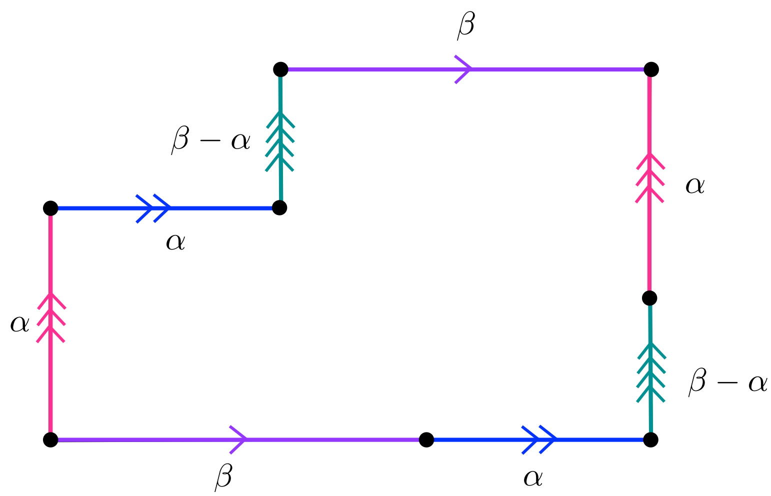
Amie Wilkinson Exercises

Lecture 3

(1) Let $f\colon X \to X$ be a homeomorphism of a compact metric space.
   (a) Prove that if f is transitive, then for every continuous $\psi\colon X \to \mathbb{R}$, if $\psi \circ f = \psi$, then ψ is constant.
   (b) Prove that the converse is false. **Hint:** Glue together two copies of the cat map.

(2) The polygon below, when the edges are glued according to the arrows depicted, gives a surface (the lengths of the edges are labeled in terms of two parameters, α and β).



   (a) Tile the plane with copies of this polygon in a way that respects the labeling of the edges.
   (b) Show that the surface obtained by this gluing is a torus. For what values of α, β is this the standard torus obtained by gluing opposite edges of a unit square?
   (c) Let $\lambda > 1$ be the golden mean, that is, the larger solution to

$$x^2 - x - 1 = 0.$$

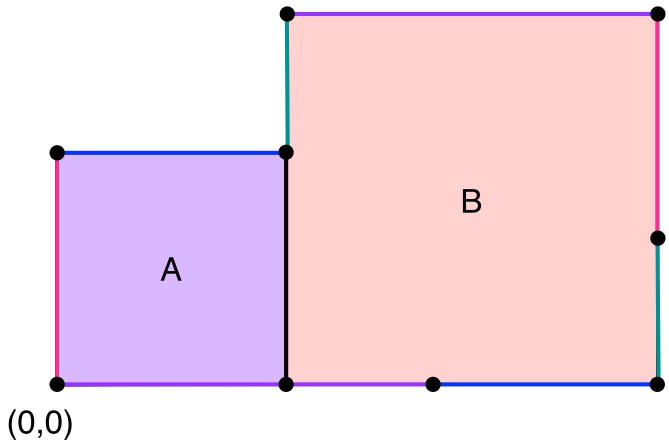
   Consider the tiling of the plane from part (a), where the polygon is divided into squares A and B as shown below. Suppose that

$$\alpha = \frac{1}{\sqrt{1+\lambda^2}}, \ \beta = \frac{\lambda}{\sqrt{1+\lambda^2}}.$$

   Apply the matrix

$$C = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda^{-1} \end{pmatrix}$$

   to A and B and depict their image in the tiling. Show that C defines an automorphism of the square torus. What matrix represents C with respect to the standard lattice for the square torus?


B
A
(0,0)