Jacobs University
School of Engineering and Science
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Spring Term 2009

Problem Set 3

Analysis II

Homework Problems

3.1. Find the following indefinite integrals:

(a) $\int \arcsin x\,dx$; (b) $\int \ln x\,dx$; (c) $\int \frac{dx}{a^2-x^2}$; (d) $\int \frac{dx}{a^3+x^3}$.

3.2. Find all the functions $f(x)$ satisfying the following properties:

$$f'(x) = \frac{2x^3+7x}{(x^2+3)^3}, \qquad f(0) = -\frac{1}{3}$$

3.3. Compute the following Riemann integrals:

(a) $\int_0^2 \frac{x^5}{\sqrt{x^2+1}}\,dx$; (b) $\int_0^{\pi/4} \tan^{2n} x\,dx$; (c) $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$.

3.4. Using Riemann integrals of appropriate functions, find the following limits:

(a) $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{3n}\right)$;

(b) $\lim_{n\to\infty}\frac{1}{n}\sqrt[n]{(n+1)(n+2)\cdots(n+n)}$;

($\star$) $\lim_{n\to\infty}\left(\sin\frac{n}{n^2+1^2}+\sin\frac{n}{n^2+2^2}+\cdots+\sin\frac{n}{n^2+n^2}\right)$.

3.5. Define *Gamma function* $\Gamma(x)$ in the following way:

$$\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}\,dt$$

Show that

(a) $\Gamma(x)$ is well-defined for all $x>0$ (i.e., the integral converges for any $x>0$);

(b) $\Gamma(x+1) = x\Gamma(x)$;

(c) $\Gamma(n) = (n-1)!$ for $n\in\mathbb{N}$;

3.6. Which of the following integral converge ($\alpha>0$)?

(a) $\int_1^{\infty} \sin x^{\alpha}\,dx$; (b) $\int_1^{\infty} e^{\sin x}\frac{\sin 3x}{x^{\alpha}}\,dx$; ($\star$) $\int_0^{\infty} e^{\cos x}\frac{\sin(\sin x)}{x}\,dx$.

Due Date: Friday, March 6, at the beginning of class.