Riemannian Geometry

Hints 11-12

1. (*)

   (b) There are at least to natural approaches:
   First approach. In general, to show that a curve c is geodesic, one can check that the equation $\frac{D}{dt}c' = 0$ holds along $c(t)$.
   Second approach. In case when the metric have some obvious symmetries (as in our case), one can use the symmetry together with the uniqueness of geodesic running from the given point in the given direction.

   (d) One may use Hopf-Rinow theorem or derive statement directly from the definition and (c).

2. (a) First, compute A^n for $n \leq 4$.

3. (b) Compute X_i^n and use well-known power series.

4. (a) Use that fact that $Ad(g_1)Ad(g_2) = Ad(g_1 g_2)$.
   Then apply left-invariance of $dvol$ for a suitable function f.

5. (*)

   (a) Substitute X be Y and cancel zero terms. (Explain, why these are zeros!)

   (c) Apply the result of (b) to the vector field $X + Y$.