Exercise B1. Consider $h : \Sigma_2^+ \to \S^1$ defined by

$$h(\underline{x}) = \bigcap_{n=0}^{\infty} E_2^{-n}(\Delta_{x_n}).$$

(a) Prove that

$$\bigcap_{n=0}^{N} E_2^{-n}(\Delta_{x_n})$$

is an interval of length $\frac{1}{2^{N+1}}$.

(b) Prove that h is continuous.

(c) Prove that h is surjective.

Exercise B2. Now let $f : \S^1 \to \S^1$ be a general expanding map of $\deg(f) = 2$ and define

$$h(\underline{x}) = \bigcap_{n=0}^{\infty} f^{-n}(\Delta_{x_n}).$$

Prove that

$$\bigcap_{n=0}^{N} f^{-n}(\Delta_{x_n}) \neq \emptyset$$

is an interval.