DURHAM UNIVERSITY
Department of Mathematical Sciences

Levels 3 and 4 Mathematics modules
Course Booklet
2015 - 2016

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Contents

1 General Information

Welcome to 3H or 4H Mathematics! About 1,200 undergraduates take modules provided by the Department. This booklet provides information on third and fourth-year modules offered by the department.
It also contains summary information on key policies related to assessment and academic progress.

Full details of the department's policies and procedures are available in the departmental degree programme handbooks at http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/ , which also contains on on-line version of the course descriptions contained in this booklet.

Information concerning general University regulations, examination procedures etc., are contained in the Faculty Handbooks (`www.dur.ac.uk/faculty.handbook`) and the University Calendar, which provide the definitive versions of University policy. The Teaching and Learning Handbook (`www.dur.ac.uk/teachingandlearning.handbook`) contains information about assessment procedures, amongst other things.

You should keep this booklet for future reference. For instance, prospective employers might find it of interest. Whether you are entering the final year of your course, or the third year of a four year course, there is a good choice of options and you can study those topics which you find most interesting. You can look forward to an enjoyable year.

1.1 Useful Contacts

The first point of contact for issues referring to a particular course or module should be the relevant lecturer. For more general questions or difficulties you are welcome to consult the Course Director or your Adviser. For queries relating to teaching issues, for example registration, timetable clashes, support for disabilities or illness, you should visit the department to speak to someone in the main Maths Office (CM201), or send an email to maths.teaching@durham.ac.uk.

Head of Department: maths.head@durham.ac.uk
Director of Undergraduate Studies:
Dr Peter Bowcock (CM307, peter.bowcock@durham.ac.uk)

The Course Directors for students are determined by their programme and level of study as follows:
Students on Mathematics programmes at level one:
maths.1hcoursedirector@durham.ac.uk
Students on Mathematics programmes at level two:
maths.2hcoursedirector@durham.ac.uk
Students on Mathematics programmes at levels three and four:
maths.34hcoursedirector@durham.ac.uk
Students on Natural Sciences and Combined Honours programmes at all levels:
maths.natscidirector@durham.ac.uk
Students on programmes other than Mathematics and Natural Sciences and Combined Honours at all levels:
maths.otherprogdirector@durham.ac.uk

We may also wish to contact you! Please keep the Mathematics Office informed of your current term-time residential address and e-mail address.

1.2 Registration for 3H

You will register for the required number of modules in June. You may attend additional modules during the first few weeks of the Michaelmas Term. If you then decide that you want to change one or more of your modules you must contact maths.teaching@durham.ac.uk or visit the main Maths office (CM201). Any such change should be completed during the first three weeks of the Michaelmas Term.

1.3 Course Information

Term time in Durham is Michaelmas (10 weeks), Epiphany (9 weeks) and Easter (9 weeks). There are 22 *teaching* weeks, and the last seven weeks are dedicated to private revision, examinations and registration for the subsequent academic year.

Timetables giving details of places and times of your commitments are available on Departmental web pages and noticeboards in the first floor corridor of the Department. It is assumed that you read them!

You can access your own Maths timetable at `www.maths.dur.ac.uk/teaching/` and then clicking on the 'My Maths timetable' link.

Also, teaching staff often send you important information by e-mail to your local '@durham.ac.uk' address, and so you should scan your mailbox regularly.

1.4 Assessment

Full details of the University procedures for Examinations and Assessment may be found in Section 6 of the Learning and Teaching Handbook, http://www.dur.ac.uk/learningandteaching.handbook/. The Department's policies and procedures are described in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/ . The Department follows the marking guidelines set out by the University Senate:

Degree Class	Marking Range(%)
I	70 – 100
II(i)	60 – 69
II(ii)	50 - 59
III	40 – 49
Fail	0 – 39

With the exception of the final year project (MATH3382 Project III), Statistical Methods III (MATH3051) and Mathematics Teaching (MATH3121), assessment for third year modules is by written examination.

For Statistical Methods III (MATH 3051), 30% of the module is based on in-year exams, one at the end of the Michaelmas term and one at the end of the Epiphany term. For Mathematics Teaching (MATH 3121), 20% of the assessment is based on the school file, 30% on summative coursework, 10% on a presentation, and 40% is based on a written report of the students' placement projects.

For Project III (MATH 3382), 15% of the assessment is based on a presentation and poster, and 85% is based on a written report. All courses include either summative or formative assessed work, with assignments being set on a regular basis in lecture-based courses. The purpose of formative and summative assessment of coursework is to provide feedback to you on your progress and to encourage effort all year long.

Regular assignments are marked A-E to the following conventions:

Grade	Equivalent Mark	Quality
A	$\geq 80\%$	*Essentially complete and correct work*
B	60%—79%	*Shows understanding,* *but contains a small number of errors or gaps*
C	40%—59%	*Clear evidence of a serious attempt at the work,* *showing some understanding, but with important gaps*
D	20%—39%	*Scrappy work, bare evidence of understanding* *or significant work omitted*
E	<20%	*No understanding or little real attempt made*

Use of Calculators in Exams The use of electronic calculators is allowed in some module examinations and other module assessments. Each student taking modules offered by departments or schools within the science faculty, which specify that calculators be allowed in assessments, will be offered a calculator, free of charge, at the beginning of their course. The model will be a Casio fx-83 GTPLUS or a Casio fx-85 GTPLUS.

Calculators will become the property of students who will be responsible for their upkeep. No replacement calculators will be provided free of charge, but may be available to purchase from departments/schools, depending on availability. The specified calculator will also be generally available, in shops and online, should a replacement purchase be necessary.

Where the use of calculators is allowed in assessments, including examinations, the only models that will be allowed are either a Casio fx-83 GTPLUS or a Casio fx-85 GTPLUS. In particular, examination invigilators will refuse to allow a candidate to use any calculator other than the model(s) specified, which will be explicitly stated on the front of the examination paper. During examinations no sharing of calculators between candidates will be permitted, nor will calculators or replacement batteries be supplied by the Department or the Student Registry Office.

1.5 Academic progress

The Department is responsible for ensuring that students are coping with the courses and meeting their academic commitments.
For 3rd year modules you are required:
- for Project modules, to attend meetings arranged with the supervisor, submit your poster and draft and final written reports on time, and give a presentation as scheduled.
- to submit summative or formative assessed work on time to a satisfactory standard.
Assessed work which is graded D or E is counted as being of an unsatisfactory standard.
Attendance and submission of work is monitored through a database. It is your responsibility to ensure that your attendance is recorded by signing the relevant attendance sheets.
Students who are not keeping up with their commitments will be contacted by course directors to help get them back on track.

Persistent default will result in a formal written warning, which may be followed by the initiation of Faculty procedures.

Full details of academic progress requirements for the department are available in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/ .

1.6 Durham University Mathematical Society

MathSoc: Necessary and Sufficient

Durham University Mathematical Society, affectionately known as MathSoc, provides an opportunity for maths students (or anyone with an interest in maths) to meet away from lectures.

We arrange a variety of events throughout the year, including bar crawls, talks by guest speakers, a Christmas meal, and the highlight of the year – a trip to see Countdown being filmed! So there's something for everyone. We are currently sponsored by EY so we are able to offer many exclusive networking opportunities and careers events!

MathSoc works with the Maths Department to arrange Undergraduate Colloquia, where departmental and external lecturers give relaxed, inspiring talks on their current research. These cover a wide range of mathematical topics with previous titles including 'Dot-dots, zig-zags and plank-planks' and 'Defects of integrable field theory'. These are at a level such that anyone with an interest in maths can enjoy them and they aim to motivate an interest in an area of maths you may not have seen before.

We have our own website (durhammathsoc.wix.com/mathematicalsociety), where you will find all the most up-to-date information about the society. Here you will also find our second-hand book list, which has many of the books needed for courses for much cheaper than you will find them in the shops. Last year people saved up to £50 by using this service!

If you would like any more information about either the society itself, or advice on any other aspect of the maths course, please do not hesitate to get in touch with any of our friendly exec members listed below or via the society email address (mathematical.society@durham.ac.uk). You can also find us on Facebook by searching for "The Official Durham MathSoc Group".

To join:

Come and see our stand at the freshers' fair, or email us at any time: it costs only £7 for life membership, or £4 for a year. You can sign up on the Durham Students Union Website (http://www.durhamsu.com/groups/mathematical). You can also find us on Facebook by searching for "The Official Durham MathSoc Group" and follow @DUMathsoc on Twitter!

This year's Exec is:

President	Eleanor Kershaw-Green (e.l.kershaw-green@durham.ac.uk)
Secretary	Fleur Goff-Beardsley (fleur.goff-beardsley@durham.ac.uk)
Treasurer	Becca Rickwood (r.j.rickwood@durham.ac.uk)
Social Secretary	Gina Cuomo (g.m.cuomo@durham.ac.uk)
Publicity Officer	Becky Harvey (r.k.harvey@durham.ac.uk)

1.7 Disclaimer

The information in this booklet is correct at the time of going to press in May 2015. The University, however, reserves the right to make changes without notice to regulations, programmes and syllabuses. The most up-to-date details of all undergraduate modules can be found in the Faculty Handbook on-line at `www.dur.ac.uk/faculty.handbook/`.

1.8 Booklists and Descriptions of Courses

The following pages contain brief descriptions of the level 3 and 4 modules available to you.These supplement the official descriptions in the module outlines in the faculty handbook which can be found at
`http://www.dur.ac.uk/faculty.handbook/module_search/?search_dept=MATH&search_level=3` or
`http://www.dur.ac.uk/faculty.handbook/module_search/?search_dept=MATH&search_level=4`. Note that the official module outlines contain information on module pre- and co-requisites, excluded combinations, assessment methods and learning outcomes. The descriptions which follow supplement this by providing a list of recommended books and a brief syllabus for each module.

For some modules you are advised to buy a particular book, indicated by an asterisk; for others a choice of titles is offered or no specific recommendation is given. There are also suggestions for preliminary reading and some time spent on this during the summer vacation may well pay dividends in the following years.

Syllabuses, timetables, handbooks, exam information, and much more may be found at `www.maths.dur.ac.uk/teaching/`, or by following the link 'teaching' from the Department's home page (`www.maths.dur.ac.uk`). These syllabuses are intended as guides to the modules. The definitive information on course content and expected learning outcomes is in the official module outlines.

1.8.1 ALGEBRAIC GEOMETRY III AND IV – MATH3321/MATH4011 (38 lectures)

Dr A. Bouganis

Algebraic geometry studies polynomials in several variables. It does so by means of a dictionary between algebraic notions (rings, ideals etc) and geometric notions (curves, surfaces, tangent spaces etc). The resulting theory is remarkably rich, bringing powerful algebraic tools to bear on geometric problems, and bringing geometric insight to problems in algebra. The subject is central in modern mathematics and borders on areas as diverse as number theory, differential geometry and mathematical physics.

This course is concerned with complex plane curves. Bezout's theorem determines their intersection behaviour, and has elegant applications to the geometry of their tangents and their singular points. It also gives rise to special behaviour possessed by curves of particular low degrees: for example a plane cubic has a natural group structure and can be parametrised by elliptic functions.

Studying the field of rational functions on a variety (= curve, surface, three manifold etc) leads to the idea of 'birational geometry', in which the variety can have quite different manifestations in spaces of different dimensions. A famous example is the cubic surface in 3-space, whose geometry is equivalent to that of 6 points in the plane. The birational geometry of a curve is much simpler than that of higher dimensional varieties and is governed by the Riemann-Roch theorem. This involves the 'genus' of the curve, which (over the complex numbers) can be interpreted as the number of handles if the curve is viewed as a real orientable surface.

Recommended Books

1. Reid: **Undergraduate algebraic geometry**, LMS Student Texts 12, Cambridge 1988, ISBN 0-521-35662-8
2. Kirwan, **Complex algebraic curves**, LMS Student Texts 23, Cambridge 1992, ISBN 0-521-42353-8
3. Gibson, **Elementary Geometry of Algebraic Curves**, CUP 1998
4. Harris, **Algebraic geometry - a first course**, GTM 133, Springer 1992
5. Fulton, **Algebraic curves**, Benjamin 1969
6. Reid **Undergraduate commutative algebra**, LMS Student Texts 29, Cambridge 1995
7. Brieskorn, H. Knörrer, **Plane algebraic curves**, Birkhuser 1986
8. Clemens, **A scrapbook of complex curve theory**, Plenum 1980.

The course is based mainly on [2], and a bit on [1]. It is also largely parallel to [3], which has many examples and explanations. [4] is also excellent, and much more thorough. [5] is an old warhorse; [6] should be mentioned as a close relative of [1]. [7] and [8] are for those with stamina.

Preliminary Reading: Chapter 1 in either of [1] or [2].

4H reading material reference: M. Reid, **Undergraduate algebraic geometry**, LMS, Student Texts 12, Cambridge 1988, Chapter II, paragraph 3 "Affine Varieties and the Nullstellensatz".

Aim: To introduce the basic theory of algebraic varieties and birational geometry, with particular emphasis on plane curves.

Term 1 (20 lectures)

Plane Curves: Affine and projective plane curves over a field. Conics, Pappus' Theorem. Unique factorisation in polynomial rings. Study's lemma, irreducibility. Singular points, tangents, points of inflection. Dual plane, linear systems of curves.

Bezout's Theorem: Resultants, weak form of Bezout. Applications to Pascal's theorem, Cayley-Bacharach theorem, group law on a cubic. Intersection multiplicity, strong form of Bezout.

Term 2 (18 lectures)

Bezout's Theorem: Applications. Flexes. Hessian, configuration of flexes of a cubic. Elliptic curves, Weierstrass normal form.

Complex Curves as Real Surfaces: Basic topology and manifolds; degree-genus formula. Resolution of singularities and non-singular models.

1.8.2 ANALYSIS III AND IV – MATH3011/4201 (38 lectures)

Dr W. Klingenberg

This module gives a rigorous development and applications of a cornerstone of analysis, namely integration as developed though measure theory. The methods and results are fundamental to many areas of pure and applied mathematics.

The course starts with elements of the theory of Lebesgue Measure and Integration, with the aim of providing a rigorous basis for defining the most important examples of complete normed and inner product vector spaces, namely the Lebesgue spaces of integrable functions on the real line.

It proceeds with the study of elementary properties of Banach and Hilbert space.

Thirdly, it introduces Fourier series and its convergence theorems. These occur as a special case of the spectral theory of Sturm-Liouville problems, a boundary value problem for an ordinary differential equaltion of second order that has wide applications in physics and engineering.

Recommended Books

1. R. G. Bartle, The Elements of Integration and Lebesgue Measure, Wiley-Interscience (1995)

2. H.L. Royden, P.M. Fitzpatrick, Real Analysis, Pearson (2010)

3. H. Dym, H. McKean, Fourier Series and Integrals, Academic Press (1985)

4. Al-Gwaiz, Sturm-Liouville Theory and its Applications, (Springer Undergraduate Mathematics Series) (2007)

Term 1 (20 lectures)

Set theory: countable and uncountable sets; axiom of choice, the real numbers; **Metric spaces**: completeness; separability; compactness; **Continuous functions**: equicontinuity and Ascoli's theorem; the Stone-Weierstrass theorem; **Measure**: outer measure; measurability and sigma-algebras; Borel sets; Lebesgue measure; **Measurable functions**: Properties of measurable functions; approximation by simple functions and by continuous functions; **Integration** : construction and properties of the integral; convergence theorems;

Term 2 (18 lectures)

Special properties of functions on the real line : absolute continuity; differentiation and integration; **Elementary properties of Banach and Hilbert spaces**: the Lebesgue spaces L^p, the space of continuous functions $C(X)$; completeness and the Riesz-Fischer theorem; orthonormal bases of L^2; **Basic harmonic analysis**: Basic properties of Fourier series, convergence in L^2;

4H Reading material: Will be announced at the end of term 1.

1.8.3 BAYESIAN STATISTICS III AND IV – MATH3341/MATH4031 (38 lectures)

Dr I. Jermyn / Dr J. Cumming

This course provides an overview of theoretical, algorithmic, and practical aspects of the Bayesian approach to statistical inference, introduced for simple models in Statistical Concepts II. Prior knowledge and statistical models for data are combined to provide a flexible and powerful probabilistic approach to knowledge representation. The Bayesian approach has become an important practical tool for data analysis and modelling of complex situations, thanks largely to the use of Markov Chain Monte Carlo methodology made possible by the steady increase in power of readily available computers.

The first term will discuss the rational basis for Bayesian inference, statistics, the role of exchangeability, exponential families and conjugate families of prior distributions, and will then start to look at the need for more complex models, and the theory and practice of how to build and represent such models for realistic situations.

The second term will continue the study of models with large numbers of variables and complex dependencies, and will then examine the powerful computational techniques that have rendered such models useful in practice. In particular, it will study Markov chain Monte Carlo methods for generating random values from multivariate distributions.

Recommended Books

The first book is a general introduction to the theory of Bayesian statistics. The second presents the foundational view of probability theory that is used in the course. It is extremely lucid, with many examples, and is a full of well-argued ideas and trenchant opinions. The third is available for free on the web, and contains useful material on graphical models and Monte-Carlo algorithms. The fourth comes more from a machine learning perspective, and is also very useful for graphical modelling and Monte-Carlo. Solutions to many exercises are available on the web. The fifth discusses Markov chain Monte Carlo in detail and some aspects of Bayesian modelling. None of the books is completely suitable by itself as a text for this module. Selected material will be made available via DUO in due course.

P. M. Lee, Bayesian statistics: an introduction (2nd ed.), Arnold, 1997, ISBN 0340677856.

E. T. Jaynes, Probability Theory: the Logic of Science, Cambridge University Press, 2003, ISBN 978-0521592710.

D. J. C. MacKay, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2007, ISBN 978-0521642989.

C. M. Bishop, Pattern Recognition and Machine Learning, Springer, 2007, ISBN 978-0387310732.

D. Gamerman and H.F. Lopes, Markov chain Monte Carlo : stochastic simulation for Bayesian inference (2nd ed.), Chapman & Hall/CRC, 2006, ISBN 1-58488-587-4.

Level IV reading material reference: to be specified by the lecturer in due course.

Outline of course **Bayesian Statistics III/IV**

Aim: To provide an overview of the theoretical basis for Bayesian statistics and of practical Bayesian statistical methodology together with important applications.

Term 1: (20 lectures)

Review: Bayesian paradigm; Conditional independence and conjugacy; Manipulation of multivariate probability distributions.

Foundations: Rational basis for Bayesian inference and decision theory; Exchangeability; Parametric modelling.

Exponential families: Regular exponential families; Canonical representation; Maximum entropy; Sufficiency; Conjugacy; Expectation.

Hierarchical modelling: Motivation; Latent variables; Random effects; Conjugacy and semi-conjugacy.

Term 2: (18 lectures)

Bayesian graphical modelling: Directed acyclic and undirected graphs; Bayesian and Markov networks; Moral graph; Separation theorem.

Computation: Monte Carlo; Markov Chain Monte Carlo: Markov chains, equilibrium distribution, Gibbs sampling; Metropolis-Hastings: Metropolis random walk, independence sampler, Gibbs sampling; Other algorithms.

Practicalities: Specification of prior beliefs; Analysis and interpretation of MCMC output.

Model comparison: Bayes factors; Criteria for model choice.

1.8.4 CONTINUUM MECHANICS III AND IV – MATH3101/MATH4081 (38 lectures)

Dr A. R. Yeates

From our cytoplasm, to pouring drinks and climate dynamics, we are surrounded (indeed made up of) fluid and elastic materials. In this course we will study the equations modeling such materials and their solutions (e.g., sound, water waves great and small, flows in pipes and around wings, etc.), with emphasis on fluid mechanics.

We start with a general kinematic description of fluid flow, followed by the equations governing the motion of an ideal fluid ("dry water"); specific examples of these are given and their limitations discussed. We will then consider the more realistic Navier–Stokes equations and the equations for a compressible fluid.

Recommended Books

D.J. Acheson, **Elementary Fluid Mechanics**, Oxford, ISBN 0198596790
G. Falkovich, **Fluid Mechanics: A Short Course for Physicists**, CUP, ISBN 1107005752
A.R. Paterson, **A First Course in Fluid Dynamics**, CUP, ISBN 0521274249
G.K. Batchelor, **An Introduction to Fluid Dynamics**, CUP, ISBN 0521663962
A.J. Chorin and J.E. Marsden, **A mathematical introduction to fluid mechanics**, Springer, ISBN 0387904069

None of these is specifically recommended for purchase, but all contain much of the course material from different points of view. Note that Falkovich is available electronically through the University Library.

Preliminary Reading

M. Samimy et al, **A gallery of fluid motion**, CUP, ISBN 9780521535007
Or see the APS online gallery of fluid motion (`http://gfm.aps.org/`).
In addition, you are recommended to revise vector calculus (grad, div, curl, and line/surface/volume integrals) as this will be used heavily.

4H reading material: TBA

Aim: To introduce a mathematical description of fluid flow and other continuous media to familiarise students with the successful applications of mathematics in this area of modelling. To prepare students for future study of advanced topics.

Term 1 (20 lectures)

Kinematics: continuum hypothesis, velocity field, material derivatives, particle paths and streamlines, vorticity and compressibility.

Dynamics: derivation of the Navier–Stokes and Euler equations, vorticity and circulations.

Compressible flows: sound waves, linearisation and solution techniques

Term 2 (18 lectures)

Navier-Stokes equations: boundary layers, scaling and the Reynolds number.

Water waves: surface gravity waves, shallow water equations, weakly nonlinear systems (solitons), shocks.

1.8.5 DECISION THEORY III – MATH3071 (38 lectures)

Dr C. Caiado / Prof F. Coolen

Decision theory concerns problems where we have a choice of decision, and the outcome of our decision is uncertain (which describes most problems!). Topics for the course typically include the following (contents may vary a bit from year to year):

(1) Introduction to the ideas of decision analysis.

(2) Decision trees - how to draw and how to solve.

(3) Representing decision problems using influence diagrams.

(4) Quantifying rewards as utilities - informal ideas, formal construction, relevance to statistical analysis and multi-attribute utility. Von Neumann-Morgenstern theory of maximum expected utility.

(5) Alternative decision criteria and reflection on normative versus descriptive theories (including prospect theory).

(6) Applications, e.g. in health, industry and insurance.

(7) Bargaining problems: Nash theory for collaborative games.

(8) Group decisions and social choice: Arrow's theory on social welfare functions, Harsanyi's utilitarianism, further developments.

(9) Game theory: two-person zero-sum games, brief discussion of more complex games and applications in e.g. biology.

(10) Further topics related to recent developments in research and applications.

Recommended Books

M. Peterson, **An Introduction to Decision Theory**, Cambridge University Press, 2009.
J.Q. Smith, **Decision Analysis - A Bayesian Approach**, Chapman & Hall, 1988.
D.V. Lindley, **Making Decisions**, (2nd edition), Wile, 1985.
S. French, **Decision Theory: An Introduction to the Mathematics of Rationality**, Ellis Horwood, Chichester, 1986.
M.H. DeGroot, **Optimal Statistical Decisions**, McGraw-Hill.
R.T. Clemen, **Making Hard Decisions**, (2nd edition), Duxbury Press, 1995.

Preliminary Reading

While any of the recommended books would make a good preparation for the course (except De Groot, which is rather too technical) you are advised to look at **Making Decisions** by Lindley, which gives an interesting and fairly painless introduction to some of the most important ideas. Peterson is relatively cheap and covers a substantial part of the topics in the course, however it has slightly more emphasis on philosophical aspects and less on mathematics than our presentation.

Aim: To describe the basic ingredients of decision theory, for individuals and for groups, and to apply the theory to a variety of interesting and important problems.

Term 1 (20 lectures)

Introduction to Decision Analysis: Decision trees, sequential decisions; uncertainties and values, solution by backward induction; perfect information and cost of information; representation by influence diagrams.

Utility: Von Neumann - Morgenstern theory of maximum expected utility; utility of money and risk aversion; multi-attribute utility; relevance to statistical analysis.

Alternative theories: Alternative decision criteria, prospect theory.

Applications: Some examples of applications in e.g. health, industry and insurance.

Term 2 (18 lectures)

Bargaining: Nash theory for collaborative games; alternative theories.

Group Decisions and Social Choice: Arrow's theory on social welfare functions; Harsanyi's theory on utilitarianism; alternative theories and recent developments.

Game Theory: Two-person zero-sum games; brief discussion of non-constant sum games and other more complex games, and of applications in e.g. biology.

Further topics: Selection of topics related to recent developments in research and applications.

1.8.6 DIFFERENTIAL GEOMETRY III – MATH3021 (38 lectures)

Prof J. Parker

Differential geometry is the study of curvature. Historically it arose from the application of the differential calculus to the study of curves and surfaces in 3-dimensional Euclidean space. Today it is an area of very active research mainly concerned with the higher-dimensional analogues of curves and surfaces which are known as n-dimensional differentiable manifolds, although there has been a great revival of interest in surfaces in recent years. Differential geometry has been strongly influenced by a wide variety of ideas from mathematics and the physical sciences. For instance, the surface formed by a soap film spanning a wire loop is an example of a minimal surface (that is, a surface whose mean curvature is zero) but the ideas and techniques involved in analysing and characterising such surfaces arose from the calculus of variations and from Riemann's attempt to understand complex analysis geometrically. The interplay of ideas from different branches of mathematics and the way in which it can be used to describe the physical world (as in the case of the theory of relativity) are just two features which make differential geometry so interesting.

In order to keep the treatment as elementary and intuitive as possible this level III course will be almost entirely devoted to the differential geometry of curves and surfaces, although most of the material readily extends to higher dimensions. The techniques used are a mixture of calculus, linear algebra, and topology, with perhaps a little material from complex analysis and differential equations. The course will follow the notes of Woodward and Bolton, and the book by Do Carmo is also be very suitable. You are recommended to buy either of these. Spivak's books are interesting, well written and useful for reference. If you like the format of the Schaum series, then you may find Lipschutz's book helpful.

Recommended Books

M. do Carmo, **Differential Geometry of Curves and Surfaces**, Prentice-Hall.
S. Lipschutz, **Differential Geometry**, Schaums Outline Series, McGraw-Hill.
M. Spivak, **Differential Geometry**, Vols. II & III, Publish or Perish.
* L.M. Woodward and J. Bolton, **Differential Geometry** (Preliminary Version). Duplicated copies of this are available for students from the Departmental Office.

Preliminary Reading

Try looking at the first two chapters of do Carmo or Woodward and Bolton, or Chapters 1-3 of Spivak II. If you cannot get hold of these, then any book which deals with the differential geometry of curves and surfaces in Euclidean 3-space should be useful.

Outline of course **Differential Geometry III**

Aim: To provide a basic introduction to the theory of curves and surfaces, mostly in 3-dimensional Euclidean space. The essence of the module is the understanding of differential geometric ideas using a selection of carefully chosen interesting examples.

Term 1 (20 lectures)

Curves: Plane curves. Arc length, unit tangent and normal vectors, signed curvature, Fundamental theorem. Involutes and evolutes. Gauss map, global properties. Space curves. Serret-Frenet formulae. Fundamental theorem. [Global properties]

Surfaces in $\mathbb{R}^n$: Brief review of functions of several variables including differential (with geometric interpretation) and Inverse Function Theorem. Definition of regular surface in and Coordinate recognition lemma. [Change of coordinates.] Curves on a surface, tangent planes to a surface in $\mathbb{R}^n$.

First Fundamental Form: Metric, length, angle, area. Orthogonal and isothermal coordinates. [Orthogonal families of curves on a surface.] [Some more abstract notions such as 'metrics' on open subsets of $\mathbb{R}^n$ can be introduced here.]

Mappings of Surfaces: Definitions, differential, expressions in terms of local coordinates. [Conformal mappings and local isometrics. Examples (conformal diffeomorphisms and isometries of $\mathbb{R}^2$, S^2 and helicoid; conformal diffeomorphism of $S^1(a) \times S^1(b) \subseteq \mathbb{R}^4$ onto torus of revolution in $\mathbb{R}^3$; $\mathbb{R}P^2$ as Veronese surface in $\mathbb{R}^5$)].

Term 2 (18 lectures)

Geometry of the Gauss Map: Gauss map and Weingarten map for surfaces in $\mathbb{R}^3$, second fundamental form, normal (and geodesic) curvature, principal curvatures and directions. Gaussian curvature K and mean curvature H. Second fundamental form as second order approximation to the surface at a point. Explicit calculations of the above in local coordinates. Umbilics. Compact surface in $\mathbb{R}^3$ has an elliptic point.

Intrinsic Metric Properties: Christoffel symbols, Theorema Egregium of Gauss. Expression of K in terms of the first fundamental form. Examples. [Intrinsic descriptions of K using arc length or area]. Mention of Bonnet's Theorem. [Surfaces of constant curvature].

Geodesics: Definition and different characterisations of geodesics. Geodesics on surfaces of revolution. Geodesic curvature with description in terms of local coordinates.

Minimal Surfaces: Definition and different characterisations of minimal surfaces. Conjugate minimal surfaces and the associated family. [Weierstrass representation.]

Gauss-Bonnet Theorem: Gauss-Bonnet Theorem for a triangle. Angular defect. Relationship between curvature and geometry. Global Gauss-Bonnet Theorem. Corollaries of Gauss-Bonnet Theorem. [The Euler-Poincaré-Hopf Theorem.]

1.8.7 DYNAMICAL SYSTEMS III – MATH3091 (38 lectures)

Dr P. Heslop / Dr B. van Rees

Dynamical systems is the mathematical study of systems which evolve in time. One classical example is Newtonian dynamics, but the applicability is in fact much wider than this.

This course mainly deals with systems described by (coupled) ordinary differential equations. We start by studying the local behaviour or solutions, both in time and in the neighbourhoods of fixed points. Although one might think that such systems could be solved in closed form with sufficient effort, Poincaré realised a century ago that there are fundamental obstacles to global exact solvability and that most dynamical systems are in fact unsolvable in closed form for all time. This leads us to develop methods to study the global qualitative behaviour of such systems, finishing with a brief introduction to chaotic dynamical systems.

Recommended Books

D.K. Arrowsmith and C.M. Place, **Dynamical Systems**, Chapman & Hall 1992, ISBN 0412390809

P.G. Drazin, **Nonlinear Systems**, CUP 1992, ISBN 0521406684

P. Glendinning, **Stability, Instability and Chaos**, CUP 1994, ISBN 0521425662

F. Verhulst, **Nonlinear Differential Equations and Dynamical Systems**, Springer (2nd Edition) 1996, ISBN 3540609342

M.W. Hirsch, S. Smale, R.L. Devaney **Differential Equations, Dynamical Systems, and an Introduction to Chaos**, Academic Press 2003, ISBN 0123497035 These are all paperback introductory undergraduate texts with plenty of examples. Their style, content and order of topics all differ, but each includes something on most of the course material.

Any one is likely to be helpful and there is no best buy. Instead arrange with your friends to get different ones, and share.

Aim: To provide an introduction to modern analytical methods for nonlinear ordinary differential equations in real variables.

Term 1 (20 lectures)

Introduction: Smooth direction fields in phase space. Existence and Uniqueness Theorem and initial-value dependence of trajectories. Orbits. Phase portraits. Equilibrium and periodic solutions. Orbital derivative, first integrals.

Linear Autonomous Systems: Classification of Linear Systems in two and higher dimensions. The exponential map.

Nonlinear systems near equilibrium Hyperbolic fixed points, Stable and Unstable Manifolds. Stable-Manifold Theorem, Hartman-Grobman Theorem.

Stability of fixed points Definitions of stability, Orbital derivative, first integrals, Lyapounov functions and Stability Theorems.

Term 2 (18 lectures)

Local Bifurcations: Classification for 1d systems, Some 2d examples, Robustness of bifurcations, Example of a global bifurcation

Orbits and Limit Sets: Omega-limit sets, Poincare-Bendixson Theorem, Index theorem, Absorbing sets and limit cycles

Lorenz system and Introduction to Chaos

1.8.8 ELECTROMAGNETISM III – MATH3181 (38 lectures)

Prof. S. F. Ross

Electromagnetism constitutes the nineteenth century's greatest contribution to our understanding of the fundamental laws of nature. It provides a unified mathematical description of electricity, magnetism and light and contains the seeds of special relativity.

The range of phenomena and applications of electromagnetism is immense. The course deals with the most important of these, building from basic to more complex situations in a step-by-step fashion.

The major topics covered are:

(i) electrostatics
(ii) magnetostatics
(iii) time-dependent fields and electromagnetic waves
(iv) special relativistic formulation of electromagnetism

Recommended Books

The recommended text for this course is:

D. J. Griffiths, **Introduction to Electrodynamics**, Pearson 2008.

There are a large number of other texts on this subject. One example at a similar level is

I.S. Grant and W.R. Philips, **Electromagnetism**, Wiley 2008;

It may also be interesting to look at

R.P. Feynman, R.B. Leighton and M. Sands, **Feynman Lectures on Physics** Vol. 2, Addison-Wesley 1971;

W.J. Duffin, **Electricity and Magnetism**, W.J. Duffin Publishing, 2001;

Many other books are available in college and university libraries.

Preliminary Reading

A prerequisite for the course is *a good knowledge of vector analysis*, as covered in the 2H Analysis in Many Variables course. To refresh your memory, you could read the early chapters of Spiegel's **Vector Analysis** (or equivalent).

Volume 1 of the **Feynman Lectures on Physics** is excellent background reading for more general aspects of the course, especially the sections on relativity and electromagnetic radiation.

Aim: To appreciate classical electromagnetism, one of the fundamental physical theories. To develop and exercise mathematical methods.

Term 1 (20 lectures)

Electrostatics: From experiment: Coulomb's law and vector superposition of forces due to different charges. Units: discussion of the possibilities, choice. Electric fields due to point charges and to volume and surface distributions. Electric field **E** expressed in terms of electrostatic potential ϕ . Differential equations for the electrostatic field. Electric dipoles. Multipole expansion of the electrostatic potential. Energy for a charge distribution ρ in an external field. Electrostatic energy. Perfect conductors; the force per unit area on a conductor in an electrostatic field.

(It may be necessary to review some or all of the following topics from mathematical methods: vector algebra, vector analysis, solid angle, delta functions, Green's functions, Fourier integral decompositions.)

Term 2 (18 lectures)

Magnetostatics:

From experiment: the force between current-carrying loops. Current density **j** and conservation of charge; integral form and continuity equation. Magnetic field **B** due to a current-carrying loop; extension to volume distribution of current. Differential equations for **B**. Vector potential **A** and its differential relations. Force density on a current density in a magnetic field. Magnetic dipoles and the gyromagnetic ratio. Permeable media, magnetisation and the phenomenological **H**.

Time-dependent Fields and Maxwell's Equations :

From experiment: Faraday's law. Maxwell's equations with microscopic sources and in simple media. Potentials and gauge invariance. Wave equation. Energy and momentum conservation (including energy density, Poynting vector, stress tensor). Plane waves. Polarisation. Retarded potentials.

Special Relativistic Formulation of Electromagnetism: Invariances of Maxwell's equations: the equations with microscopic sources expressed as tensor equations in Minkowski spacetime. Relativistic equation of motion for a charged particle in an external electromagnetic field.

1.8.9 GALOIS THEORY III – MATH3041 (38 lectures)

Prof. V. Abrashkin

The origin of Galois Theory lies in attempts to find a formula expressing the roots of a polynomial equation in terms of its coefficients. Evarist Galois was the first mathematician to investigate successfully whether the roots of a given equation could in fact be so expressed by a formula using only addition, subtraction, multiplication, division, and extraction of n-th roots. He solved this problem by reducing it to an equivalent question in group theory which could be answered in a number of interesting cases. In particular he proved that, whereas all equations of degree 2, 3 or 4 could be solved in this way, the general equation of degree 5 or more could not. Galois Theory involves the study of general extensions of fields and a certain amount of group theory. Its basic idea is to study the group of all automorphisms of a field extension. The theory has applications not only to the solution of equations but also to geometrical constructions and to Number Theory.

Recommended Books

*I. Stewart, **Galois Theory**, Chapman and Hall, ISBN 0412345404; £23.99
E. Artin, **Galois Theory**, Dover, ISBN0486623424; about £6
J. Rotman, **Galois Theory**, Springer
M.H. Fenrick, **Introduction to the Galois Correspondence**, Birkhaser
D.J.H. Garling, **A Course in Galois Theory**, Cambridge University Press
P.J. McCarthy, **Algebraic Extensions of Fields**, Chelsea
H.M. Edwards, **Galois Theory**, Springer
B.L. Van der Waerden, **Modern Algebra**, Vol. 1, Ungar.

Aim: To introduce the way in which the Galois group acts on the field extension generated by the roots of a polynomial, and to apply this to some classical ruler-and-compass problems as well as elucidating the structure of the field extension.

Term 1 (20 lectures)

Introduction: Solving of algebraic equation of degree 3 and 4. Methods of Galois theory.

Background: Rings, ideals, a ring of polynomials, fields, prime subfields, factorisation of polynomials, tests for irreducibility.

Field Extensions: Algebraic and transcendental extensions, splitting field for a polynomial, normality, separability.

Fundamental Theorem of Galois Theory: Statement of principal results, simplest properties and examples.

Term 2 (18 lectures)

Galois Extensions with Simplest Galois Groups: Cyclotomic extensions, cyclic extensions and Kummer Theory, radical extensions and solvability of polynomial equations in radicals.

General Polynomial Equations: Symmetric functions, general polynomial equation and its Galois group, solution of general cubic and quartic, Galois groups of polynomials of degrees 3 and 4, non-solvability in radicals of equations of degree ≥ 5.

Finite Fields: Classification and Galois properties, Artin-Schreier Theory, construction of irreducible polynomials with coefficients in finite fields.

1.8.10 GENERAL RELATIVITY III AND IV – MATH3331/MATH4051 (38 lectures)

Prof R. Ward

Towards the end of the last century, it became clear that Newtonian mechanics breaks down in three different areas: the world of the very small, the world of the very fast, and the world of the very massive. To cope with these, physicists invented three different extensions of Newtonian mechanics: quantum mechanics, special relativity and general relativity.

Before beginning General Relativity, we briefly review special relativity. General Relativity involves curved spacetime, and gravity is embodied in it. We sketch the notions of differential geometry which allow us to discuss curvature in four dimensions. We study Einstein's field equations with applications to the classical tests of general relativity, black holes and cosmological models.

Recommended Books

Start by browsing some of the links on the course web page, and by looking at one or more non-technical books such as:
R. Geroch, **General Relativity from A to B**, University of Chicago Press 1981
K.S. Thorne, **Black Holes and Time Warps: Einstein's Outrageous Legacy**, W.W. Norton, New York 1995
R.M. Wald, **Space, Time and Gravity: the Theory of the Big Bang and Black Holes**, University of Chicago Press 1992.

The books below are all worth looking at. In the lectures, we will follow notation adopted by Wald.

Relatively easy:
J.Hartle, **GRAVITY: An introduction to Einstein's General Relativity**, Addison Wesley 2003
L.P. Hughston and K.P. Tod, **An Introduction to General Relativity**, Cambridge 1994
B.F. Schutz, **A First Course in General Relativity**, Cambridge 1985 W. Rindler, **Introduction to Special Relativity**, Oxford 1991
R. d'Inverno, **Introducing Einstein's Relativity**, Oxford 1992

Somewhat harder:
S. Carroll, **Lecture Notes on General Relativity**, Addison Wesley 2004
http://pancake.uchicago.edu/∼carroll/notes/
C.W. Misner, K.S. Thorne and J.A. Wheeler, **Gravitation**, Freeman 1973
R.M. Wald, **General Relativity**, University of Chicago 1984

Preliminary Reading:

As preliminary reading, Hartle is to be recommended for a more intuitive approach, and the first 3 chapters of Houghston & Tod, the first 4 chapters of Schutz, or the final 3 chapters of Rindler's Introduction to Special Relativity, would be useful.

4H reading material references:
R. Geroch and G.T. Horowitz, **Global structure of spacetimes**, in the book **General Relativity: An Einstein centenary survey**, ed. by S.W. Hawking and W. Israel, Cambridge U. Press 1979
S. W. Hawking and G.F.R. Ellis, **The Large Scale Structure of Space-Time**, New York: Cambridge U. Press, 1975

Aim: To appreciate General Relativity, one of the fundamental physical theories. To develop and exercise mathematical methods.

Term 1 (20 lectures)

Introduction to General Relativity: differences between GR & SR, gravity as geometry, equivalence principle.

Special Relativity: spacetime diagrams, line element, vectors and tensors, electromagnetism, stress-energy tensor.

Differential Manifolds: spacetime is a manifold, coordinates and coordinate transformations, tangent vectors, tensors revisited.

Metric: distance relationships, light cones, Riemann normal coordinates.

Covariant Derivative: inadequacy of partial derivatives, parallel transport, connection coefficients, differentiating tensors, metric connection, geodesics.

Curvature: Riemann tensor, characterisation of flat space, parallel transport around closed curves, commutation formulae, Bianchi identity, Einstein tensor, geodesic deviation.

Term 2 (18 lectures)

General Relativity: equivalence principle, physics in curved spacetime, Einstein's equations, linearized theory and Newtonian limit, Einstein-Hilbert action.

Black Holes: spherical symmetry, Schwarzschild solution, geodesics, solar-system applications, event horizon and Kruskal coordinates, black hole formation.

Cosmology: isotropy and homogeneity, FRW metric, examples of cosmologies, Hubble law, particle horizons.

4H reading material: to be announced

1.8.11 MATHEMATICAL BIOLOGY III – MATH3171 (38 lectures)

Dr B. Chakrabarti

Mathematical Biology is one of the most rapidly growing and exciting areas of applied mathematics. Over the past decade research in Biological sciences has evolved to a point that experimentalists are seeking the help of applied mathematicians to gain quantitative understanding of their data. Mathematicians can help biologists to understand very complex problems by developing models for biological situations and then providing a suitable solution. However, for a model to be realistic cross-talk between these two disciplines is absolutely essential.

Mathematics is now being applied in a wide array of biological and medical contexts and professionals in this field are reaping the benefits of research in these disciplines. Examples range from modelling physiological situations e.g. ECG readings of the heart, MRI brain scans, blood flow through arteries, tumor invasion and others. At the cellular level we can ask quantitative questions about the process by which DNA gets "transcribed" (copied) to RNA and then "translated" to proteins, the central dogma of molecular biology. Mathematical Biology also encompasses other interesting phenomena observed in nature, e.g. swimming behavior of microorganisms, spread of infectious diseases, and emergence of patterns in nature. In this course we shall examine some fundamental biological problems and see how to go about developing mathematical models that describe the biological situation with definite predictions that can then be tested to validate the model.

Recommended Books

J.D. Murray, **Mathematical Biology I. An Introduction**, Springer, ISBN 0387952233; (about £20)
J.D. Murray, **Mathematical Biology II. Spatial models and biomedical applications**, Springer, ISBN 0387952284; (about £57)
N. F. Britton, **Essential Mathematical Biology**, Springer, ISBN 185233536X; (about £ 30)
Lee A. Segel **Modelling dynamic phenomena in molecular and cellular biology**, Cambridge University Press, ISBN 052127477X; about £20
J. Keener and J. Sneyd **Mathematical Physiology**, Springer, ISBN 0387983613; (about £50)
L. A. Segel, and L. Edelstein-Keshet **A primer of Mathematical Models in Biology**, Cambridge University Press, ISBN 9781611972498 (about £50)
K. Sneppen, **Models of Life: Dynamics and Regulation in Biological Systems**, Cambridge University Press, ISBN 978-1-107-06910-3 (about £ 60)

None of these books covers the course entirely. However, the course is covered by all. Segel's book is easiest to read followed by Britton. Murray's are excellent. Keener and Sneyd is a very good account of more medical applications of mathematics.

Aim: Study of non-linear differential equations in biological models, building on level 1 and 2 Mathematics.

Term 1 (20 lectures)

Introduction to the Ideas of Applying Mathematics to Biological Problems

Reaction Diffusion Equations and their Applications in Biology:

Diffusion of insects and other species. Hyperbolic models of insect dispersal and migration of a school of fish. Modelling the life cycle of the cellular slime mold Dictyostelium discoideum, and the phenomenon of chemotaxis. Glia aggregation in the human brain and possible connection with Alzheimer's disease.

Term 2 (18 lectures)

ODE Models in Biology: Enzyme kinetics and the chemostat for bacteria production

The Formation of Patterns in Nature: Pattern formation mechanisms, morphogenesis. Questions such as how does a Diffusion driven instability and pattern formation (Turing instability)

Epidemic Models and the Spread of Infectious Diseases: Epidemic models. Spread of infectious diseases. Simple ODE model. Spatial spread of diseases.

Epidermal and Dermal wound healing • Basic models, Derivation of equations, Stability analysis and travelling wave solutions.

1.8.12 MATHEMATICAL FINANCE III AND IV – MATH3301/4181 (38 lectures)

Dr C. Caiado / Dr N. Georgiou

Finance is one of the fastest developing areas in the modern banking and corporate world. This, together with the sophistication of modern financial products, results in a need for new mathematical models and modern mathematical methods. The demand from financial institutions for well-qualified mathematicians is great.

The first part of this course discusses discrete time financial models. Main concepts of financial mathematics such as forward contracts, call and put options, completeness, self-financing and replicating strategies, arbitrage and risk-neutral measures will be introduced. Pricing European and American options under Cox-Ross-Rubinstein (CRR) Binomial model will be discussed. The Black-Scholes model will be derived as a limit of CRR model. In the second part of the course, Ito's integral and ito's formula will be introduced. Feynman kac formula, Girsanov and martingale representation theorems will be discussed in detail. Derivation of the Black-Scholes PDE and its solution via heat equation will be shown. As time permits, pricing of Barrier and other path dependent options will also be discussed.

Recommended Books

A. Etheridge, **A Course in Financial Calculus**. Cambridge University Press, 2002.

N.H.Bingham and R.Kiesel, **Risk Neutral Valuation**, Springer 1998.

M.Baxter and A. Rennie, **Financial Calculus**, Cambridge University Press, 1996.

T. Bjork, **Arbitrage in Continuous Time**, Oxford University Press, 1999.

E. Shreve, **Stochastic Calculus for Finance (Volumes I and II)**, Springer Finance, 2004.

R.J.Elliot and P.E.Kopp, **Mathematics of Financial Markets**, Springer, 1999.

J.C. Hull, **Options, Futures and Other Derivatives**, Pearson Intern'l Ed., ISBN 978 0136015864.

P. Wilmott, S. Howison and J. Dewynne, **The mathematics of financial derivatives: a student introduction**, CUP, ISBN 0521497892.

Preliminary Reading

Sections 1.3, 1.4, 1.5, 2.10, 4.1, 4.4 of J.C. Hull, **Options, Futures and Other Derivatives**, Pearson Intern'l Ed., ISBN 978 0136015864.

4H reading material references

Will be assigned later.

Aim: To provide an introduction to the mathematical modelling of financial derivative products.

Term 1 (20 lectures)

Introduction to options and markets. Interest rates and present value analysis, asset price random walks, pricing contracts via arbitrage, risk neutral probabilities.

The Black-Scholes formula. The Black-Scholes formula for the geometric Brownian price model and its derivation.

More general models. Limitations of arbitrage pricing, volatility, pricing exotic options by tree methods and by Monte Carlo simulation.

Term 2 (18 lectures)

Introduction to stochastic calculus.

The Black-Scholes model revisited. The Black-Scholes partial differential equation. The Black-Scholes model for American options.

Finite difference methods.

1.8.13 MATHEMATICS TEACHING III – MATH3121 (38 lectures/seminars/school placement)

Mr Chris Goy

Module Prerequisites: three Mathematical Science modules taken in year 2, at least two of which are at Level 2.
Module Co-requisites: two level 3 Mathematical Sciences modules.
Module Cap: Maximum 20 students – see **Selection Process**.

The course examines the organisation, delivery and assessment of mathematics teaching in this country. Students will have the opportunity to consider current trends in pedagogy and engage in topical debates on current educational issues, including the purpose and methods of assessment. There will be a strong research led theme and students will be encouraged to reflect in depth upon what works in the classroom on a practical level.

In Term 1, in addition to the seminars that will cover a large range of issues relevant to the teaching and learning of mathematics, students will have the opportunity to visit schools and observe mathematics lessons. Students will prepare an individual School File containing a report and analysis of their observations.

In Term 2, students do a project on Assessment that comprises 50% of the module mark in total. As part of this project they will undertake an extended school placement (12 hours pupil contact over 6 weeks), working as a classroom assistant with a particular class. During this time, the student will prepare an examination paper that the pupils in the class will sit. The student will mark the papers and prepare a feedback report for the class-teacher at an individual pupil level, all of which contributes to their project as a whole. In the final week of Epiphany Term students will give a short demonstration of their project. The final write up will be completed over the Easter holidays.

The assessment of the course will be through the School File (20%), continuous assessment in Term 1 (30%) and through the demonstration (10%) and write-up (40%) of the students' placement projects.

Recommended Reading: Knowledge of the structure, and general content, of national curriculum mathematics would be useful to inform school visits. These include:

DfE (2011) *The National Curriculum for Mathematics* [available at
`http://www.education.gov.uk/schools/teachingandlearning/curriculum/secondary/b00199003/mathematics`]
Vorderman C., Budd C., Dunne R., Hart M., Porkess R. (2011) *A World Class Education for all our Young People* (a.k.a. The Vordermann Report) [available at
`http://www.tsm-resources.com/pdf/VordermanMathsReport.pdf`]
Nuffield Foundation *Is the UK an Outlier? An international comparison of upper secondary mathematics* (2010) and *Towards universal participation in Post-16 Mathematics* (2012) [available at http://www.nuffieldfoundation.org/towards-universal-participation-post-16-mathematics]

There will be some assigned reading for sessions during Term 1. The continuous assessment element will comprise some structured questions exploring how students think and learn in specific mathematical contexts, together with some general assignments exploring wider issues.

Aims: To encourage students to reflect on how mathematics is taught and learnt; to help them understand how mathematics teaching fits into a wider education system; to develop students' group and presentational skills; to help inform students' future career decisions.

Brief Synopsis of Seminar Themes:

The Educational Landscape (2): What's currently out there, and why does it change so quickly? Why Education matters. Mathematics teaching from an advanced standpoint.

Academic Writing (2): A brief guide to academic writing, referencing, and the avoidance of plagiarism.

Mathematics Teaching and Learning (2): What to look for in the school visit - practicalities and advice. What constitutes a good or outstanding lesson? How do you plan a maths lesson? The psychology of learning mathematics, and how we can use it to make us better teachers. The Art of Engagement.

Mathematics Teaching Resources (6): An extended examination of what works well and why. N.B. These seminars will have a significant "student-led" element.

Assessment (4): Does it work? What are the choices? Have exams really been getting easier? Designing an exam paper. Different assessment models. Attainment v Achievement. Do league tables help or hinder? Catering for more able students. The Awarding Body perspective.

Schoo Performance and Accountability (2) League Tables, value-added measures and Ofsted.

You, the teacher (4): Why teach? What do schools want and have you got it? Skills and knowledge requirements. Developing a broader mathematical capability. Teaching experience with 6th Form students.

Selection Process

Students will be interviewed for a place on this course, and interviews will take place from Wednesday 22nd April to Friday 24th April 2015. Students wishing to express an interest in being interviewed for the course should email the course leader [chris.goy@durham.ac.uk] by Friday 13th March 2015. You will receive a time slot well before the interview date but please make sure you are available on the day in question. Interviews will be fairly short, and any preparation you do should revolve simply around why you wish to take this module.

Please note: All students taking this module must have a recent DBS certificate issued (except in exceptional circumstances) by Durham University. Application forms will be given out during interviews.

1.8.14 OPERATIONS RESEARCH III – MATH3141 (38 lectures)

Dr M. Troffaes / Prof M. Menshikov

As its name implies, operations research involves ”research on operations”, and it is applied to problems that concern how to conduct and coordinate the operations (activities) within an organization. The nature of the organization is essentially immaterial, and, in fact, OR has been applied extensively in such diverse areas as manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, and public services to name just a few.
This course is an introduction to mathematical methods in operations research. Usually, a mathematical model of a practical situation of interest is developed, and analysis of the model is aimed at gaining more insight into the real world. Many problems that occur ask for optimisation of a function f under some constraints. If the function f and the constraints are all linear, the simplex method is a powerful tool for optimisation. This method is introduced and applied to several problems, e.g. within transportation.
Many situations of interest in OR involve processes with random aspects and we will introduce stochastic processes to model such situations. An interesting area of application, addressed in this course, is inventory theory, where both deterministic and stochastic models will be studied and applied. Further topics will be chosen from: Markov decision processes; integer programming; nonlinear programming; dynamic programming.

Recommended Books

* F.S. Hillier and G.J. Lieberman, **Introduction to Operations Research**, 8^{th} ed., McGraw-Hill 2004, ISBN 007123828x
This excellent book (softcover, £43.99) covers most of the course material (and much more), and the course is largely developed around this textbook. It is well written and has many useful examples and exercises. It also contains some PC software that can be used to gain additional insight in the course material. This is the best textbook we know of for this course (the 6^{th} and 7^{th} edition are equally useful).

There are plenty of books with ’operations research’, ’mathematical programming’, ’optimisation’ or ’stochastic processes’ in the title, and most of these contain at least some useful material. Be aware though that this course is intended to be an introduction, and most books tend to go rather quickly into much more detail.

Preliminary Reading
The first two chapters of the book by Hillier and Lieberman provide an interesting introduction into the wide area of operations research, and many of the other chapters start with interesting prototype examples that will give a good indication of the course material (restrict yourself to the chapters on topics mentioned above). Appendices 2 and 4 of this book could be read to refresh your understanding of matrices and the notion of convexity. Most other books within this area (see above) have introductory chapters that provide insight into the topic area, and the possible applications of OR.

Aim: To introduce some of the central mathematical models and methods of operations research.

Term 1 (20 lectures)

Introduction to Stochastic Processes: Stochastic process; discrete-time Markov chains.

Markov Decision Processes: Markovian decision models; the optimality equation; linear programming and optimal policies; policy-improvement algorithms; criterion of discounted costs; applications, e.g. inventory model.

Inventory Theory: Components of inventory models; deterministic models; stochastic models.

Further Topics Chosen From: Dynamic programming: Characteristics; deterministic and probabilistic dynamic programming.

Queueing theory: Models; waiting and service time distributions; steady-state systems; priority queues.

Integer programming: Model; alternative formulations; branch-and-bound technique; applications, e.g. knapsack problem, travelling salesman problem.

Nonlinear programming: Unconstrained optimisation; constrained optimisation (Karush-Kuhn-Tucker conditions); study of algorithms; approximations of nonlinear problems by linear problems; applications.

Term 2 (18 lectures)

Introduction to Operations Research: Role of mathematical models, deterministic and stochastic OR.

Linear Programming: LP model; convexity and optimality of extreme points; simplex method; duality and sensitivity; special types of LP problems, e.g. transportation problem.

Networks: Analysis of networks, e.g. shortest-path problem, minimum spanning tree problem, maximum flow problem; applications to project planning and control.

1.8.15 PARTIAL DIFFERENTIAL EQUATIONS III AND IV – MATH3291/MATH4041 (38 lectures)

The topic of partial differential equations (PDEs) is central to mathematics. It is of fundamental importance not only in classical areas of applied mathematics, such as fluid dynamics and elasticity, but also in financial forecasting and in modelling biological systems, chemical reactions, traffic flow and blood flow in the heart. PDEs are also important in pure mathematics and played a fundamental role in Perelman's proof of the million dollar Poincaré conjecture.

In this module we are concerned with the theoretical analysis of PDEs (the numerical analysis of PDEs is covered in the course Numerical Differential Equations III/IV). We will study first-order nonlinear PDEs and second-order linear PDEs, including the classical examples of Laplace's equation, the heat equation and the wave equation. For some simple equations with appropriate boundary conditions we will find explicit solutions. For equations where this is not possible we will study existence and properties of solutions. We will see for example that solutions of the heat equation have very different properties to solutions of the wave equation.

Recommended Books No particular book is recommended for purchase. The following are useful references:

L.C. Evans, Partial Differential Equations, 2nd edition, AMS 2010 (Chap. 1–3);

J. Ockendon, S. Howison, A. Lacey and A. Movchan, Applied Partial Differential Equations, revised edition, Oxford 2003;

E.C. Zachmanoglou and D.W. Thoe, Introduction to Partial Differential Equations with Applications, Dover 1986.

Preliminary Reading For the flavour of the course look at Chapter 1 of Ockendon et al. or Chapter III of Zachmanoglou and Thoe.

Reference for 4H Additional Reading Specified parts of the book by Ockendon et al.

Aim: To develop a basic understanding of the theory and methods of solution for partial differential equations.

<u>**Term 1**</u> (20 lectures)

Introduction: examples of important PDEs, notation, the concept of well-posedness.

First-order PDEs and characteristics: the transport equation, general nonlinear first-order equations.

Conservation laws: models of traffic flow and gases, shocks and rarefactions, systems of conservation laws.

Second-order linear PDEs: examples and classification (elliptic, parabolic, hyperbolic).

Poisson's equation: fundamental solution.

<u>**Term 2**</u> (18 lectures)

Laplace's equation: mean value formula, properties of Harmonic functions, maximum principle, Green's functions, energy method.

The heat equation: fundamental solution, maximum principle, energy method, infinite speed of propagation, properties of solutions.

The wave equation: solution formulas, energy method, finite speed of propagation, properties of solutions.

1.8.16 PROJECT III – MATH3382

The aim is to take a topic in mathematics or statistics and to communicate the ideas, by a poster, a short presentation, and a written report, to an audience at the level of 3H. The emphasis is on communication of material at the 3H level.

The first term consists of independent guided study within a small group. Each group consists normally of 3 or 4 students. The second term consists of preparing an individual written report. During the second term, students give a group-linked presentation, accompanied by individual posters. The poster and presentation are assessed and count for 15% of the module mark.

There are no lectures. However, there is a short series of workshops. These address what constitutes effective communication in mathematics and statistics: how to present mathematical material, how to use computer packages to write mathematical reports, how to lay out reports, how to convey statistical information effectively, and so forth.

The topic supervisor guides the technical content and organizes tutorial meetings at least once per fortnight.

This module is a double 3H option so you should devote on average one-third of your efforts to it. The report deadline is the first Friday of the Easter term.

For details of topics and supervisors visit the projects webpage
`www.maths.dur.ac.uk/Ug/projects/index.html` and follow the prominent link.

To take this option, submit your preferred topics via the web link before the deadline given there. Definite project allocations will be announced before the start of the new academic year.

Please note that a supervisor may not accept 5 or more students, and a topic with fewer than 3 takers may be withdrawn.

1.8.17 QUANTUM MECHANICS III – MATH3111 (38 lectures)

Dr A. Donos / Dr M. Zamaklar

Quantum theory has been at the heart of the enormous advances that have been made in our understanding of the physical world over the last century, and it also underlies much of modern technology, e.g., lasers, transistors and superconductors. It gives a description of nature that is very different from that of classical mechanics, in particular being non-deterministic and 'non-local', and seeming to give a crucial role to the presence of 'observers'.

The course begins with a brief historical and conceptual introduction, explaining the reasons for the failure of classical ideas, and the corresponding changes needed to describe the physical world. It then introduces the defining postulates and reviews the necessary mathematical tools – basically linear vector spaces and second order differential equations. Applications to simple systems, which illustrate the power and characteristic predictions of the theory, are then discussed in detail. Some problems are solved using algebraic methods, others involve solutions of partial differential equations.

Recommended Books

G. Auletta, M. Fortunato and G. Parisi, **Quantum Mechanics**, CUP 2009, ISBN 0521869633; £42.
C. Cohen-Tannoudji, B. Diu, F. Laloë, **Quantum Mechanics**, Hermann 1977, ISBN 2705658335; £39.
F. Mandl, **Quantum Mechanics**, Wiley 1992, ISBN 0471931551; £25.
L.I. Schiff, **Quantum Mechanics**, McGraw Hill 1969, ISBN 0070856435; £39.
A. Messiah, **Quantum Mechanics**, 2 Volumes, North Holland 2000, ISBN 0486409244; £20
P. Dirac, **The Principles of Quantum Mechanics**, OUP 4th Ed. 1981, ISBN 0198520115; £25
R. Shankar, **"Principles of quantum mechanics", New York : Plenum, c1980 ISBN 8181286863**
Landau and Lifshitz, **Quantum Mechanics. (Non-relativistic Theory)** Butterworth-Heinemann 1982, ISBN 0080291406; £35.

Any of the above covers most of the course. There are many other books which are quite similar.

Preliminary Reading

Because quantum theory is conceptually so different from classical mechanics it will be useful to have read descriptive, non-mathematical, accounts of the basic ideas, for example from the introductory chapters of the books above, or from general 'popular' accounts e.g., in Polkinghorne, 'The Quantum World'; Pagels, 'The Cosmic Code'; Squires, 'Mystery of the Quantum World', etc.

Aim: To give an understanding of the reasons why quantum theory is required, to explain its basic formalism and how this can be applied to simple situations, to show the power in quantum theory over a range of physical phenomena and to introduce students to some of the deep conceptual issues it raises.

Term 1 (20 lectures)

Problems with Classical Physics: Photo-electric effect, atomic spectra, wave-particle duality, uncertainty principle.

Formal Quantum Theory: Vectors, linear operators, hermitian operators, eigenvalues, complete sets, expectation values, commutation relations. Representation Theory, Dynamics: Schrödinger and Heisenberg pictures.

Spectra of Operators: Position operator, Harmonic Oscillator, Angular momentum.

Term 2 (18 lectures)

Waves and the Schrödinger Equation: Time-dependent and time-independent Schrödinger equation. Probability meaning of $|\Psi|^2$. Currents. Plane-waves, spreading of a wave packet.

Applications in One Dimension: Square well, harmonic oscillator, square barrier, tunnelling phenomena.

Three dimensional problems Three-dimensional harmonic oscillator, Hydrogen atom, other systems.

1.8.18 REPRESENTATION THEORY III AND IV – MATH3371/MATH4241 (38 lectures)

Dr J. Funke

The central topic in the course are group representations, which are, quite literally, representations of groups as groups of matrices. Representation theory is one of the central areas in mathematics with many applications within pure mathematics, say in number theory, but also in physics (eg quantum mechanics) and chemistry (eg crystallography).

In the first term, we will cover the representation theory of finite groups. This topic is particularly neat and enjoys a very satisfying calculus of so-called group characters. This provides a striking solution to the problem of determining all the representations of a finite group.

The other main topic of investigation is the representation theory of some classical matrix groups (linear Lie groups) such as $SL_2(\mathbb{R})$, the 2 by 2 matrices of determinant 1, $SO(3)$, the group of rotations in 3-space. To study those we will associate to these groups their so-called Lie algebras. These are vector vector spaces and their representation theory can be studied solely by algebraic means. Via the exponential map one can then obtain representations for the matrix groups.

Prequisites

Algebra II MATH 2581

Recommended Books

R. Berndt, **Representations of Linear Groups, An Introduction Based on Examples from Physics and Number Theory**, Vieweg Verlag, ISBN 3834803197
W. Fulton & J. Harris, **Representation Theory. A first Course**, Springer Verlag, ISBN 0387974954
V.E. Hill, **Groups and Characters**, Chapman & Hall 2000, ISBN 1584880384
G.James & M.Liebeck, **Representations and characters of groups**, CUP, ISBN 0521445906
Y. Kosmann-Schwarzbach, **Groups and Symmetries**, Springer-Verlag, ISBN9780387788654
J.-P. Serre, **Linear Representations of finite groups**, Springer-Verlag, ISBN 0387901906

For the representation theory of finite groups we will mainly follow the treatment in Serre and Fulton-Harris. For the second term, we mainly use the books by Kosmann-Schwarzbach and Fulton-Harris.

Preliminary Reading

Revise 1H Linear Algebra and the group theory from 2H Algebra.

4H extra reading

The extra reading material will be taken from the book by Fulton-Harris. Depending on interest we can either cover the Frobenius-Schur theory of the representation theory of the symmetric group or of $SL_2(\mathbb{F}_p)$ and $GL_2(\mathbb{F}_p)$ over the finite field of p elements.

Term 1 (20 lectures)

Aim: To develop and illustrate the representation theory and that of complex characters of finite groups.

- Representation theory of finite groups: Basic notions, Schur's Lemma, Unitarizable representations, Maschke's theorem, tensor products.
- Character theory and tables: Revise conjugacy classes. Character of a representation and its properties, irreducible characters. Orthogonality of irreducible characters. Number and degree of irreducible characters. Construction of the character table of a finite group, orthogonality properties. Plancherel formula, Fourier inversion.
- Induced representations: Construction and basic properties. Frobenius reciprocity, Basics of the Mackey machine.
- Modules over the group algebra.
- Representations of abelian groups, dihedral groups, quaternion group, S_3, S_4, S_5, A_4, A_5, finite Heisenberg group. (throughout the term).

Term 2 (18 lectures)

Aim: To develop and illustrate the representation theory of classical Lie groups and Lie algebras via highest weight theory.

- Linear Lie groups and their Lie algebras, exponential map.
- Lie group and Lie algebra representations. (Finite)-dimensional representations of $\mathbb{R}$ and S^1. Relationship with Fourier series.
- Finite-dimensional representations of $SL_2(\mathbf{C})$ and its Lie algebra $\mathfrak{sl}_2(\mathbf{C})$ via highest weight theory. Representations of $GL_2(\mathbf{C})$.
- Representations of $SU(2)$ and $SO(3)$. Spherical harmonics.
- Finite-dimensional representations of $SL_3(\mathbf{C})$ and its Lie algebra $\mathfrak{sl}_3(\mathbf{C})$. Outlook to $SL_n(\mathbf{C})$ and $\mathfrak{sl}_n(\mathbf{C})$.
- Time permitting: Representation theory of the Heisenberg group and Lie algebra.

1.8.19 SOLITONS III – MATH3231 (38 lectures)

Prof P. Dorey / Dr P. Heslop

Solitons occur as solutions of certain special nonlinear partial differential equations. In general, nonlinear equations cannot be handled analytically. However, soliton equations (which arise naturally in mathematical physics) have beautiful properties; in particular, they admit solutions that one can write down explicitly in terms of simple functions. Such solutions include solitons: localized lumps which can move around and interact with other lumps without changing their shape. The course will describe a number of these soliton equations, explore their solutions, and study solution-generating techniques such as Bäcklund transformations and inverse scattering theory.

Recommended Books

*P.G. Drazin and R.S. Johnson, **Solitons: An Introduction**, CUP 1989; ISBN 0521336554
R. Remoissenet, **Waves Called Solitons**, Springer 1999, ISBN 3540659196
G.L. Lamb, **Elements of Soliton Theory**, Wiley 1980
M. Toda, **Theory of Nonlinear Lattices**, Springer [2nd Ed] 1988
G.B. Whitham, **Linear and Nonlinear Waves**, Wiley 1974
T. Dauxois and M. Peyrard, **Physics of solitons**, CUP 2006

The book by Drazin and Johnson covers most of the course, and is available in paperback.

Preliminary Reading

Read chapter 1 of Drazin and Johnson, and study the exercises for that chapter (note that the answers are given at the end of the book!).

Explore the website `www.ma.hw.ac.uk/solitons/` (in particular the movies and the pages on Scott-Russell's soliton).

Outline of course **Solitons III**

Aim: To provide an introduction to solvable problems in nonlinear partial differential equations which have a physical application. This is an area of comparatively recent development which still possesses potential for growth.

Term 1 (20 lectures)
Nonlinear Wave Equations: Historical introduction; John Scott Russell and the first experimental observation of a solitary wave (1834); Korteweg and de Vries' wave equation (1905). Soliton scattering. Properties of nonlinear wave equations; dispersion, dissipation: dispersion law for linear equations; group velocity and phase velocity. Examples.
Travelling Wave Solutions: Derivation of the sine-Gordon equation as the limit of the motion under gravity of a set of rigid pendulums suspended from a torsion wire. Travelling wave solutions: D'Alembert's solution recalled, and travelling wave solutions of KdV and sine-Gordon equations.
Topological lumps and the Bogomolnyi argument: sine-Gordon and ϕ^4 examples.
Conservation Laws in Integrable Systems: Conservation laws for the wave equation, sine-Gordon and KdV equations. Relation to conservation of soliton number and momentum in scattering.
Bäcklund Transformations for Sine-Gordon Equation: The Liouville equation and its solution by Bäcklund transformations of solutions of the free field equation. Generation of multisoliton solutions of sine-Gordon by Bäcklund transformations. Theorem of permutability. Two-soliton and breather solutions of sine-Gordon. Lorentz-invariant properties. Asymptotic limits of two-soliton solutions. Discussion of scattering.
Bäcklund Transformations for KdV Equation: The same analysis repeated for the KdV equation.
Hirota's Method: Hirota's method for multisoliton solutions of the wave and KdV equations.

Term 2 (18 lectures)
The Inverse Scattering Method: Discussion of the initial value problem; motivation for the inverse scattering method. The inverse scattering method for the KdV equation. Lax pairs. Discussion of spectrum of Schrdinger operator for potential; Bargmann potentials. Scattering theory; asymptotic states, reflection and transmission coefficients, bound states. Marchenko equations. Multisoliton solutions of KdV.
Integrability: Hamiltonian structures for the KdV equation. Integrability. Hierarchies of equations determined by conservation laws.
Toda equations: The Toda molecule and the Toda chain. Conservation laws for the Toda molecule.

1.8.20 STATISTICAL METHODS III – MATH3051 (36 lectures + 6 R practicals)

Dr J. Einbeck / Dr P. Craig

The course introduces widely used statistical methods. The course should be of particular interest to those who intend to follow a career in statistics or who might choose to do a fourth year project in statistics. Having a particular emphasis on the intersection of theory and practice, the learning objective of the course includes the ability of performing hands-on data analysis using the statistical programming language R. Therefore, three computer practicals will be held in each of Michaelmas and Epiphany term. Towards the end of each term, a practical examination component will be held, each of which contributes 15% towards the total examination mark.

Topics include: statistical computing using `R`; multivariate analysis (in particular, principal component analysis); regression (linear model: inference, prediction, variable selection, influence, diagnostics, outliers,); analysis of designed experiments (analysis of variance); extensions to transformed, weighted, and/or nonparametric regression models;

There is no one recommended book but the books below more than cover the course material; in particular those by Weisberg and Krzanowski provide (in conjunction) a good coverage in an accessible style. The book by Kutner et al. is quite voluminous but worth of consideration for those who prefer a detailed step-by-step description of the methods.

Recommended Books

M.J. Crawley, **The R book**, Wiley 2007, ISBN 0470510242.
W.J. Krzanowski, **Principles of Multivariate Analysis: A User's Perspective**, OUP Oxford 2000, ISBN 0198507089.
M. Kutner, C. Nachtsheim, J. Neter, W. Wasserman and W. Li, **Applied Linear Statistical Methods** (several editions with different combinations of authors 1964–2005), ISBN 007310874X.
K.V. Mardia, J.T. Kent, and J.M. Bibby, **Multivariate Analysis**, Academic Press 1979, ISBN 0-12-471250-9.
T. Raykov, **Basic Statistics - an introduction with R**, Rowman & Littlefield Publishers 2012, Access via Durham MyiLibrary, ISBN 9781283833837.
J.A. Rice, **Mathematical Statistics and Data Analysis**, Brooks/Cole 2006, 3rd ed. ISBN 0495110892 (a used, i.e. cheap, second edition is also appropriate).
S. Weisberg, **Applied Linear Regression**, Wiley 2005, ISBN 0471879576.

Preliminary Reading

Chapters 3.1-3.4, 8.1-8.4, 12, and 14 in Rice's book **Mathematical Statistics and Data Analysis**, Brooks/Cole 2006.

Aim: To provide a working knowledge of the theory, computation and practice of statistical methods, with focus on the linear model.

Term 1 (19 lectures + 3 R practicals)

Basics: Statistical computing in R, matrix algebra, multivariate probability and likelihood, multivariate normal distribution.

The linear model: Assumptions, estimation, inference, prediction, analysis of variance, designed experiments, model selection.

Term 2 (17 lectures + 3 R practicals)

Regression diagnostics: influence, outliers, lack-of-fit.

Introduction to multivariate analysis: Variance matrix estimation, Mahalanobis distance, principal component analysis; dimension reduction.

Extensions: Basics of transformed, weighted, and/or nonparametric regression models.

1.8.21 STOCHASTIC PROCESSES III AND IV – MATH3251/MATH4091 (38 lectures)

Dr O. Hryniv / Prof M. Menshikov

A stochastic process is a mathematical model for a system evolving randomly in time. For example, the size of a biological population or the price of a share may vary with time in an unpredictable manner. These and many other systems in the physical sciences, biology, economics, engineering and computer sciences may best be modelled in a non-deterministic manner.

More technically, a stochastic process is a collection of random quantities indexed by a time parameter. Typically these quantities are not independent, but have their dependency structure specified via the time parameter. Specific models to be covered include martingales, branching processes, Markov chains in continuous time, and Poisson processes.

Recommended Books

G. R. Grimmett and D. R. Stirzaker, **Probability and random processes**, 3rd ed., OUP, 2001.
S. M. Ross, **Introduction to probability models**, Academic Press, 2003.
S. Karlin and H. M. Taylor, **A first course in stochastic processes**, Academic Press, 1975.
W. Feller, **An introduction to probability and its applications**, Volume I., Wiley, 1968.
J. R. Norris, **Markov chains**, Cambridge University Press, 1998.
R. Durrett, **Essentials of Stochastic Processes**, Springer, 1999.

Aim: This module continues on from the treatment of probability in 2H modules MATH2151 or MATH2161. It is designed to introduce mathematics students to the wide variety of models of systems in which sequences of events are governed by probabilistic laws. Students completing this course should be equipped to read for themselves much of the vast literature on applications to problems in mathematical finance, physics, engineering, chemistry, biology, medicine, psychology and many other fields.

Term 1 (20 lectures)

Probability Revision: Conditional expectation, sigma fields, generating functions.

Branching processes and their applications. Survival probability. Critical branching process.

Discrete Parameter Martingales: The upcrossing lemma, almost sure convergence, the backward martingale, the optional stopping theorem.

Applications of Martingales: Examples, applications of martingale theory to discrete time processes, including branching processes and discrete time Markov chains.

Term 2 (18 lectures)

General Renewal Theory: The renewal equation and limit theorems in the continuous case, excess life, applications. The renewal-reward model.

Poisson Processes: Poisson process on the line, relation to exponential distribution, marked/compound Poisson processes, Cramér's ruin problem, spatial Poisson processes.

Continuous Time Markov Chains: Kolomogorov equations, birth and death processes, simple queueing models, Jackson networks.

Topics Chosen From: Random graphs, Brownian motion, percolation theory, interacting particle systems.

4H reading material: To be announced.

1.8.22 TOPOLOGY III – MATH3281 – (38 lectures)

Dr V. Kurlin / Prof J. Hunton

Topology is a mathematical theory which explains many interesting natural phenomena: it helps to study equilibrium prices in economics, instabilities in behaviour of dynamical systems (for example, robots), chemical properties of molecules in modern molecular biology and many other important problems of science. Topological methods are used in most branches of pure and applied mathematics.

The course is an introduction to topology. We shall start by introducing the idea of an abstract topological space and studying elementary properties such as continuity, compactness and connectedness. A good deal of the course will be geometrical in flavour. We shall study closed surfaces and polyhedra, spend some time discussing winding numbers of planar curves and their applications. Orbit spaces of group actions will provide many interesting examples of topological spaces.

Elements of algebraic topology will also appear in the course. We shall study the fundamental group and the Euler characteristic and apply these invariants in order to distinguish between various spaces. We shall also discuss the concept of homotopy type; this material will be further developed in the course 'Algebraic Topology IV'.

Recommended Books

M.A. Armstrong, **Basic Topology**, Springer-Verlag 1983, ISBN 3540908390
R. H. Crowell and R. H. Fox, **Introduction to Knot Theory**, 1963.
W. Fulton, **Algebraic topology, a first course**, 1995

Preliminary reading

To get an idea of what topology is about one may consult the first two chapters of Armstrong. Any of the following two books may also serve as an enjoyable introduction to topology:

W. G. Chinn and N.E. Steenrod, **First Concepts of Topology**, Random House, New York, 1966
C.T.C. Wall, **A Geometric Introduction to Topology**, Addison Wesley, Reading, Mass., 1972.

Aim: To provide a balanced introduction to Point Set, Geometric and Algebraic Topology, with particular emphasis on surfaces.

Term 1 (20 lectures)

Topological Spaces and Continuous Functions: Topological spaces, limit points, continuous maps, homeomorphisms, compactness, product topology, connectedness, path-connected spaces, quotient topology, graphs and surfaces.

Topological Groups and Group Actions: Orthogonal groups, connected components of $O(n)$, the concept of orientation, topological groups, quaternions, group actions, orbit spaces, projective spaces, lens spaces.

Term 2 (18 lectures)

Elements of geometric topology:

Classification of graphs up to homeomorphism. Connected sums of surfaces. Orientable and non-orientable surfaces. The topological classification of compact surfaces. Polyhedra, triangulations of topological spaces, the topological invariance of the Euler characteristic, properties of the Euler characteristic.

Basic concepts of homotopy theory: The winding number of loops and applications. Homotopic maps and homotopy equivalence of spaces. Classification of graphs up to homotopy.

1.8.23 ADVANCED QUANTUM THEORY IV – MATH4061 (38 lectures)

Dr M. Zamaklar / Dr K. Peeters

The course will introduce Quantum Field Theory (QFT) by bringing together concepts from classical Lagrangian and Hamiltonian mechanics, quantum mechanics and special relativity. It also provides an elementary introduction to string theory, both as a simple two-dimensional QFT and as a way to go beyond QFT concepts.

The course will begin with a reminder of classical field theory concepts. We then go on to a discussion of free and interacting quantum field theories in the operator formalism, explain Feynman diagrams and the computation of scattering amplitudes. Throughout, simple examples will be used to emphasise the conceptual ingredients rather than the computational technicalities.

In the second term, we begin with an explanation of the path integral formulation of quantum theory, both in relativistic quantum mechanics and in quantum field theory. String theory follows next, and we will discuss its spectrum, symmetries and dualities. We will see how self-consistency conditions lead to a preferred number of spacetime dimensions and the existence of gauge fields and gravitons (providing a connection to general relativity). The last part of the course is about scale dependence, renormalisation and the renormalisation group.

The course is complementary to the PHYS4181 Particle Theory module in the sense that it focuses more on conceptual and mathematical foundations. It should prove interesting and useful especially to students who want to continue to pursue an interest in what is currently understood of the fundamental nature of matter.

Recommended Books

Typed lecture notes will be provided. In addition, there are many QFT books and string theory texts so you will find no shortage of material in the library. Some suggestions: Anthony Zee, **Quantum Field Theory in a Nutshell**, Princeton 2010, ISBN 0691140340.
Brian Hatfield, **Quantum Field Theory of Point Particles and Strings**, Perseus 1999, ISBN 0201360799.
Mark Srednicki, **Quantum Field Theory**, Cambridge 2007, ISBN 0521864496.
Barton Zwiebach, **A First Course in String Theory**, Cambridge 2004, ISBN 0521831431.
Michael Green, John Schwarz & Edward Witten, **Superstring theory, Volume 1, Introduction**, Cambridge 1988, ISBN 0521357527.

Aim: To introduce quantum field theory using the operator formalism as well as path integrals, and to apply it to string theory, developing it sufficiently to show that its spectrum includes all elementary particles thus unifying the fundamental forces.

Term 1 (20 lectures)

Action principles and classical theory: Review of Lagrangian formulation of classical field theory. Symmetries, equations of motion, Lagrangian and Hamiltonian methods, Noether's theorem.

Quantisation of free scalar fields: Multi-particle quantum mechanics, canonical quantisation of free scalar fields, Fock space, anti-particles, propagators, causality.

Interacting quantum fields: Evolution operators, perturbative expansion, Wick's theorem, Feynman diagrams in position and momentum space, LSZ reduction, scattering matrix, cross sections.

Term 2 (18 lectures)

Path integrals: Relativistic particle in the world-line formulation, generating functionals, diagrammatic expansion, zero-dimensional quantum field theory.

String theory: World-line action for free relativistic particle, formulation with intrinsic metric. Nambu-Goto action and Polyakov action and equations of motion, symmetries, boundary conditions, simple classical solutions, quantisation, Virasoro algebra, physical states, connection to general relativity, T-duality.

Renormalisation and scale dependence: Regularisation methods, renormalisation, power counting, renormalisation group flow.

1.8.24 ALGEBRAIC TOPOLOGY IV – MATH4161 (38 lectures)

Dr A. Lobb / Dr D. Schuetz

The basic method of Algebraic Topology is to assign an algebraic system (say, a group or a ring) to each topological space in such a way that homeomorphic spaces have isomorphic systems. Geometrical problems about spaces can then be solved by 'pushing' them into algebra and doing computations there. This idea can be illustrated by the theory of fundamental group which is familiar from the Topology III course.

In the course 'Algebraic Topology' we meet other, more sophisticated theories such as singular homology and cohomology. The course begins with an introduction to cell complexes, a convenient class of topological spaces which contain many important examples and which are amenable to homology calculations. Properties of singular homology are studied and used to prove important theorems such as the Brouwer Fixed Point Theorem and the Jordan Curve Theorem.

In the second term cohomology is introduced, which appears formally very similar to homology, but adds useful structure to the theory. Applied to manifolds, the full power of these theories lies in their interplay, which culminates in the duality theorems of Poincaré and Alexander. As an application we show that non-orientable surfaces such as the real projective plane and the Klein bottle cannot be embedded in 3-dimensional Euclidean space.

Recommended Books

Allen Hatcher, **Algebraic topology**, Cambridge university press, 2002; This book is available free of charge from `www.math.cornell.edu/~hatcher`
M.A. Armstrong, **Basic topology**, Springer-Verlag, 1983
G. Bredon, **Topology and Geometry**, Springer-Verlag, 1993
A. Dold, **Lectures on Algebraic topology**, Springer-Verlag, 1980
W. Fulton, **Algebraic topology (a first course)**, Springer - Verlag, 1995.
E. Spanier, **Algebraic topology**, 1966

Aim: The module will provide a deeper knowledge in the field of topology (a balanced introduction having been provided in Topology III (MATH3281))

Term 1 (20 lectures)

Homotopy properties of cell complexes: Cell complexes, main constructions (mapping cones and cylinders, products), examples.

Elements of homological algebra: Chain complexes, homology, chain homotopy, exact sequences, Euler characteristic.

Homology theory of topological spaces: Singular homology of topological spaces, homotopy invariance, Mayer - Vietoris sequences, relation between homology and the fundamental group, geometric interpretation of homology classes, homology groups of cell complexes.

Applications: Brouwer Fixed Point Theorem, Jordan Curve Theorem, Invariance of Domain.

Term 2 (18 lectures)

Cohomology theory of topological spaces: Singular cohomology of topological spaces, Statement of the Universal Coefficient Theorem, cup products and ring structure, Künneth formula.

(Co)homology of manifolds: Fundamental classes, orientations in terms of homology, intersection numbers, Poincaré duality.

Applications: Alexander duality, Non-embeddability of non-orientable closed surface in Euclidean 3-space.

1.8.25 PROJECT IV – MATH4072

Each project is two terms' work for a group of typically 2 to 4 students who collaborate on an open-ended mathematical or statistical problem. Each student writes an individual report on an aspect of the work, and gives an oral presentation with accompanying poster during the Epiphany term. The presentation and poster are assessed and count towards 15% of the module mark.

There are no lectures; the emphasis is on developing understanding in depth and encouraging creativity through reading, discussion and calculation. The Project supervisor gives general guidance and organizes tutorial meetings at least once a fortnight.

A project is equivalent to two 4H options so you should devote on average one-third of your effort to it. The report deadline is the first Friday of the Easter term.

The Project topics and supervisors, with further details, are available via the projects webpage `www.maths.dur.ac.uk/Ug/projects/index.html` and follow the prominent link.

Choose your preferred topics, via the web link, before the deadline given there. Definite project allocations will be announced before the start of the new academic year.

1.8.26 RIEMANNIAN GEOMETRY IV – MATH4171 (38 lectures)

Dr A. Felikson / Dr P. Tumarkin

This course is based on the course Differential Geometry III as a prerequisite.

In the first term we introduce Riemann's concept of a manifold as a space with locally Euclidean coordinates and an intrinsic method of measuring distances and angles and discuss it in several examples. We also introduce geodesics via an variational approach and discuss parallel transport of vector fields along curves.

The second term is concerned with the notion of curvature and its influence on the geometry of the underlying space. We study spaces of constant curvature, present a global curvature comparison theorem (Bonnet-Myers) and discuss applications.

The aim of this course is to develop acquaintance with more general geometric spaces than just the Euclidean space and to the geometric meaning of curvature.

Possible Books:

M. P. Do Carmo: **Riemannian Geometry**, Birkhäuser, ISBN 0-8176-3490-8

F. Morgan: **Riemannian Geometry: A Beginner's Guide**, Jones and Bartlett Publishres, ISBN 0-86720-242-2

S. Gallot, D. Hulin, J. Lafontaine: **Riemannian Geometry**, Springer, ISBN 0-387-52401-0

J. Lee: **Riemannian Manifolds, An Introduction to Curvature**, Springer, ISBN 0-387-98271-X

J. Cheeger, D. G. Ebin: **Comparison Theorems in Riemannian Geometry**, Elsevier Science, ISBN 0-444-107649

Aim: The aim of this course is to develop acquaintance with more general geometric spaces than just the Euclidean space and to the geometric meaning of curvature.

Term 1 (20 lectures)

- From submanifolds to **abstract manifolds** via examples: surfaces of revolution, projective space, Grassman manifolds, hyperbolic space, matrix groups
- Tangent vectors and **tangent space**, computations in examples (e.g. in matrix groups), vectorfields and computations of Lie brackets
- examples of **Riemmanian manifolds**
- length of curves, Riemannian manifolds as metric spaces
- local and global properties of **geodesics**, first variation formula, **Levi-Civita connection**, parallel transport, discussion of examples (e.g. hyperbolic space, matrix groups)
- geodesics on surfaces of revolution, Clairaut's theorem

Term 2 (18 lectures)

- different curvature notions and computations thereof: **Riemmanian curvature tensor**, sectional curvature, Ricci curvature, scalar curvature
- spaces of constant curvature, the Cartan-Ambrose-Hicks theorem
- integration on Riemannian manifolds, volume calculations in spaces of constant curvature
- the second variation formula and **Bonnet-Myers theorem** as a global comparison result
- applications (e.g., the n-torus does not admit a metric of positive curvature; the global 2-dimensional Gauß-Bonnet theorem from differential geometry implies same statement for 2-torus)