Assignment 1-2
Starred problems due on Friday, 21 October

1.1 In this exercise we recall basic theorems of Euclidean geometry.
For each Theorem or Corollary in the handout do the following:

(a) draw the diagram;

(b) Decide whether the statement is new for you or you have already seen this statement before (say, in school)?

(c) list the statements which look new to you.
Remark: Please, prepare the list in the form "E6, E17, E16:cor, E20:converse". We will collect this info anonymously during Lecture 3.

2.1 (*) Let $Isom^+(\mathbb{E}^2) \subset Isom(\mathbb{E}^2)$ be a group of orientation-preserving isometries of $\mathbb{E}^2$.
Show that $Isom^+(\mathbb{E}^2)$ is generated by rotations.

(*Recall*: a group G is generated by the set of elements S, if for every element of $g \in G$ there exist finitely many elements $g_1, \dots, g_{n(g)}$ of G such that $g = g_{n(g)} \circ \dots \circ g_1$).

2.2 Show that a composition of a rotation and a translation is a rotation by the same angle. How to find the centre of the new rotation?

2.3 A *glide reflection* is a composition of a reflection with respect to a line and a translation along the same line. Show that every composition of 3 reflections in $\mathbb{E}^2$ is either a glide reflection or a reflection.

2.4 (*) List all finite order elements of the group $Isom(\mathbb{E}^2)$. Justify your answer.

2.5 Let t_a be a translation by the vector a and let $R_{\alpha,z}$ be a rotation by angle α around $z \in \mathbb{C}$. What can you say about the isometry $f = R_{\alpha,z} \circ t_a \circ R_{-\alpha,z}$?

2.6 Give an example of an isometry $f : \mathbb{E}^2 \to \mathbb{E}^2$ and a set $A \subset \mathbb{E}^2$ for which

(a) $f(A) \subset A$, $f(A) \neq A$;

(b) A is a bounded set, $f(A) \subset A$, $f(A) \neq A$.

2.7 (*) Let $x = (x_1, x_2)$ be a point in $\mathbb{E}^2$ and $a = (a_1, a_2)$ be a vector. Consider the line given by the equation $\langle x, a\rangle = 0$, i.e. the set of points $\{(x_1, x_2) \mid a_1x_1 + a_2x_2 = 0\}$.
Show that the transformation

$$f : x \mapsto x - 2\frac{\langle x, a\rangle}{\langle a, a\rangle}a$$

(a) is an isometry;

(b) preserves the line $\langle x, a\rangle = 0$ pointwise;

(c) is a reflection with respect the the line $\langle x, a\rangle = 0$.

(d) What is the geometric meaning of $\frac{\langle x,a\rangle}{\langle a,a\rangle}a$?
(It should help you to see that f is the reflection even without any computations).

2.8 (Mirror on the wall)
Assume you are 2m tall and looking at the wall mirror from 1m away. How long the mirror should be so that you could see both your toes and your head? How does the answer depend on your hight? on the distance to the mirror?