Anna Felikson, Durham University **Geometry**, 19.3.2023

Feedback 17-18

For all questions:

As usually: please, please support your solutions with diagrams!!!.

- **Question 17.4:**
  - Most solutions of the question started with something like "let's use the classification of isometries" - which is the best strategy here. However, later many students fall into different traps:
    - One should remember there are orientation-preserving isometries and orientation-reversing ones;
    - There are two types of "translations" in $\mathbb{H}^2$: the parabolic and the hyperbolic ones!
    - All orientation-preserving isometries in $\mathbb{H}^2$ have a fixed point in $\mathbb{H}^2$ or on the absolute. But only the elliptic one have fixed points inside $\mathbb{H}^2$. (Fixed points on the boundary do not help to conclude f is identity, as distances are only defined for two points lying inside $\mathbb{H}^2$).
- **Question 17.5:**
  - While computing in the hyperboloid model, remember
    - to use the pseudo-scalar product $(\mathbf{u}, \mathbf{v}) = u_1v_1 + u_2v_2 - u_3v_3$.
    - to square (u, v) in the numerator of $Q = \frac{(\mathbf{u},\mathbf{v})^2}{(\mathbf{u},\mathbf{u})(\mathbf{v},\mathbf{v})}$
    - Also, remember that when computing in the vector model, it is not necessary to have $\langle a, a\rangle = 1$ (as soon as you do not forget about the denominator in Q). So, you can use the vector which is more convenient for computations (i.e. for example $(1, 1, 0)$ rather than something with $\sqrt{2}$).
  - Remember, if l_1 and l_2 are two intersecting lines forming an angle θ then the composition of reflections in these lines is a rotation by the angle 2θ!
- **Question 18.2:**
  - When working with circles, horocycles of equidistant curves, remember:

    they are not straight lines!

    In particular, congruence of triangles applies to triangles formed of straight lines, the same holds for any statements about sum of angles.