs due to need for implicit solver
  - Difficulty of partitioning due to
    - Channelized flow
    - Long horizontal Wells, give  nonlocal connections



- Methods using simplexes
  - Aspect ration imply to many grids





# Our view on specific challenges for reservoir simulation

- Large aspect ratio
  - Reservoirs: 10 km laterally , 50-200 m vertically
- Discontinuities:
  - Permeability
  - Relative permeability
  - Capillary pressure
- Grid and model parameter are strongly connected
  - strange grids, general polyhedral cells
- Coarse grid
  - Grid cells typically 100m laterally , 4  m  vertically
  - Transport hyperbolic
- Strong coupling between "elliptic" and "hyperbolic" variables
  - Large scale:  gravity
  - Smaller scale: capillary pressure
- Non local connections:
  - Wells or fast flowing channels
  - Parallelization

# Conclusion: What is needed

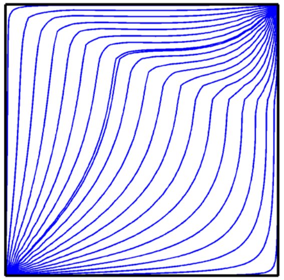
- Research should focus on:
  - Methods for general challenging grid with generic implementation
  - Methods which  work for  elliptic, parabolic and hyperbolic problems
  - Methods for strongly coupled problems
  - Tensor Mobilities
- Specific purpose simulators
  - Codes using modern methods for correctly simplified systems
- Accept for simplifications
  - In reservoir simulation an fully implicit solve using TPFA and mobility upwinding is ofhen assumed to be the truth.
- Work flows including:
  - Simple models
  - Numerical (specific) upscaled/reduced models
  - Trusted simulations/"Full physics simulations."
- Open source
  - Simulators to challenge industry simulators
  - Implementations of current research
- Open Data
  - Real reservoir models as benchmark

# More advanced operator splitting

$$\nabla \cdot \vec{v} = q, \qquad \vec{v} + \lambda K \big[\nabla p - \lambda_w \rho_w + \lambda_n \rho_n)\vec{g}\big] = 0$$

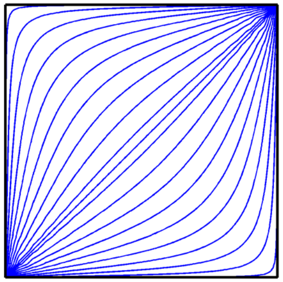
$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \big[ f_w (\vec{v} + \lambda_n (\rho_w - \rho_n) K \vec{g}) \big] = q_w.$$

$$\vec{v} = \vec{v}_a + \vec{v}_c$$



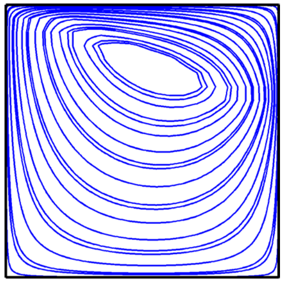
$$\nabla \cdot \vec{v} = q$$

$$\vec{v} + \lambda K [\nabla p - \omega(S) \vec{g}] = 0$$



$$\nabla \cdot \vec{v_a} = q$$

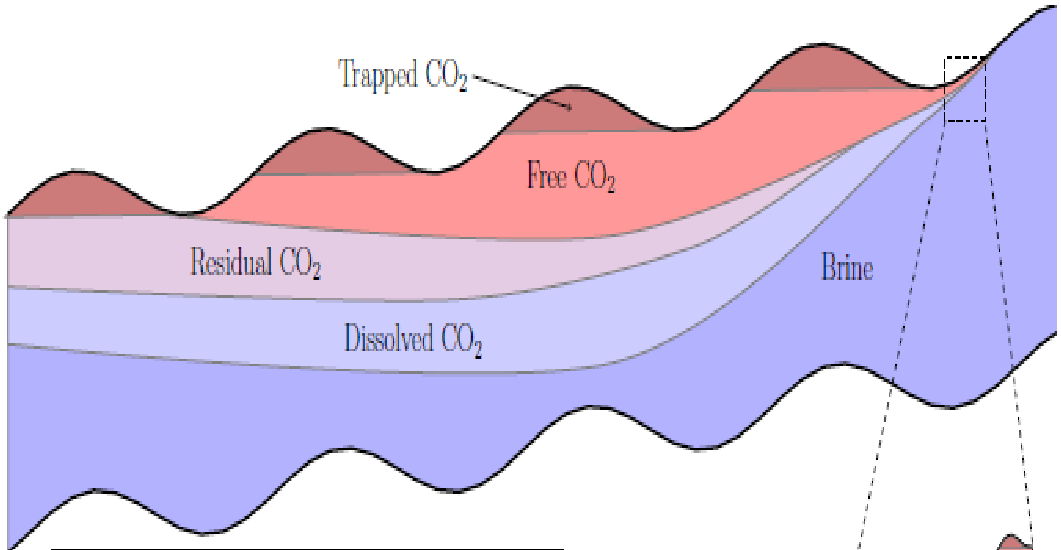
$$\vec{v_a} + \lambda K [\nabla p_a] = 0$$



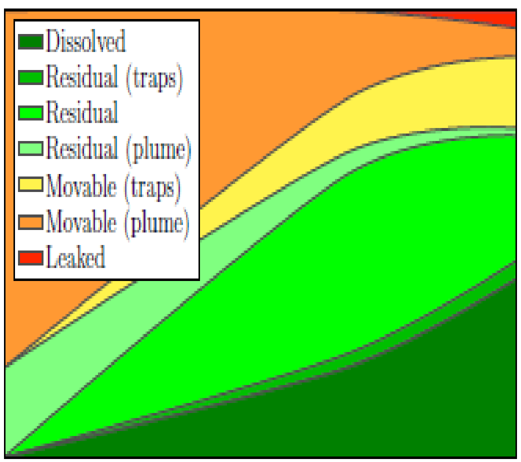
$$\nabla \cdot \vec{v_c} = 0$$

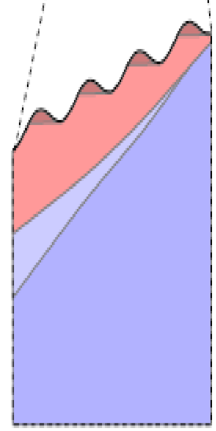
$$\vec{v_c} + \lambda K [\nabla p_c - \omega(S) \vec{g}] = 0$$

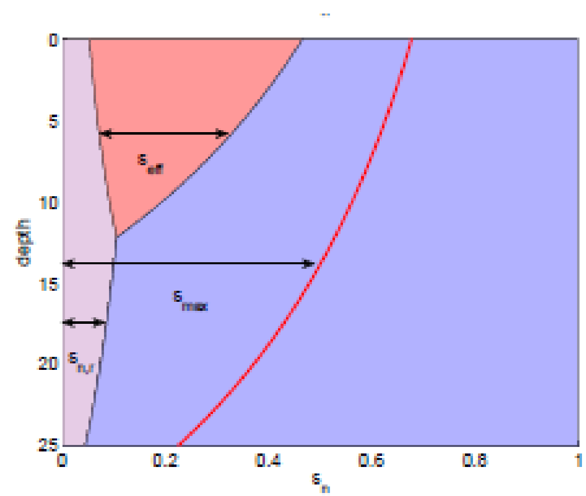
# Vertical equilibrium calculations: inventory



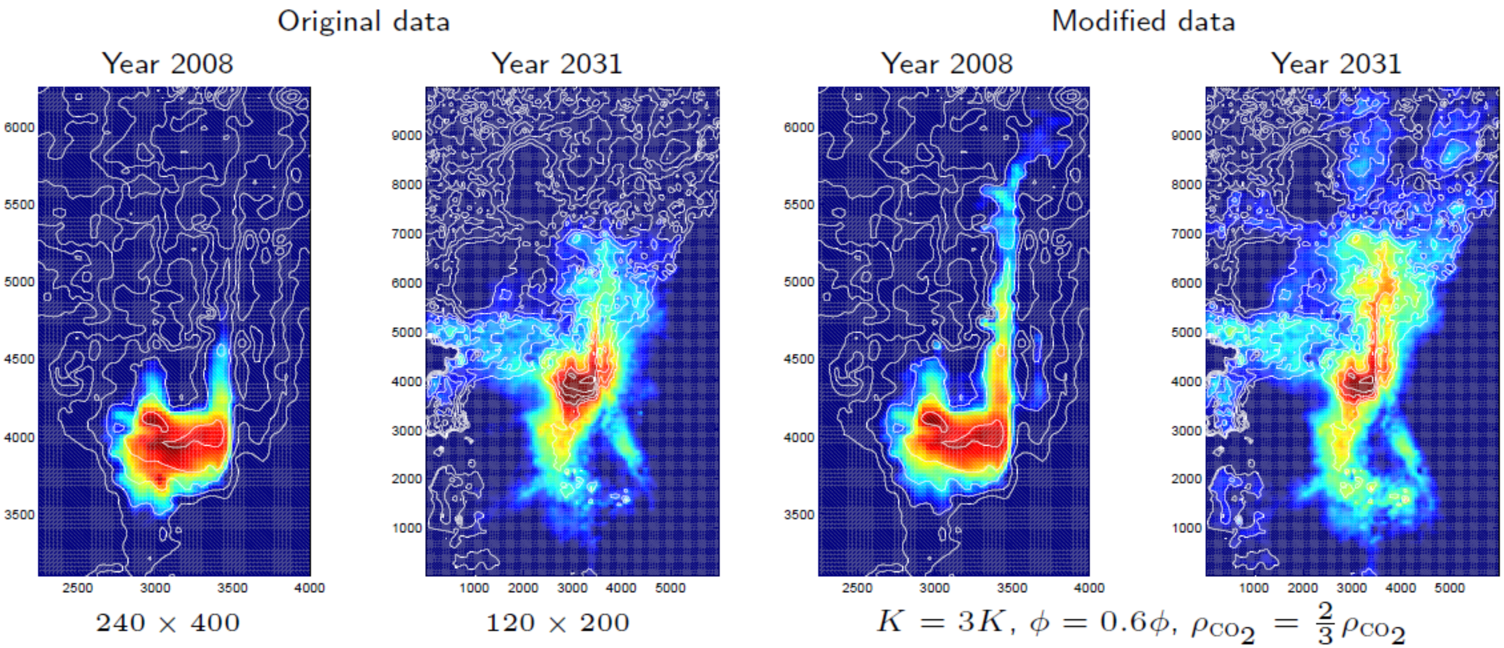

- Phase model:
  - incompressible
  - compressible
  - dissolution
- Relative permeability models
  - sharp interface
  - capillary fringe
  - detailed hysteric model
  - upscaling of subscale variations

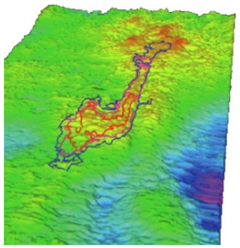


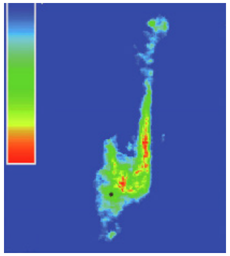





# Simulation of Sleipner Layer 9



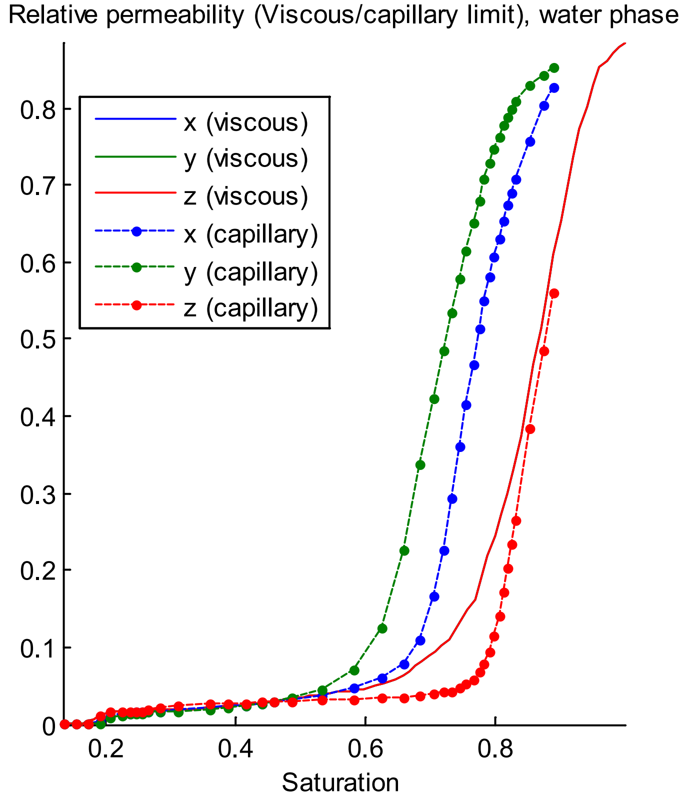



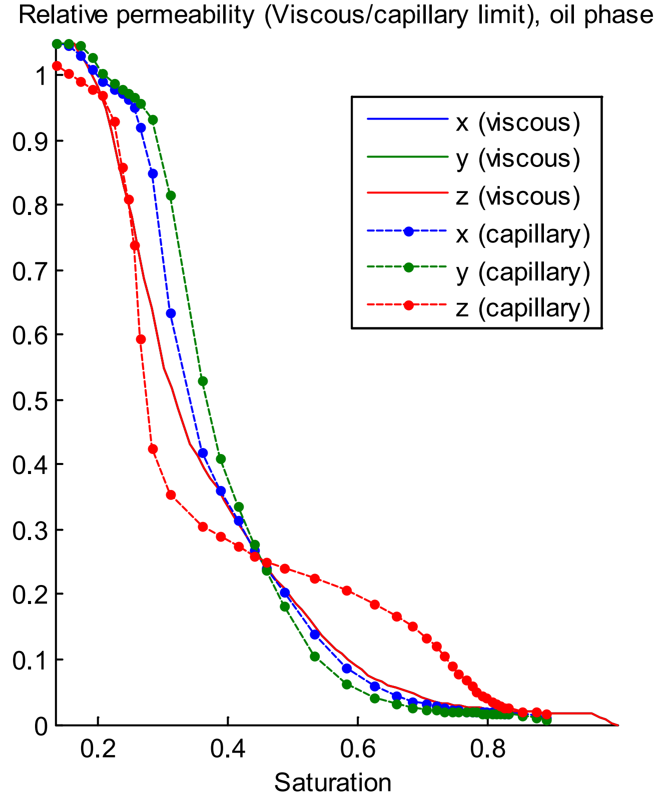


## Experience

- Depth-integrated models are highly efficient and sufficiently accurate to predict long-term plume migration
- Often more accurate than unresolved 3D simulations
- Gravity dominated flow highly sensitive to small changes in top surface

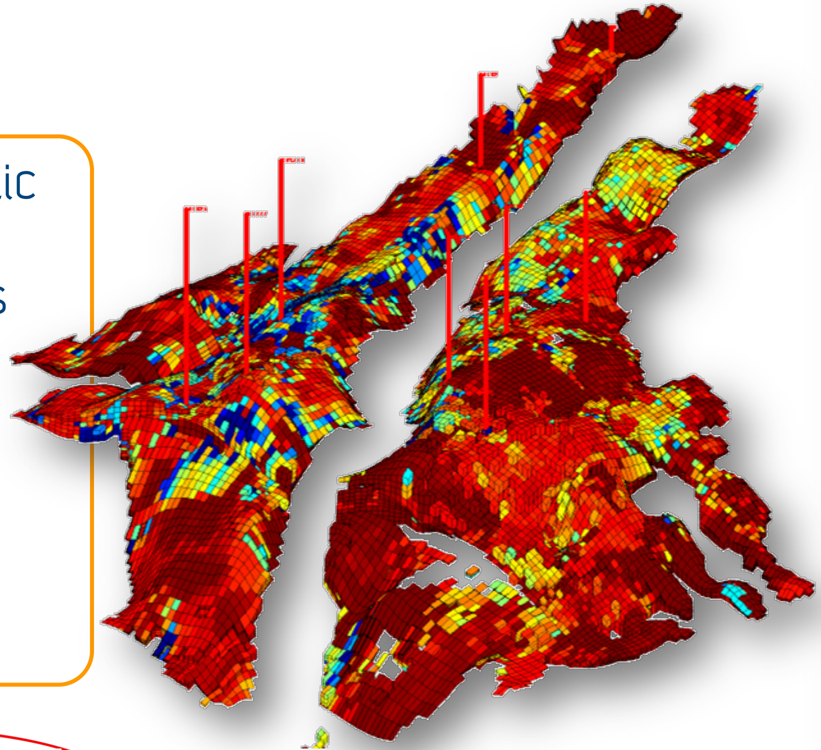
# Relperm upscaling:



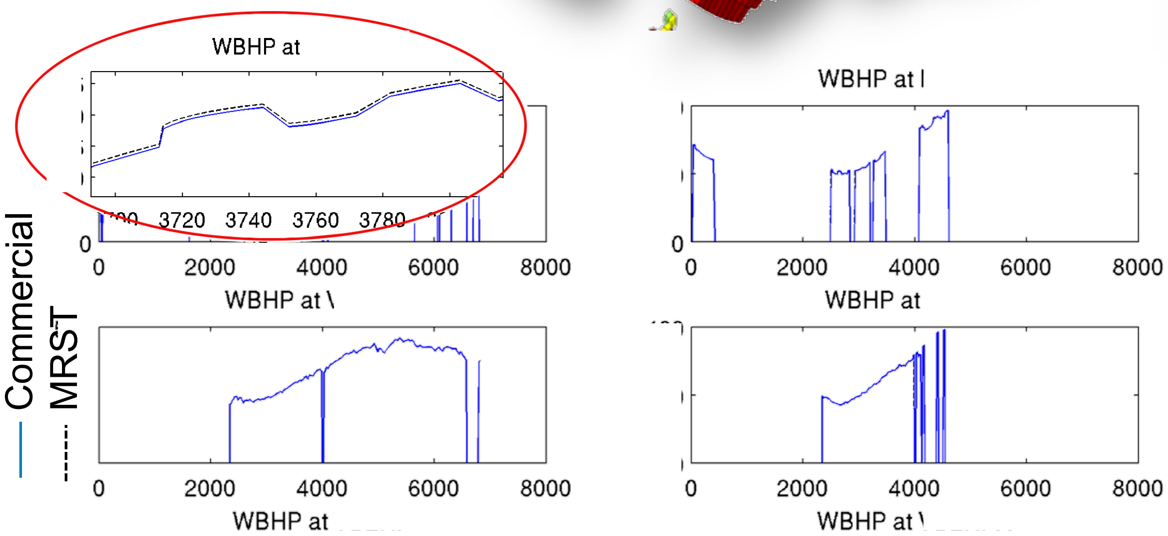



# Fully implicit code

- Based on automatic differentiation for autoamtic generation of Jacobians
- Gradients obtained through adjoint simulations
- Current models
  - Oil/water (+ polymer/surfactant)
  - Oil/gas
  - 3-phase black oil (live oil/dry gas)



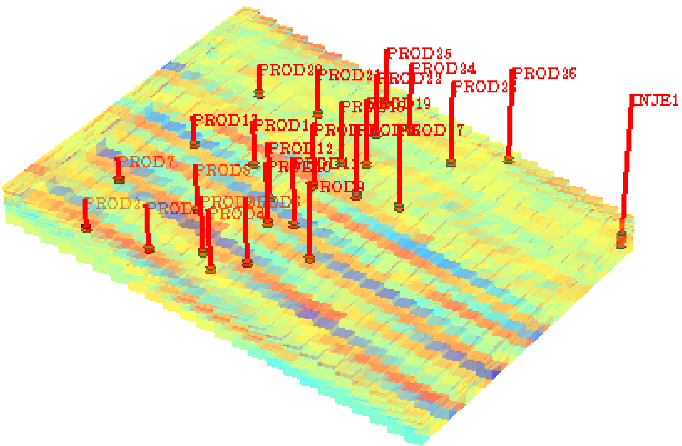
- Benchmarked against commercial simulator on real field black oil model
- ~20 years of historic data
- Virtually identical results



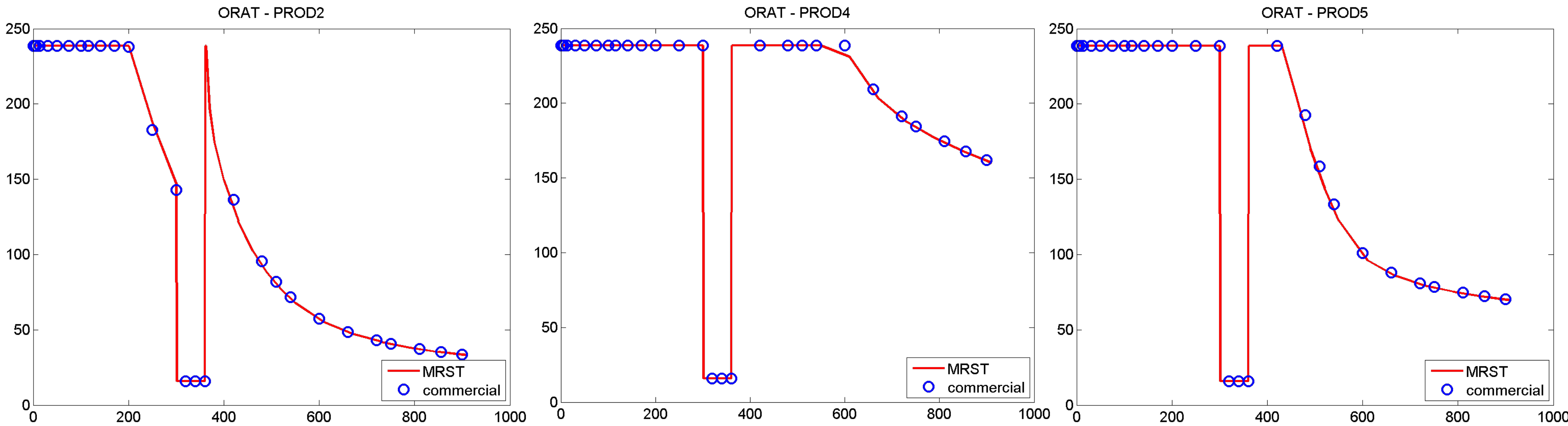
# Numerical Example (Black oil)

### SPE9 – 3 phase black-oil

- 1 water injector, rate-controlled – switches to bhp
- 25 producers, oil-rate controlled – most switch to bhp
- Appearance of free gas due to pressure drop
- Almost perfect match between MRST and commercial simulator

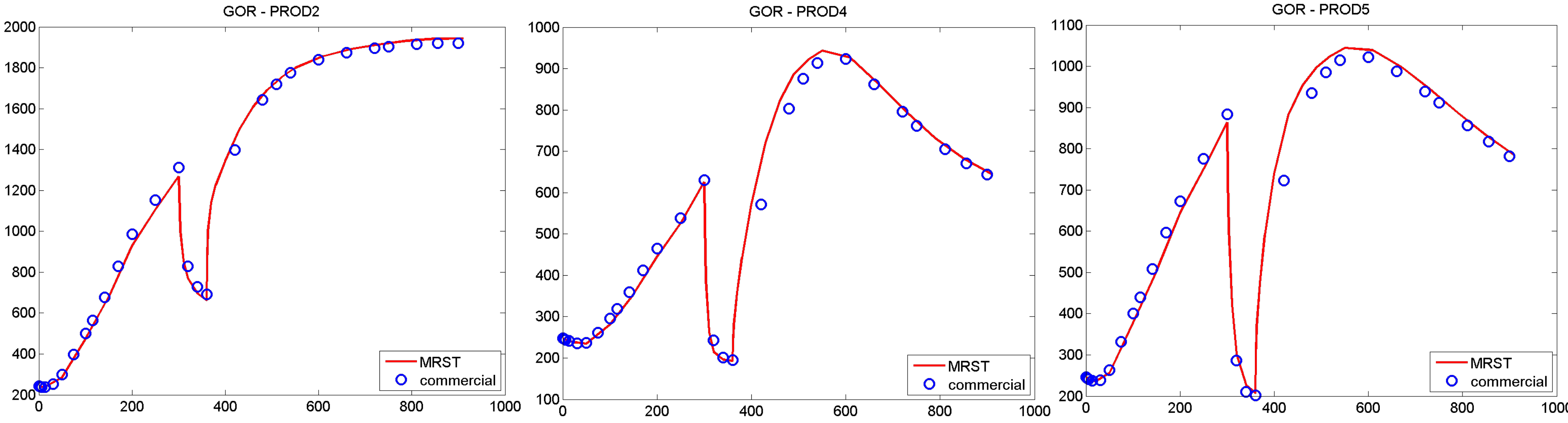



## Oil rates at producers 1, 3 and 4

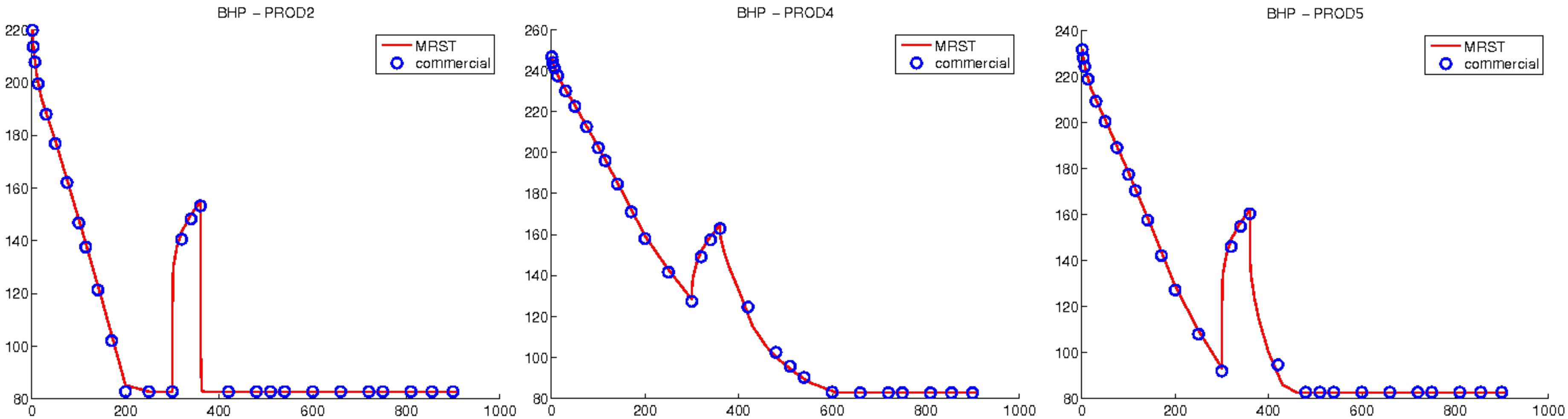


# Numerical Example (Black oil)

## GOR at a producer 1, 3 and 4




## BHP at producers 1, 3 and 4

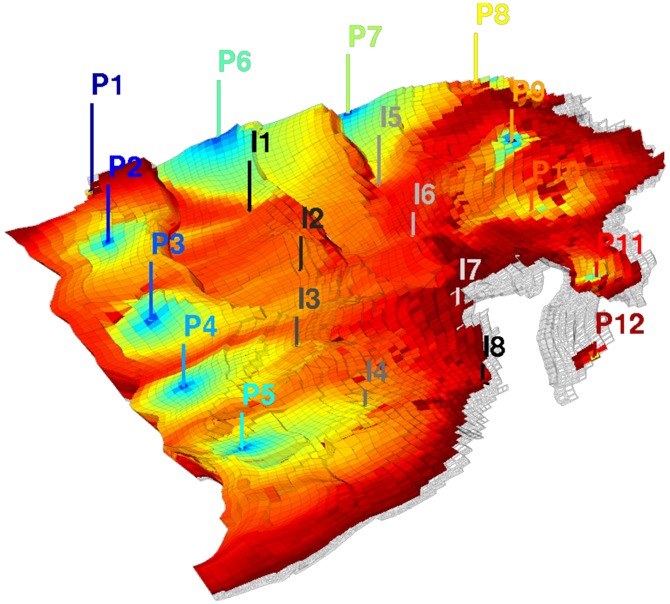


# Background: time-of-flight (TOF) and tracer equations

In this context: TOF and stationary tracer equations are solved efficiently after a single flow (pressure) solve:

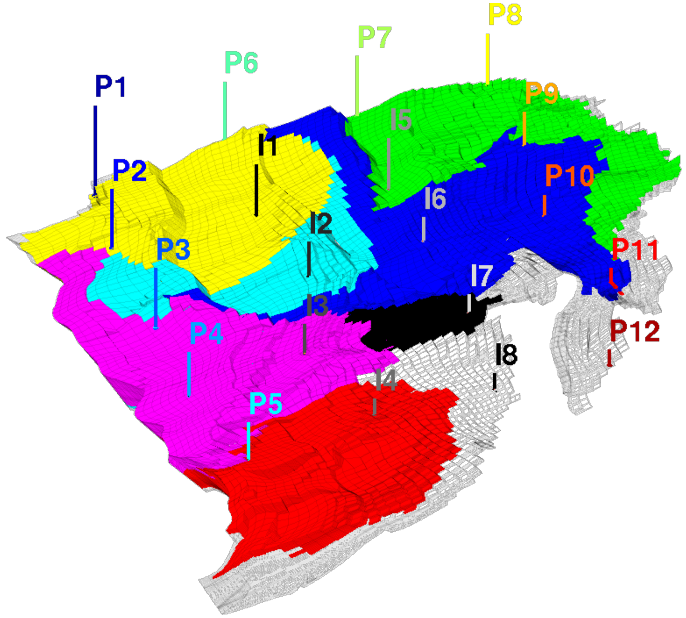
TOF: the times it takes for a *particle* to travel from

- injector to a given location
- a given location to a producer




Stationary tracer: portion of volume that *eventually* will

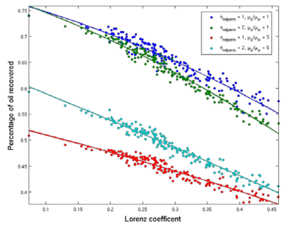
- arrive at a given producer
- come from a given injector



# Diagnostics based on time-of-flight (TOF) and tracers

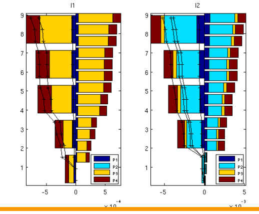
## Efficient ranking of geomodels

- Reduce ensamble prior to (upscaling and) full simulation
- Need measures that correlate well with e.g., receovery prediction



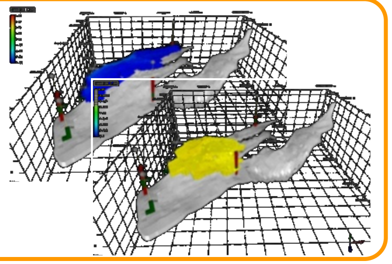

## Validation of upscaling

- Use allocation factors for assessing quality of upscaling



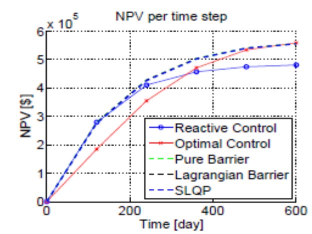
## Visualization

- See flow-paths, regions of influence, interaction regions etc
- Immediately see effect of new well-placements, model updates etc.



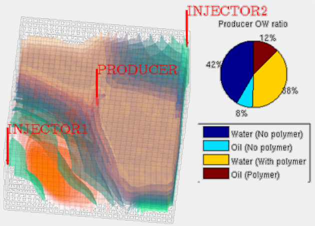
## Optimization

- Use as proxies in optimization to find good initial guesses.
- Need measures that correlate well to objective (e.g, NPV)

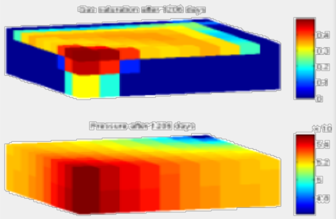



SINTEF

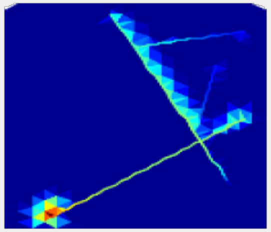
# MRST add-on modules



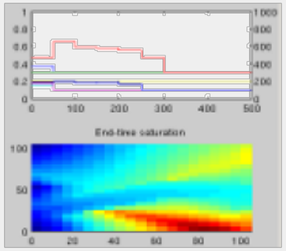

Fully implicit solvers (AD and gradients)



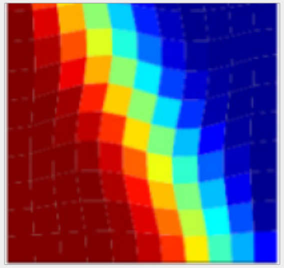
IMPES black-oil solvers



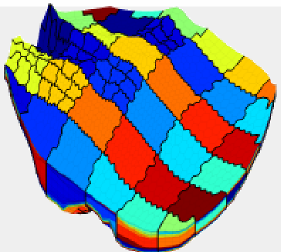
Discrete fracture models



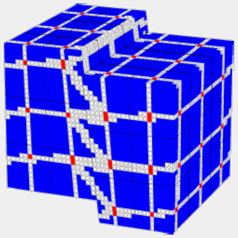

Adjoint methods



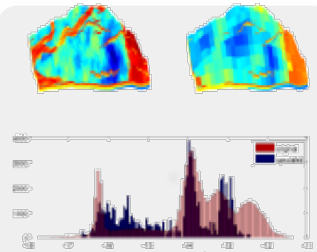
MPFA methods



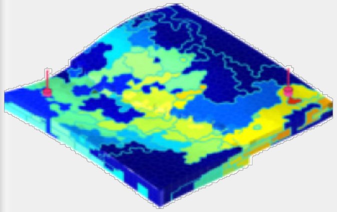
Multiscale mixed finite elements



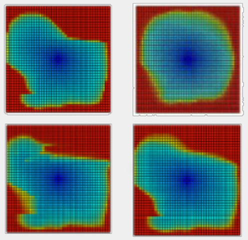
Multiscale finite volumes



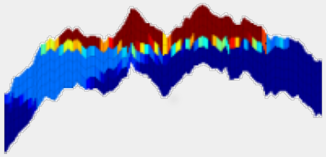
Single and two-phase upscaling



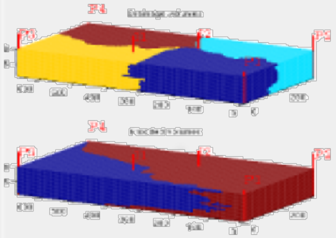
Grid coarsening



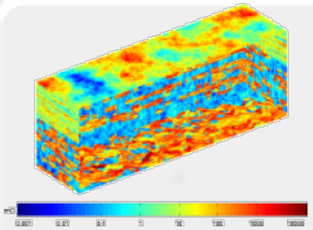
Ensamble Kalman filter



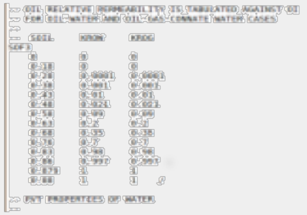
CO2 laboratory



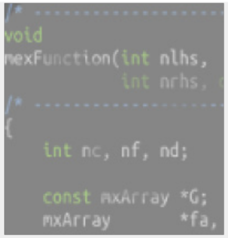
Flow diagnostics



Data sets (e.g. SPE 10)



Industry standard input formats



C-accelerated routines

# Fit-for-purpose reservoir simulation

Flexible simulators that are easy to extend with new functionality and scale with the requirement for the accuracy and computational budget

**accuracy +speed + robustness + access to gradients + model tuning**

seconds | minutes | hours

**Diagnostics/proxies**

- Physics.-based proxies
- Not accurate but qualitatively correct
- Optimization: fast response enables extensive search
- Characterization: ranking of model ensembles

**Upscaling**

- Traditional upscaling
- Mulitscale methods
- Model-reduction techniques
- Training runs to calibrate upscaling/model reduction
- Case-based upscaling enables more aggressive coarsening

**Fully implicit**

- Automatic differentiation: rapid development of new time-consuming but robust fully-implicit simulators
- Fast simulation methods (educated simplifications)
- Sensitivities: adjoint

# Black-oil model

$$\frac{d}{dt}(\phi b_o s_o) + \nabla \cdot (b_o v_o) - b_o q_o = 0$$

$$\frac{d}{dt}(\phi(b_g r_v s_g + b_o s_o)) + \nabla \cdot (b_g r_v v_g + b_o v_o) - (b_g r_v q_g + b_o q_o) = 0$$

$$\frac{d}{dt}(\phi(b_g s_g + b_o r_s s_o)) + \nabla \cdot (b_g v_g + b_o r_s v_o) - (b_g q_g + b_o r_s q_o) = 0$$

$$v_j = -\frac{k_{rj}}{\mu_j} K(\nabla p_j - \rho_j g \nabla z)$$

Water equation discretized in time:

$$\frac{V}{\Delta t}(\phi^{n+1} b_w^{n+1} s_w^{n+1} - \phi^n b_w^n s_w^n) + \nabla \cdot (b_w^{n+1} v_w^{n+1}) - b_w^{n+1} q_w^{n+1} = 0$$

Matlab code:

```
eqs{2}         = (pv/dt).*( pvMult.*bW.*sW - pvMult0.*f.bW(p0).*sW0 ) + s.div(bWvW);
eqs{2}(wc)   = eqs{2}(wc) - bWqW;
```