Jacobs University
School of Engineering and Science
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Spring Term 2009

Problem Set 1

Analysis II

Homework Problems

1.1. Show that a uniformly continuous function $f : M \to \mathbb{R}$ is continuous on M.

1.2. Suppose $f, g : M \to \mathbb{R}$ are uniformly continuous.

(a) Is it true that $f + g, fg$ are uniformly continuous?

(b) Does the answer to (a) changes if f, g are bounded?

(c) Does the answer to (a) changes if M is a closed interval?

In the following problems $\alpha : [a, b] \to \mathbb{R}$ is an increasing function, $f : [a, b] \to \mathbb{R}$ is bounded.

1.3. Let f be continuous and non-negative on $[a, b]$, $f \in \mathcal{R}(\alpha)$. Show that if $\int\limits_a^b f\,d\alpha = 0$ then $f(x) = 0$ for all $x \in [a, b]$.

1.4. Suppose $f(x) = \alpha(x) = \operatorname{sgn}(x)$. Show that f is not integrable with respect to α on any interval $[a, b]$ if $a < 0 < b$.

1.5. Let f be continuous on $[a, b]$, and $a < c < b$. Define $\alpha(x)$ by

$$\alpha(x) = \begin{cases} 0 & \text{if } x < c \\ 1 & \text{if } x \geq c \end{cases}$$

Show that $\int\limits_a^b f\,d\alpha = f(c)$

1.6. Let α be continuous. Prove that

(a) if the number of discontinuity points of f is finite then $f \in \mathcal{R}(\alpha)$.

(b) if f is monotonic then $f \in \mathcal{R}(\alpha)$.

Due Date: Wednesday, February 18, at the beginning of class.