Jacobs University
School of Engineering and Science

Spring Term 2010

Linear Algebra II, Homework 9

Due Date: Thursday, April 29, in class.

Problems marked ($\star$) are bonus ones.

9.1. Show that ellipse, hyperbola, and parabola on the plane are projectively equivalent.

9.2. In 3-dimensional space four planes pass through a line l, and a line m intersects all the four planes (but not the line l). Show that the cross-ratio of the intersection points of m with the planes doe depend on the choice of m.

9.3. ($\star$) Let $\rho = [x_1, x_2, x_3, x_4]$ be cross-ratio of four points. Show that the value

$$\frac{(\rho^2 - \rho + 1)^3}{\rho^2(\rho - 1)^2}$$

is invariant under permutations of points x_1, x_2, x_3, x_4.

9.4. Show that the natural map $t : L_1 \times L_2 \to L_1 \otimes L_2$ is bilinear.

9.5. Show that for any factorizable $x \in L_1 \otimes L_2$ there is a unique representation

$$x = x_1 \otimes x_2 \qquad x_1 \in L_1, x_2 \in L_2$$

up to transformation $x_1 \to \lambda x_1$, $x_2 \to \lambda^{-1} x_2$.