Jacobs University
School of Engineering and Science

Spring Term 2010

Linear Algebra II, Homework 8

Due Date: Thursday, April 22, in class.

Problems marked ($\star$) are bonus ones.

8.1. Show that for skew-symmetric 4×4 matrix $A = (a_{ij})$ its Pffafian can be written as

$$\mathrm{Pf}\ A = -a_{12}a_{34} + a_{13}a_{24} - a_{14}a_{23}$$

8.2. Let $A \in Sp_{2r}$.

(a) Show that $\chi_A(\lambda) = \lambda^{2r}\chi_A(1/\lambda)$.
Hint: Use the fact $A = J^{-1}(A^t)^{-1}J$, where J is the Gram matrix of a symplectic form in a symplectic basis.

(b) Show that for every eigenvalue λ of A, the numbers $\bar{\lambda}$, $\frac{1}{\lambda}$, and $\frac{1}{\bar{\lambda}}$ are also eigenvalues of A.

8.3. Let L be a linear space, $M_1, M_2 \subset L$ are linear subspaces. For any linear space M denote by $P(M)$ the projectivization of M. Show that

(a) $P(M_1) \cap P(M_2) = P(M_1 \cap M_2)$;

(b) $P(M_1 + M_2)$ coincides with the projective span $\overline{P(M_1) \cup P(M_2)}$ of $P(M_1) \cup P(M_2)$.

8.4. Let P_1 be a projective line, and $P_2 \notin P_1$ be a point in projective space $\mathbb{FP}^4$. Find $\overline{P_1 \cup P_2}$.

8.5. (a) Show that the system of linear equations

$$\sum_{j=0}^{n} a_{ij}x_j = 0, \qquad 1 \le i \le m$$

defines a projective subspace in $\mathbb{FP}^n$.

(b)($\star$) Show that any projective subspace in $\mathbb{FP}^n$ can be defined by a system of linear homogeneous equations.