Exercises - 1

1. List all clusters in the cluster structure associated with the homogeneous coordinate ring of $Gr_2(5)$.
2. Describe all elements of an additive basis of homogeneous polynomials in the Plücker coordinates x_{ij} of degree 2 in the homogeneous coordinate ring $\mathbb{C}[Gr_2(5)]$.
3. Check that the medial graph of triangulation of pentagon changes according to the mutation rule under flip.

- **Definition**. An integer $n \times n$ matrix B is *left (right) skew-symmetrizable* if there is an integer diagonal $n \times n$ matrix D such that DB (BD, correspondingly) is skew-symmetric.

4. Show that any left skew-symmetrizable matrix is also right skew-symmetrizable.
5. Check that the form $\omega = \sum_{ij} b_{ij} \frac{dx_i}{x_i} \wedge \frac{dx_j}{x_j}$ is closed, i.e. $d\omega = 0$. Show that distribution $Ker(\omega)$ is integrable, i.e. there is a submanifold S in $\mathbb{A}^n$ of dimension $\dim Ker(\omega)$ such that the tangent space $T_x(S)$ at each point $x \in S$ coincides with $Ker(\omega)|_x$.
6. Let rational functions $\{f_i(x_1, \dots, x_n)\}_{1 \le i \le n}$ be generators of the field of rational functions $\mathbb{C}(x_1, \dots, x_n)$, $(\omega_{ij})_{1 \le i,j \le n}$ be a skew-symmetric matrix. Then, $\{f_i, f_j\} = \omega_{ij} f_i f_j$ determines a Poisson bracket on $\mathbb{A}^n$.
7. Check that diagram mutation is well defined.
8. Check that the mutation class of the following matrix
$$\begin{pmatrix} 0 & 2 & -4 \\ -1 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$
is finite.
9. Let $F^\bullet, G^\bullet$ be two transversal complete flags in $\mathbb{R}^4$. Compute the number of connected components of the set of complete flags in $\mathbb{R}^4$ transversal to both $F^\bullet$ and $G^\bullet$.