Anna Felikson, Durham University **Geometry**, 3.2.2023

Feedback 11-12

For all questions:
a solution is much easier to read and understand when it is supplied with a diagram!
I expect, you might want me to be able to understand your solutions...

In general, I thought this was a more difficult set of questions (this was my experience from previous years) - but the results of most of those who submitted are really good. Thanks to everyone for that!

- **Question 11.3:**
  - Part (a) is an easy Euclidean question, almost everybody did very well with it (and the only reason to include it was that it was a useful tool for part (b)).
  - It is almost clear in the question like part (b) that one should use something like triple transitivity. Then the tricky thing is to decide what and where to map. Those, who started with mapping one of the intersection points to ∞ arrived to the required answer very quickly. Some of others mapped three points to 1, -1 and i, say. With this start one still can finish the question, but this is much more difficult. Conclusion: it may make sense to spend a couple of extra minutes on thinking whether your choice of the map is optimal. In particular, whether it would be beneficial to send that or this point to infinity.
  - In some solutions the map chosen to send an intersection point to ∞ was an inversion. Inversion is NOT a Möbius transformation. So, if you start with inversion, one should compose with another inversion or reflection to make sure the result is Möbius and not anti-Möbius.
  - In some solutions, it was assumed that Möbius transformations take centres of the circles to centre of the circles - which is wrong!
  - Minor thing: when speaking about Möbius transformations it is more natural to use complex coordinates on the plane than two real coordinates (because the linear-fractional map uses the complex coordinate).
- **Question 11.4:** This was an easy question to do using similar triangles.
- **Question 12.2:**
  - While explaining the construction, please use some steps or items: then it is easier to read!
  - The algorithm (and the proof also!) is easier to describe and to understand when there is an informative diagram accompanying it (it should not necessarily be a neat deagram, but it is good when it is labelled). One can write solutions without diagrams, but then they need to be even more precise and more detailed.