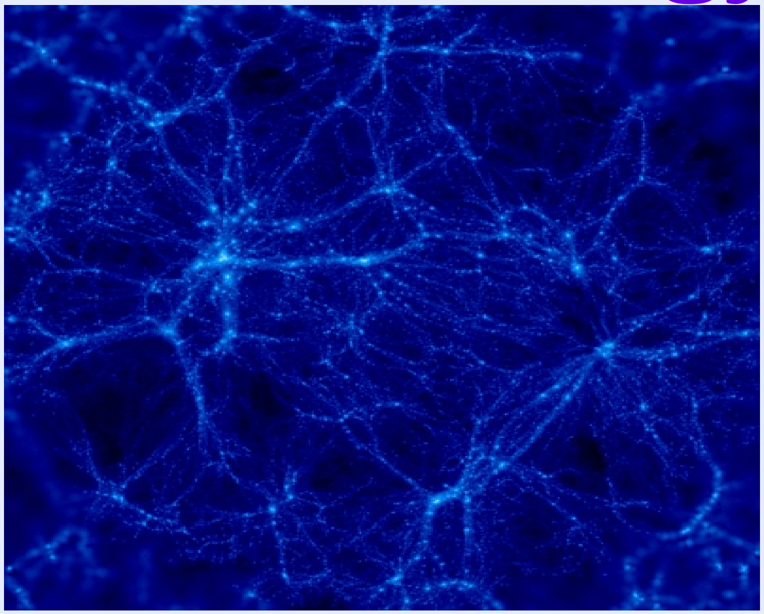
N-Body Simulations and models of Dark Energy



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Introduction

- N-Body simulations
  Motivation

- Method
  initial conditions: according to structure formation model
  assign all particles to a grid : $\rho_{i,j,k}$
  solve Poisson's equation on grid
  differentiate to get forces $F_{i,j,k} = -\nabla\phi_{i,j,k}$
  interpolate $F_{i,j,k}$ back to particle positions

Dark Energy Models

Evidence for DE
General Quintessence models
Early Dark Energy models

N-Body Simulations

- Allows us to model growth of structure in the universe
- Important tool for accurate predictions in highly non-linear regime e.g Hierarchical structure formation
- Determine properties of the primordial perturbations from inflation
- Useful for exploring how galaxies formed and theories of structure collaspe
- Measuring bias: the relation between large scale distribution of mass and the distribution of observable objects

Initial Conditions

- Placing particles in box
  - Grid -particles distributed on lattice -introduces artificial small scale periodicity
  - "Glass"state -particles advanced from random positions using opposite sign of gravity
- Zeldovich approximation to move particles. Primordial fluctuations nearly scale invariant and Gaussian

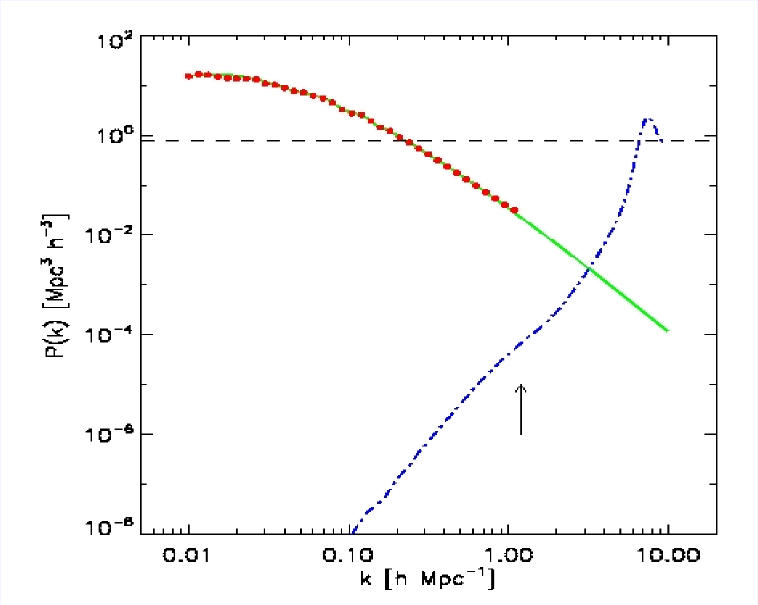
$$\vec{x} = \vec{q} - D(t)\,\vec{S}(q) \qquad \vec{q} = \text{initial coords}, D(t) = \text{growth factor}$$

- Displacement field $\vec{S}$ related to pre-calculated Power spectrum

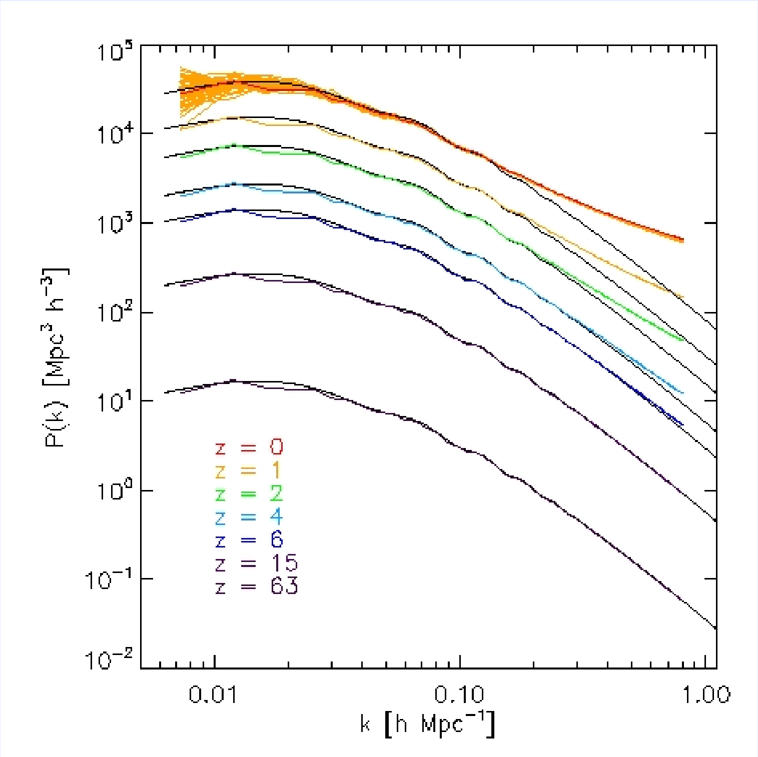
$$\frac{k^3}{2\pi^2} P(k,z) = \delta_H^2 \left(\frac{ck}{H_0}\right)^{n+3} T^2(k,z) \frac{D_1(z)^2}{D_1(0)^2}$$

- Overdensity $\delta = \frac{\rho - <\rho>}{<\rho>}$
- Transfer function $T(k, z)$ evolution of δ through horizon crossing and rad/matter transition
- $D(z)$ scale independent growth at late times

$$\ddot{\delta} + 2\,H\dot{\delta} - (3/2)\,H^2\,\Omega_m\delta = 0$$



Angulo et. al 2007



Angulo et. al 2007

- simultaneously solve collisionless Boltzmann and Poisson equation.

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

f = distribution function

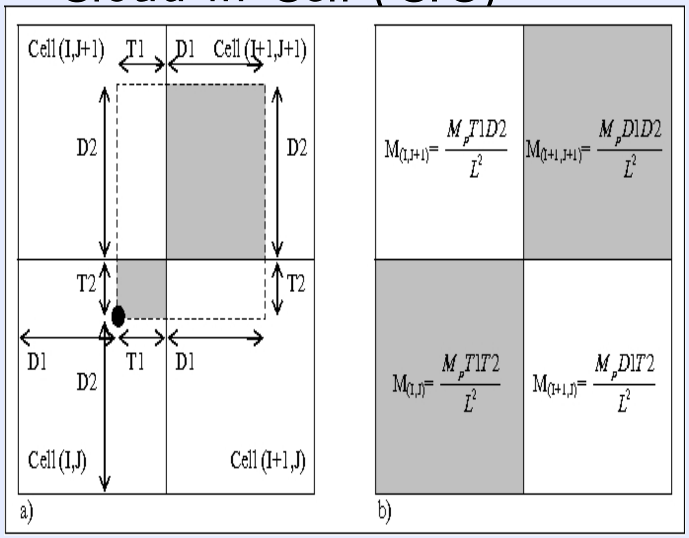
$$\nabla^2 \Phi(r) = 4\,\pi\, G\, \rho(r)$$

- Reduce problem, solve Poisson eqn for N particles
- Advance them forward using eqns of motion
  from system's Hamiltonian ($\partial f / \partial t + [f, H] = 0$)

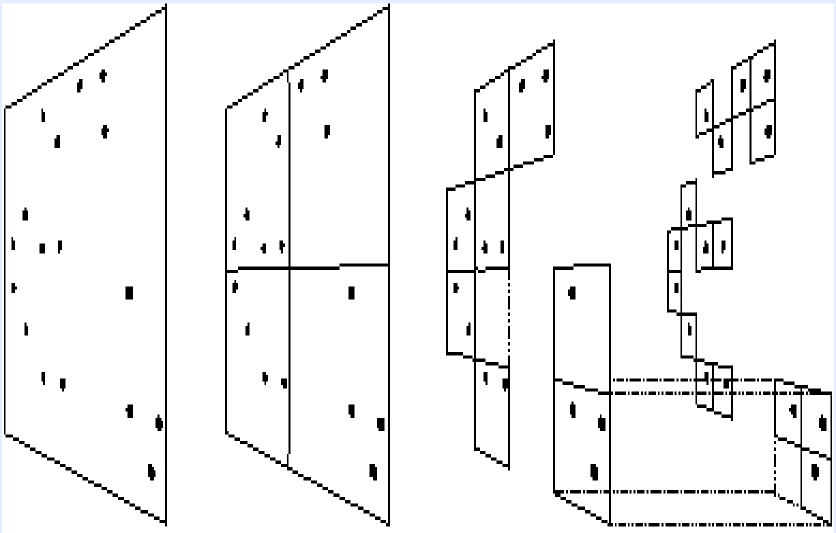
- Particle Mesh Codes-Creating $\rho\ (r)$

  Nearest Grid point (NGP)

Cloud in Cell (CIC)



CIC assignment scheme



Barnes and Hut Oct tree

$$\nabla^2\Phi(r) = 4\,\pi\,G\,\rho(r)$$

- Fast Fourier Transform $\rho(k)$
- use Green's function $-4\pi G/k^2$
- Forces on mesh:finite difference the potential

$$F = -\left(\frac{\partial\phi}{\partial x}\right)_{ijk}$$

- update particle positions and velocities

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{a^2}$$
$$\frac{d\vec{p}}{dt} = -\frac{\nabla\Phi}{a}$$

- Alternative to PM code is to use a Tree
- Particle-particle method $\rho(\vec{r}) = \sum_{i=1}^{N} m_i \delta(\vec{r} - \vec{r}_i)$
- Gadget-2: Code for cosmological N-body/SPH simulations on massively parallel computers
  PMTree code. Force split: long and short range.
  Tree code: particles which are ‘far away’act as single massive particle

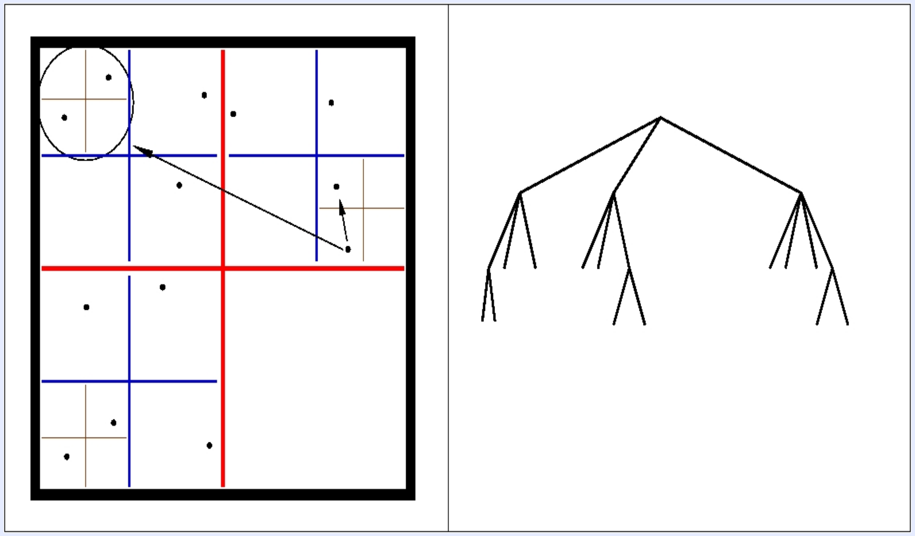


Illustration of a tree code

Resolution

- Force resolution

  Soften gravity to avoid $\Delta r = 0$ singularity. Introduce a fixed scale: ϵ

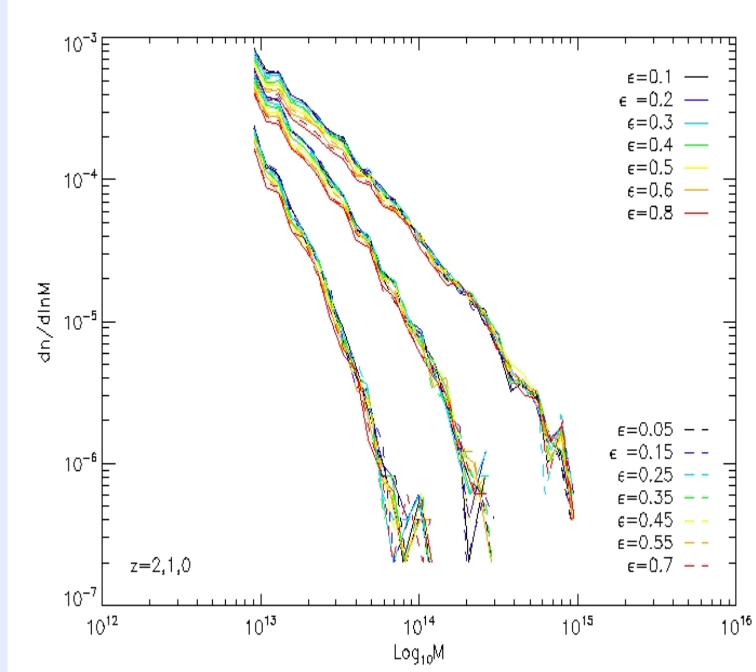
  $\vec{F}(\vec{r}) = \sum \frac{m_i m_j}{|\vec{r_i} - \vec{r_j}|^2 + \epsilon^2}$

- Mass resolution

  Set by size of box and N

  E.g. 2million particles in $L_{\text{box}} = 25\text{h}^{-1}\text{Mpc}$ with $\Omega = 0.3 \rightarrow m_p =\sim 10^8 \text{h}^{-1} M_{\text{sun}}$

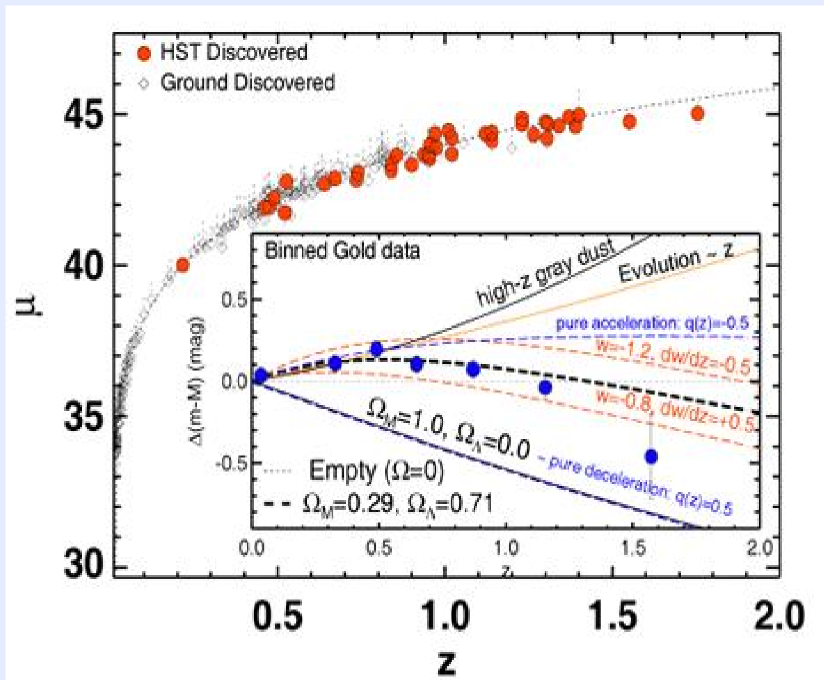
  Detail with which final object can be studied (e.g. cannot resolve dwarf galaxies $M_{\text{dwarf}} \sim 10.^7 \text{h}^{-1}\ M_{\text{sun}}$)



DarkEnergy

- Evidence for acceleration which implies dark energy

  Supernovae Ia luminosity distance and redshift observations
  Cosmic Microwave Background Radiation
  WMAP5: $\Omega_k < 0.008$ (95%C.L.), $\Omega_m = 0.274 \pm 0.015$
  Large Scale structure (LSS) e.g.clusters of galaxies $\rho_m \sim 0.3\, \rho_{\text{crit}}$



Models for Dark Energy

$$R_{\mu\nu} - 1/2\, g_{\mu\nu}\, R = 8\pi G\, T_{\mu\nu}$$

- Cosmological constant

  Cosmology $\rho_\lambda = 0.7\rho_c \sim 10.^{-48}$Gev^4
  QFT estimate $(\sim 100\text{Gev})^4$, at Planck scales ~ 120 orders difference

- Change matter Lagrangian

  Quintessence models, K-essence,Chaplygin gas,scalar-tensor theories

- Changing gravity

  F(R) theories, Brane world Cosmologies,Conformal Gravity . . .

Quintessence

Scalar field model, orginally introduced to answer "why now" problem

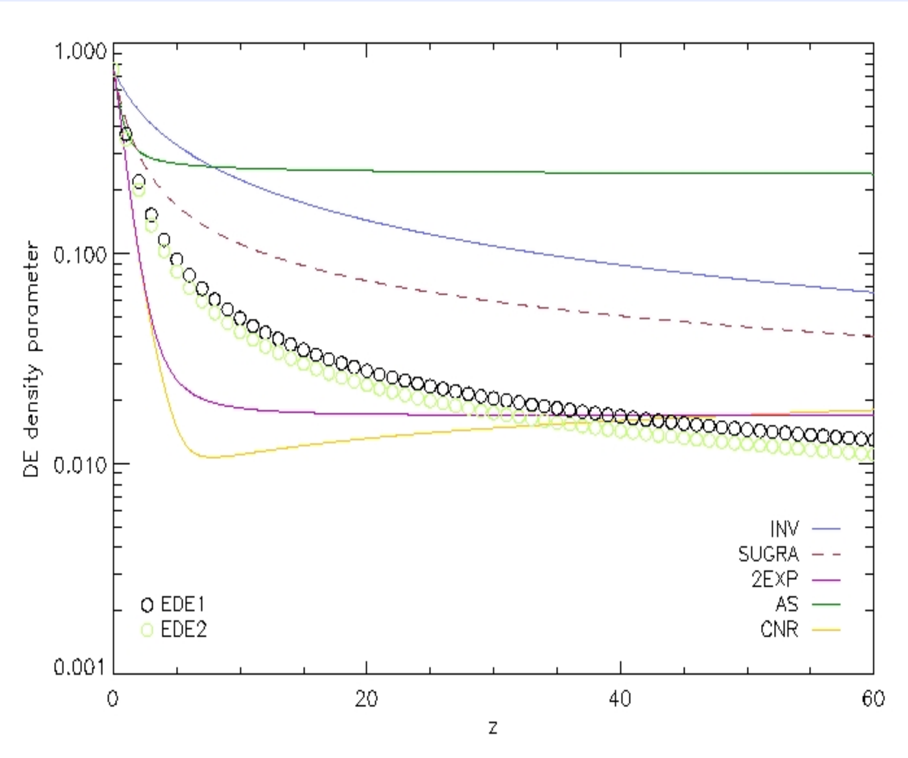
$$\ddot{\phi} + 3\,H\,\dot{\phi} + \frac{dV}{d\phi} = 0$$

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}/2 - V(\phi)}{\dot{\phi}/2 + V(\phi)}$$

- Within a flat universe, dark energy picture the expansion rate is

$$\frac{H(z)^2}{H_0^2} = \Omega_m(1+z)^3 + (1-\Omega_m)\,e^{3\,\int_0^z dln(1+z')(1+w(z'))}$$

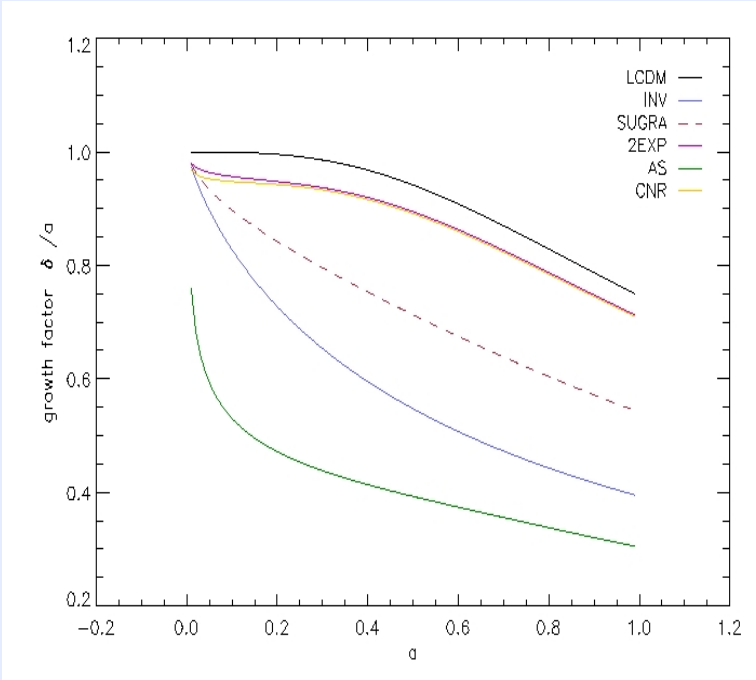
- Tracker potential $V(\phi) = M^{4+\alpha}\phi^{-\alpha}$
  Exp potential $V(\phi) = M^4 e^{\lambda\phi}$
  Scale M fixed by requiring $V(\phi \simeq Mpl) \simeq \rho_c$
- If $\alpha >> 2$ the scalar field tracks the matter but decays slower so it eventually comes to dominate



- Friedman eqn and growth rate eqn probe different parts of theory

- For any distance measurement, can find a $w(z)$ that fits it. Not neccessarily fit the growth of structure.

- Early Dark Energy model



Strong impact on structure formation expected.
Slower growth rate than ΛCDM so structure must form earlier to match present observed cluster abundances.

$$\ddot{\delta} + 2\,H\dot{\delta} - (3/2)\,H^2\,\Omega_m\delta = 0$$

Parametrisation for $w(z)$

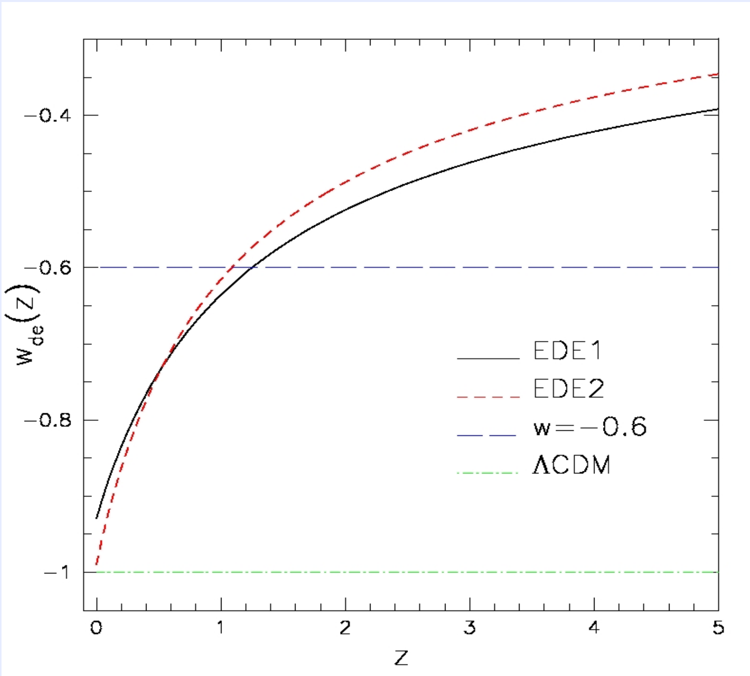
- Each model of DE its own form for w. To compare models we need a parametrisation
- Usually use $w = w_0 + w_a(1-a)$
- EDE Wetterich (2004) proposed a useful parametrisation

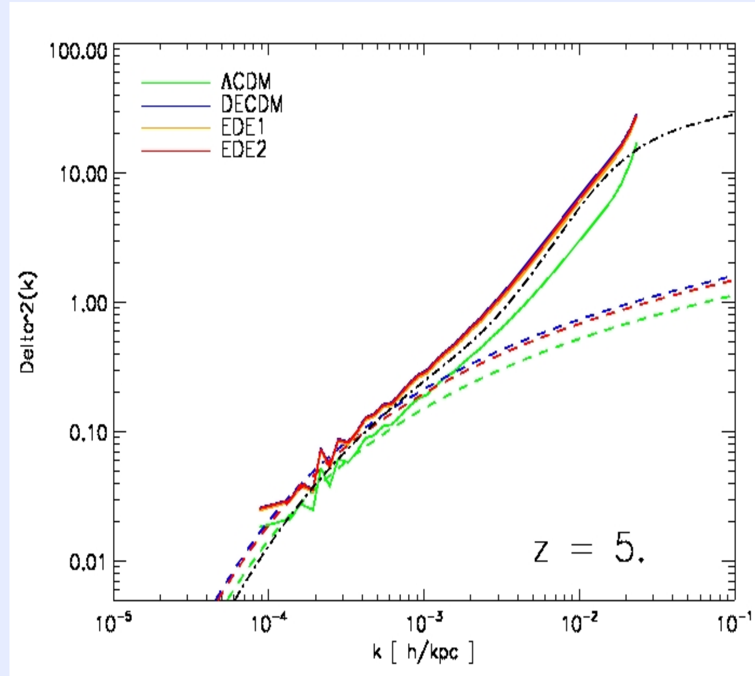
Amount of DE today: $\Omega_{de,0}$
Eqn of state parameter today: w_0
Average value of early DE: $\Omega_{de,e}$

$$w(z) = \frac{w_0}{(1+by)^2}$$

where $y = ln(1+z)$ and

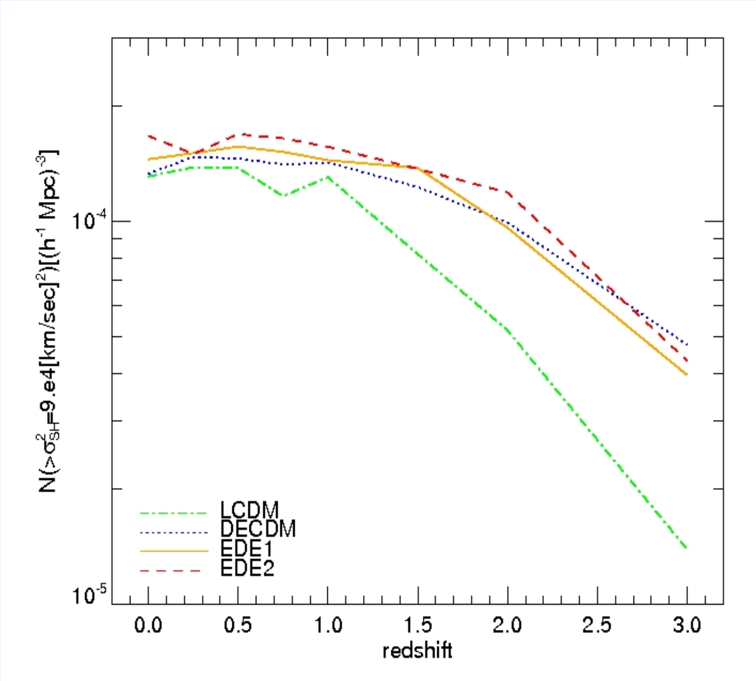
$$b = \frac{-3w_0}{ln(\frac{1-\Omega_{de,e}}{\Omega_{de,e}}) + ln(\frac{1-\Omega_m}{\Omega_m})}$$





Grossi et. al 2008

- Testable predictions: probing growth of structure at each redshift - measure of $w(z)$
- Pan-Starrs1 Medium survey: 70 deg^2, 3000 clusters in range $z = 1 - 2$
- More work is needed studing EDE models in N-Body simulations



Summary and future work

- The contribution to the energy density that leads to the observed acceleration could be more general component than a bare cosmological constant.
- Quintessence models where a scalar field, ϕ rolls down it's potential can act as an effecive cosmological constant
- The growth of structure depends sensitively on the expansion rate of the universe which in turn depends on Dark Energy eqn of state.
- Early Dark Energy models:time varying $w(z)$ gives rise to more structures at higher redshift than in ΛCDM, could also affect BAO and CMB through Integrated Sachs Wolfe effect.
- N-body simulations are a valuable tool in analysing structure growth in various Dark Energy models
- Simulations lead to significant results in restricting parameter space as well as testing analytic models (e.g. spherical collaspse model)