PROJECT IV: RATIONAL POINTS ON ALGEBRAIC VARIETIES

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Let $F(x_1, ..., x_n) \in \mathbb{Z}(x_1, ..., x_n)$ be a polynomial in n variables with integer coefficients. The question of determining when F has a non-trivial integral solution, i.e., when $F(\mathbf{x}) = 0$ for some $\mathbf{x} \in \mathbb{Z}^n \setminus (0, ..., 0)$, is one of the oldest problems in Number theory. An early result in this field is due to Lagrange in 1770, who proved that every natural number can be written as a sum of four squares, also known as the *Langrange four square theorem.* Some well studied problems which come under this umbrella include the Waring's problem, theory of Quadratic forms and the theory of more general homogeneous forms etc. In recent years the *Hardy-Littlewood Circle method* has been rather successful in answering some of these questions.

This project will be devoted to exploring the analytic aspects of this problem. The basic tools include Poisson summation formula, Dirichlet approximation theorem, bounds for exponential sums etc. The techniques learnt here are useful for other problems in Analytic Number Theory as well.

Prerequisites: A basic knowledge of real and complex analysis, and number theory is required.

References: H. Davenport, *Analytic methods for Diophantine equation and Diophantine inequalities.*
A. Karatsuba, *Basic Analytic Number Theory.*