Errata and remarks: Non-homogeneous Random Walks

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Thanks to Conrado da Costa, Chak Hei Lo, and Stas Volkov for finding some of the errors reported here.

p41 In Theorem 2.4.5, the event '$\max_{m\geq 0} X_m \geq x$' should be replaced by '$\sup_{m\geq 0} X_m > x$' (twice). For the proof, note that $\sigma_x < \infty$ if $\sup_{m\geq 0} X_m > x$.

p41 Similarly, in Corollary 2.4.6, the event should be '$\sup_{m\geq 0} X_{m\wedge\lambda_y} > x$'.

p48 In equation (2.29), the exponent on the log term should be $\gamma - 2$, not $\gamma - 1$.

p50 The proof of Theorem 2.5.2 is somewhat indirect. For a nicer argument, see e.g. the proof of Lemma 4.1 of [3].

p62 In Theorem 2.6.2, the 'a.s.' should be deleted after '$\mathbb{E}\, X_{n\wedge\tau} \geq 0$'.

p65 In the second line of Example 2.6.6, it should say '$\theta_n := \xi_{n+1} - \xi_n$'. In the line below (2.58), it should say '$\xi^{(1)}_{n\wedge\tau}\xi^{(2)}_{n\wedge\tau} - \rho(n\wedge\tau)$ is a martingale'.

p70 In Remark 2.7.2, the constant $\delta^{-1/2}$ should read $\delta^{1/2}$ (twice). Similarly after the second display on p. 73.

p75 The first display on this page has a typo on the right-hand side of the inequality; instead of '$\cdots \geq 4C\varepsilon$' it should read '$\cdots \geq 1 - 4C\varepsilon$'. Two lines later, the text should say that the event implied is $\min_{0\leq k\leq \varepsilon X_0^2} X_k > X_0/2$ (i.e., X_k rather than W_k as is printed).

p98 The first line of display (3.8) should read

$$\|\mathbf{x}+\mathbf{e}\| - \|\mathbf{x}\| = \|\mathbf{x}\| \left(\left(1 + \frac{2\mathbf{e}\cdot\mathbf{x}+1}{\|\mathbf{x}\|^2} \right)^{1/2} - 1 \right).$$

p100 In the middle line of the 3-line display after (3.14), it should read $\mathbb{P}[\sigma > r \mid \mathcal{F}_0]$ instead of $\mathbb{P}[\delta > r \mid \mathcal{F}_0]$.

p103 There's a factor of 1/2 missing from the $\nu(\nu-1)$ term on the last line of (3.17). This comes from the second-order Taylor term at the bottom of p. 104, but goes missing in the display in the middle of p. 105. The same error occurs in (3.23) and (3.28). This doesn't affect anything that follows, since it is only ever the sign of $\nu(\nu-1)$ that is important.

p108 In the first display of Example 3.5.3, the $O(x^{-2})$ terms should both be $O(x^{-1})$.

p117 On the 5th line of Example 3.6.2 $\omega(x,y)$ should be $\omega(u,v)$.

p139 In the penultimate display, there's an X_m missing inside the second maximum.

p145 Lemma 3.10.8 is erroneous. Indeed, the claim that the last exit time η_x has $\mathbb{E}\,\eta_x < \infty$ is not always true for transient Lamperti processes. The statement of the lemma should be replaced by $\mathbb{E}\sum_{n=0}^{\infty} \mathbf{1}\{X_n \leq x\} < \infty$ for all $x \in \mathbb{R}_+$, which is what is required for the proof of Theorem 3.10.1. The argument in present proof of Lemma 3.10.8 is correct to the point where it is shown $\mathbb{E}\, N < \infty$, which implies that the expected number of visits to a bounded set is finite, as required. The error comes at the top of p. 146, where it is claimed that $\eta_x \leq 2rN$, which is not the case. The question of when $\mathbb{E}\,\eta_x^{\beta} < \infty$ for transient Lamperti processes is addressed in [2].

p221 The notion of 'angular ergodicity' mentioned here is too strong; cf. the arcsine phenomenon [1, Corollary 4.15].

p284 The proof of Lemma 6.3.4 needs correcting, as the denominator in (6.50) is zero. In the last clause of the first paragraph of the proof, choose instead x_0 large enough that $|g'(x)| < 1/(4c_0)$, say. Then after (6.48), let $a = 1/(2c_0)$. Then the denominator in (6.50) is positive, and the proof works.

p286 In the first display in the proof of Lemma 6.3.5, the two error terms $O(x^{2\gamma-2})$ should be $O(x^{3\gamma-2})$.

p286 There's an erroneous term in the last display on this page. The display should read

$$D = 2\gamma x^{2\gamma-1} + \left(2 + D\gamma x^{-1}\right) x^{\gamma} \tan\alpha + 2\gamma x^{2\gamma-1} \tan^2\alpha + O(x^{3\gamma-2}).$$

Then (6.54) on the top of the next page should say

$$\begin{aligned} D &= 2\gamma x^{2\gamma-1} + \frac{2x^{\gamma}\tan\alpha + 2\gamma x^{2\gamma-1}\tan^2\alpha}{1 - \gamma x^{\gamma-1}\tan\alpha} + O(x^{3\gamma-2}) \\ &= 2\gamma x^{2\gamma-1} + 2x^{\gamma}\tan\alpha + 4\gamma x^{2\gamma-1}\tan^2\alpha + O(x^{3\gamma-2}). \end{aligned}$$

The rest of the argument is not affected.

p292 In the bullet points on this page, each but the first has an S that should be an s.

p296 The fourth line of the first display should say '$f_1(s_k^{-r}) - f_1(s) = \ldots$'.

References

[1] C.H. Lo, J. McRedmond, and C. Wallace, Functional limit theorems for random walks. arXiv:1810.06275.

[2] C.H. Lo, M.V. Menshikov, and A.R. Wade, Strong transience for one-dimensional Markov chains with asymptotically zero drifts. arXiv:2208.12955.

[3] M.V. Menshikov, A. Mijatović, and A.R. Wade, Reflecting random walks in curvilinear wedges. Chapter 26, pp. 637–675 in *In and Out of Equilibrium 3: Celebrating Vladas Sidoravicius*, Eds. M.E. Vares, R. Fernández, L.R. Fontes, C.M. Newman, Progress in Probability 77, Springer, 2021.