

KHADIM WAR EXERCISES

Let $\varepsilon > 0$ sufficiently small and $f_\varepsilon : \mathbb{T}^2 \rightarrow \mathbb{T}^2$ be a smooth map such that $\|f_\varepsilon - f_A\|_{C^1} < \varepsilon$ where f_A is given by the cat map. Prove that the following are equivalent:

- (1) $\exists \lambda, \alpha \in (0, 1)$ and a family of cone fields $\{C^s(x, \alpha), C^u(x, \alpha) : x \in \mathbb{T}^2\}$ such that

$$Df_\varepsilon^{-1}C^s(x, \alpha) \subset C^s(f_\varepsilon^{-1}(x), \lambda\alpha) \quad \text{and} \quad Df_\varepsilon C^u(x, \alpha) \subset C^u(f_\varepsilon(x), \lambda\alpha)$$

- (2) There exists n_0 such that for $x \in \mathbb{T}^2$ we have $T_x\mathbb{T}^2 = E_x^s \oplus E_x^u$, $Df_\varepsilon(E_x^\sigma) = E_{f_\varepsilon(x)}^\sigma$, for $\sigma = s, u$ and $\|Df_\varepsilon^{n_0}|_{E^s}\| < 1 < \|Df_\varepsilon^{n_0}|_{E^u}\|$.