Neutron Stars:
A Skyrmion Model

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Outline

- The Skyrme Model
- The Rational Map Ansatz
- Neutron Stars

The Skyrme Model - Introduction

- The Skyrme model is a model of baryons.
- Baryons arise as topological solitons.
- Baryon number $\leftrightarrow$ topological charge.
- Successful in modelling nuclei.

The Skyrme Model - Equations

The Skyrme Lagrangian

- $L = \int \left(\frac{F_\pi^2}{16} \mathrm{Tr}(\partial_\mu U \partial^\mu U) + \frac{1}{32e^2} \mathrm{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2) \right) \mathrm{d}^3 x.$

  - The Skyrme field, $U(x, t)$, is a $SU(2)$ valued scalar field.

The Skyrme Field Equation

- $\partial_\mu \left((\partial_\mu U) U^\dagger + \frac{1}{4} [(\partial^\nu U) U^\dagger, [(\partial_\nu U) U^\dagger, (\partial^\mu U) U^\dagger]] \right) = 0.$

The Skyrme Model - Topology

- The Skyrme boundary conditions are $U(x, t) \to \mathrm{I}$ as $|x| \to \infty$.
- The Skyrme field, $U(x, t)$, with these boundary conditions, is a map from S^3 into S^3.
- The homotopy group of a map from S^3 into S^3 is $\mathbb{Z}$.
- The topological charge, and therefore the baryon number, are integers.

The Skyrme Model - Homotopy Groups

- Consider a map $\Psi : S^n \rightarrow Y$.
- Ψ_1 and Ψ_2 are homotopic if Ψ_1 can be continuously transformed into Ψ_2.
- $\pi_n(Y)$ is the n^{th} homotopy group.
- Example: $S^1 \rightarrow S^1$ has homotopy group $\mathbb{Z}$.

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The Rational Map Ansatz - Introduction

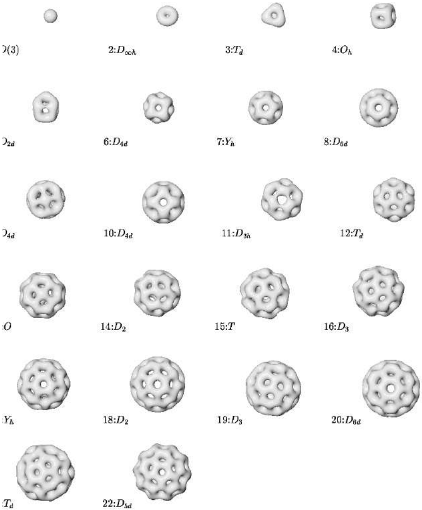
- Using polar coordinates in $\mathbb{R}^3$ define the stereographic coordinates $z = \tan\left(\frac{\theta}{2}\right) e^{i\phi}$.

The Rational Map Ansatz

- $U = \exp(i\vec{\sigma} \cdot \hat{n}_R F(r,t))$,
- $\hat{n}_R = \frac{1}{1+|R|^2}(R + \bar{R}, i(\bar{R} - R), 1 - |R|^2)$.

- $R = \frac{p(z)}{q(z)}$.
- $R = z, R = z^2, R = \frac{z^3 - \sqrt{3}iz}{\sqrt{3}iz^2 - 1}, R = \frac{z^4 + 2\sqrt{3}iz^2 + 1}{z^4 - 2\sqrt{3}iz^2 + 1}$.
- $F(0,t) = \pi, F(\infty, t) = 0$.

The Rational Map Ansatz - Solutions



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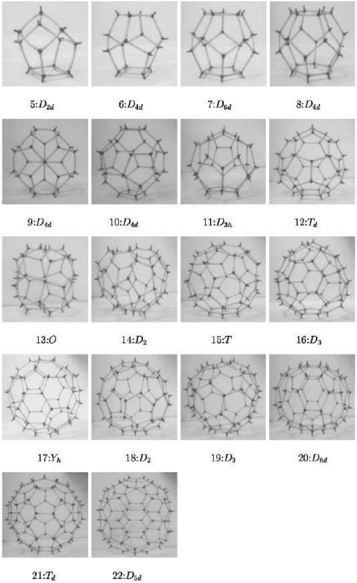
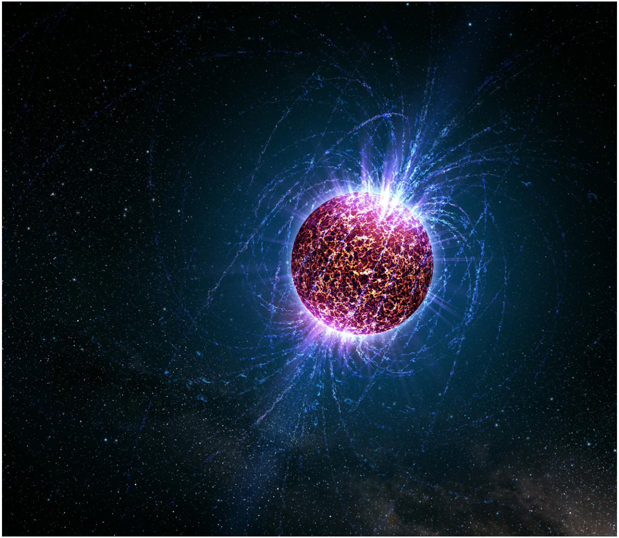


Neutron Stars - Introduction



Neutron Stars - Models

- Large baryon number, typically 10^{58}.
- Gravity must be included.
- Using the Rational map ansatz.
- Skyrme Crystals.