b mass effects in $pp \to h$ @ NNLO

18/05/2015, Durham, CPT student seminar

Outline

1. b quarks at LHC
   - 4F vs 5F scheme
   - The α_S running
   - Problems!
2. Including mass effects
   - Towards a 5F-Improved scheme
3. FONLL
4. $b\bar{b} \to h$ @ $\mathcal{O}(\alpha_S^2)$
5. Conclusions

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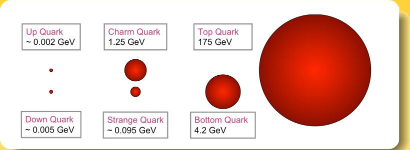

$\Lambda_{QCD} \sim 250$ MeV,
A quark Q is **heavy** $\Leftrightarrow m_Q \gg \Lambda_{QCD}$.

$m_u, m_d, m_s \ll \Lambda_{QCD} \Rightarrow$ light quarks

$m_c > \Lambda_{QCD}$ but not by much!

- b quark only quark such that

$$\Lambda_{QCD} \ll m \ll M(m_W, m_Z, m_H, m_t)$$

- b phenomenology crucially important at the LCH, from flavour physics, to Higgs characterisation and measurements and as window to New Physics.
- From a theoretical viewpoint we need better control on this kind of processes which appear as both BSM signals and SM irreducible backgrounds.
- Important examples: H and Z associated production.

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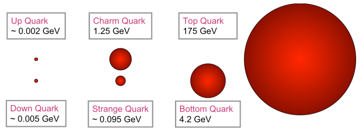

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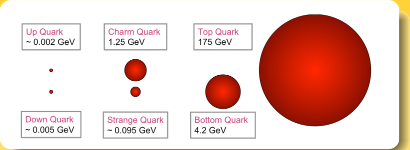

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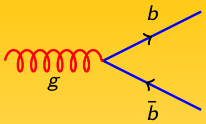
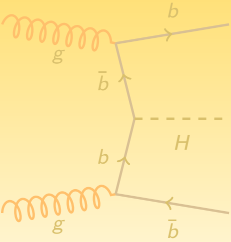


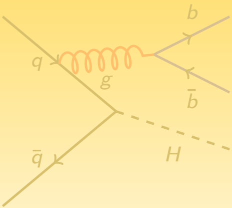
The gluon splitting is the dominant mode of production of b quarks at the LHC

Contribution to σ:

$$\sigma \propto \alpha_S \int_0^{\eta^2} \frac{\mathrm{d}\mu^2}{\mu^2 + m_b^2} \sim \alpha_S \log \frac{\eta^2}{m_b^2}$$

Can take place either in the initial state or in the final state:





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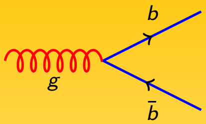

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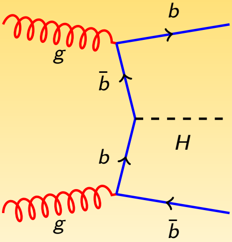

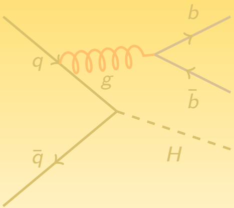

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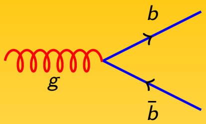

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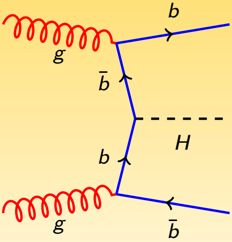

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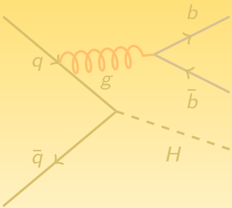

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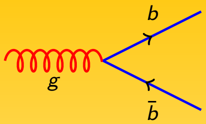

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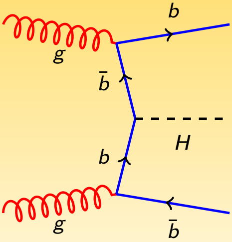

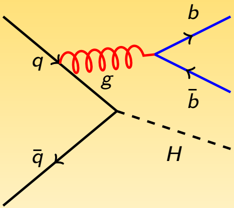

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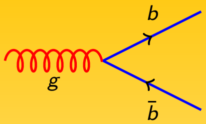

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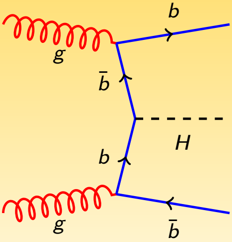

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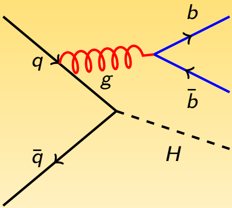

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Problem!

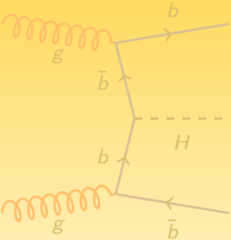
In the kinematic region $\hat{s} \gg m_b^2$

$$\alpha_S(\hat{s}) \log \frac{\hat{s}}{m_b^2} \sim \mathcal{O}(1)$$

Solution

5 flavour scheme, re-sum such logs via DGLAP eqs in b-PDF.

$$m_b = 0$$



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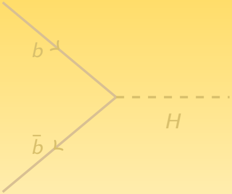

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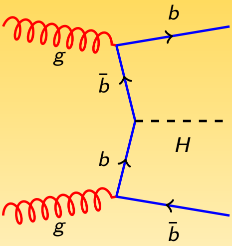

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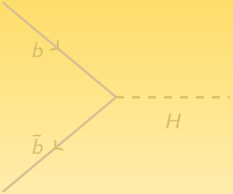

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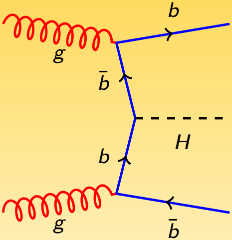

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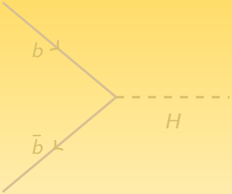

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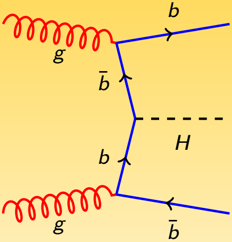

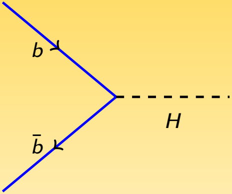

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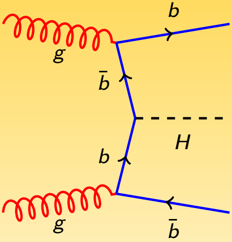

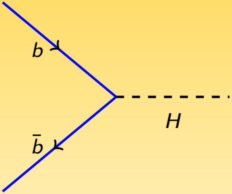

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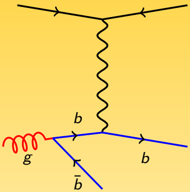

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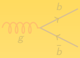

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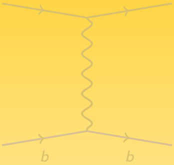

$$\lim_{m_b^2/\eta^2 \to 0} f_g \otimes \hat{\sigma}_{Xg\to b\bar{b}Y} = \underbrace{\alpha_S \log \frac{\eta^2}{m_b^2} P_{qg} \otimes f_g}_{=\tilde{b}(x,\mu^2)} \sigma_{Xb\to Y}$$

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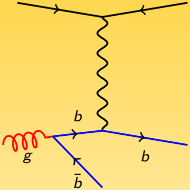

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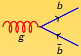

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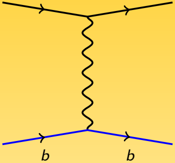

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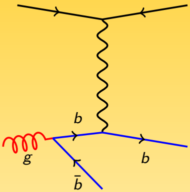

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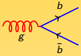

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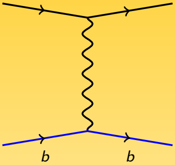

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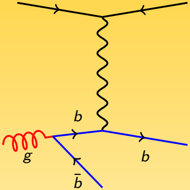
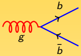
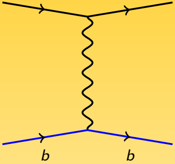


$\Longrightarrow \quad \alpha_S \log \frac{\eta^2}{m_b^2} \quad \times$ 

$\otimes$ 

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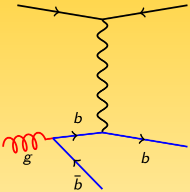

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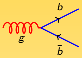

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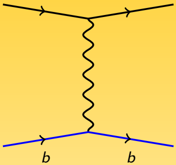

$$\lim_{m_b^2/\eta^2 \to 0} f_g \otimes \hat{\sigma}_{Xg\to b\bar{b}Y} = \underbrace{\alpha_S \log \frac{\eta^2}{m_b^2} P_{qg} \otimes f_g}_{=\tilde{b}(x,\mu^2)} \sigma_{Xb\to Y}$$

DGLAP equations:

$$\begin{cases} \dfrac{\mathrm{d}\, b(x,\mu^2)}{\mathrm{d}\log\mu^2} = \alpha_S \cdot (P_{qg} \otimes f_g)(x,\mu^2) \\ b(x, m_b^2) = 0 \end{cases} \Longrightarrow \quad b(x,\mu^2) = \alpha_S \log \frac{\eta^2}{m_b^2} (P_{qg} \otimes f_g)(x,\mu^2) + \mathcal{O}(\alpha_S^2)$$

What does that mean?

4F QCD series

Then a generic observable in the 4F scheme

$$\sigma^{(4)} = \iint \mathrm{d}x_1 \mathrm{d}x_2 f_i^{(4)}(x_1, \mu_F^2) f_j^{(4)}(x_2, \mu_F^2) \hat{\sigma}_{ij}^{(4)}(x_1, x_2, \mu_F^2)$$

where

$$\hat{\sigma}_{ij}^{(4)}(x_1, x_2, \mu_F^2) = \sum_{n=0}^{\infty} (\alpha_S)^n \hat{\sigma}_{ij}^{(4),(n)}(x_1, x_2, \mu_F^2)$$

5F QCD series

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α_S is not a constant!

RG equations

$$\frac{\mathrm{d}\alpha_S(\mu^2)}{\mathrm{d}\log(\mu^2)} = \beta(\alpha_S) = -b_0\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

b_0 actually depends on the number of *light* that can flow through the gluon loop, n_f!

Flavour dep. RG equations

$$\frac{\mathrm{d}\alpha_S^{(n_f)}(\mu^2)}{\mathrm{d}\log(\mu^2)} = \beta^{(n_f)}(\alpha_S^{(n_f)}) = -b_0^{(n_f)}\left(\alpha_S^{(n_f)}\right)^2 + \mathcal{O}(\alpha_S^3)$$

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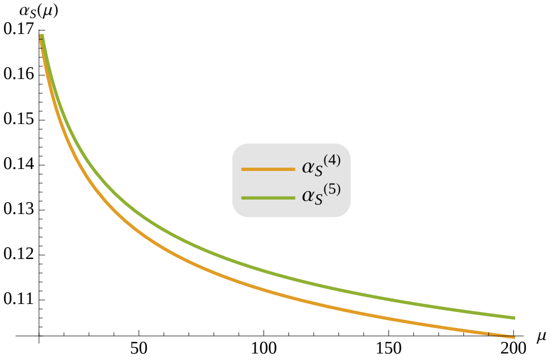
The two different runnings



- Data are more in agreement with the 5F running!

Real QCD series!

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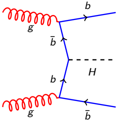
4F versus 5F scheme

4F scheme



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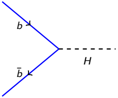


They should agree..

- Scale independence requires:

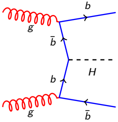
$$\sigma = \sigma^{(4)} = \sigma^{(5)}$$

$$\Rightarrow = \sum_n (\alpha_S^{(4)})^n \sigma^{(4),(n)} = \sum_n (\alpha_S^{(5)})^n \sigma^{(5),(n)}$$

... at each order the difference between the two should be of higher order in α_S...
but...

4F versus 5F scheme

4F scheme



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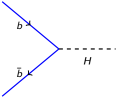

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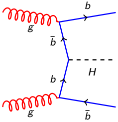

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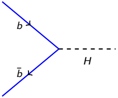

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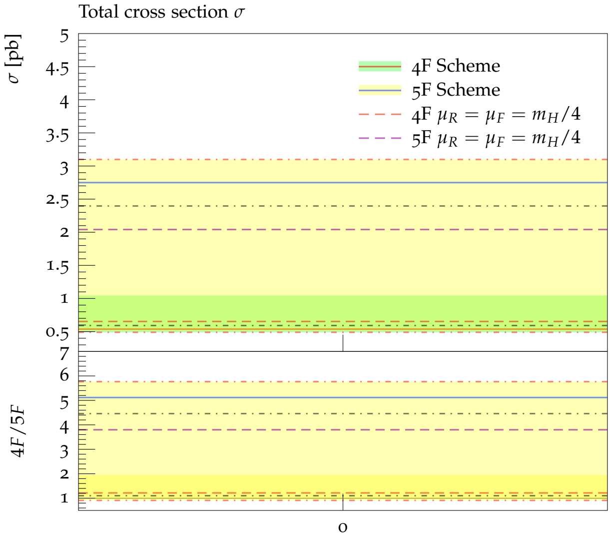
$b\bar{b} \to h$ @ LO, total XS

Yep! They should agree...



Outline

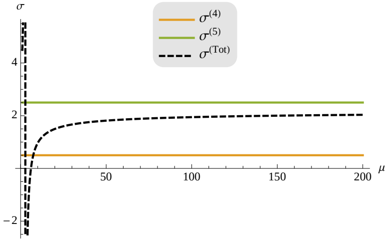
State of the art

The *Santander* matching

- Weighted average between the 4 and the 5F scheme

$$\sigma^{(\mathrm{Tot})} = \frac{\sigma^{(4)} + w\,\sigma^{(5)}}{1+w}$$

$$w = \log\frac{m_H}{m_b} - 2\,.$$



Outline

FO-NLL

"Fixed-Order-Next-to-Leading-Log"

- Originally invented for *b*-quark hadro-production
- Used to match a fixed-order (FO) with a next-to-leading-log (NLL) calculation.
- Extended to DIS and matching extended
- Match any FO with any N^{m}LL calculation, as long as you have them!
- Based on standard QCD factorization!

How does it work?

- Very simple basic idea
- 4F and 5F have many things different but also something in common $\Rightarrow$ full prediction:

$$\sigma^{(FONLL)} = \sigma^{(4)} + \sigma^{(5)} - \text{double counting}$$

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A little more details

The 5F QCD series

$$\sigma^{(5)} = \iint \mathrm{d}x_1 \mathrm{d}x_2 \sum_{ij=g,q,b} f_i^{(5)}(x_1, \mu_F^2) f_j^{(5)}(x_2, \mu_F^2) \hat{\sigma}_{ij}^{(5)} \left(x_1, x_2, \alpha_S^{(5)}(\mu^2) \right)$$

- Use DGLAP eqs to express f_b in terms of $f_{q,g}$

At the scale μ^2 with $L = \log \mu^2 / m_b^2$

$$f_b^{(5)}(x, \mu^2) = \sum_{i=q,g} \int_x^1 \frac{\mathrm{d}y}{y} f_i^{(5)}(y, \mu^2) \mathcal{C}_{bi} \left(\frac{x}{y}, \alpha_S^5(\mu^2), L \right)$$

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A deeper look into the resummation!

$\mathcal{A}$ α_S expansion

At any fixed-order N we have:

$$\mathcal{A}_{ij}^{(5)}\left(x_1, x_2, \alpha_S^{(5)}(\mu^2), L\right) = \sum_p^N \left(\alpha_S^{(5)}(\mu^2)\right)^p \underbrace{\sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2)\left(\alpha_S^{(5)}(\mu^2) L\right)^k}_{\text{b-pdf un-resummation}}$$

While

$$\mathcal{A}_{ij}^{(p),(k)}(x_1, x_2) = \sum_{l=q,g,b} \mathcal{C}_{li} \otimes \hat{\sigma}_{lj}$$

- Need to adjust the 4F scheme as well!

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The 4F scheme

The α_S Running

$$\alpha_S^{(4)}(\mu^2) = \alpha_S^{(5)}(\mu^2) + \sum_{i=2}^{\infty} c_i(L)\left(\alpha_S^{(5)}(m_b^2)\right)^i$$

The PDFs evolution

$$f_i^{(4)}(x,\mu^2) = \int_x^1 \frac{\mathrm{d}y}{y} \sum_{j=q,g} \mathcal{K}_{ij}\left(\frac{x}{y}, L, \alpha_S^{(5)}(\mu^2)\right) f_j^{(5)}(y,\mu^2)$$

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Even more!

$\mathcal{B}$ expansion!

To any fixed order N:

$$\mathcal{B}_{ij}^{(4)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}, \alpha_S^{(5)}(\mu^2)\right) = \sum_{p=0}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right)$$

- Now both 4F and 5F are expressed as a power series in the same exp parameter with the same PDFs!
- let me call $f_i(x_1, \mu^2) f_j(x_2, \mu^2) = \mathcal{L}_{ij}(x_1, x_2, \mu^2)$

$$\sigma^{(4)} = \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_{p=0}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right)$$

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The FONLL XS

$\sigma^{(FONLL)}$

$$
\begin{aligned}
\sigma^{(FONLL)} &= \sigma^{(4)} \ + \ \sigma^{(5)} \ - \ \text{double counting} \\
&= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_{p}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p \\
&\times \left\{ \mathcal{B}_{ij}^{(p)} \left(x_1, x_2, \frac{\mu^2}{m_b^2} \right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2) \left(\alpha_S^{(5)}(\mu^2) L \right)^k \right\} \\
&- \ \text{double counting}
\end{aligned}
$$

Working out the double counting piece!

The massless-limit of the massive scheme

- Terms who don't vanish for $m_b \to 0$ in the $\mathcal{B}$ must also be present in the 5F scheme
- We define the massless-limit of the massive scheme to be those scheme in which only logarithmic terms are retained in the massive scheme

It then follows that:

$$\mathcal{B}_{ij}^{(0),(p)}(x_1, x_2, L) = \sum_{k=0}^{p} \mathcal{A}_{ij}^{(p-k),(k)}(x_1, x_2) L^k$$

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The FONLL master formula

Putting everything together

$$
\begin{aligned}
\sigma^{(FONLL)} &= \sigma^{(4)} + \sigma^{(5)} - \text{ double counting} \\
&= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_{p}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p \\
&\times \left\{ \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2)\left(\alpha_S^{(5)}(\mu^2) L\right)^k \right. \\
&\left. - \sum_{k=0}^{p} \mathcal{A}_{ij}^{(p-k),(k)}(x_1, x_2) L^k \right\}
\end{aligned}
$$

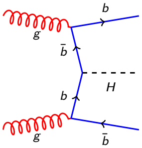
In othe words we have replaced the first N orders of the massless scheme with their known massive scheme counterparts while preserving the resummation of higher order logs!

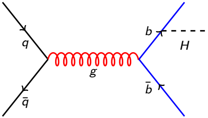
Outline

4F vs 5F scheme

4F scheme

- $\mathcal{O}(\alpha_S^2)$ is the LO

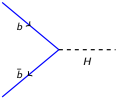




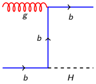
4F vs 5F scheme

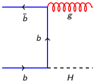
5F scheme

- 5F needs to be at least at NNLO!
- $\mathcal{O}(\alpha_S^0)$:

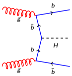


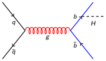
- $\mathcal{O}(\alpha_S)$: (1-loop)+





- $\mathcal{O}(\alpha_S)$ (2-loop)+ (1-loop)+ ... +

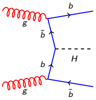


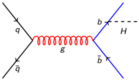


FONLL to $b\bar{b} \to h$

Without going into much details ...

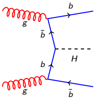
- Take the 5F scheme - the leading log part of:

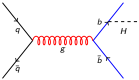




$+$ all the b initiated up to $\mathcal{O}(\alpha_S^2)$

- Add them back in the 4F scheme!

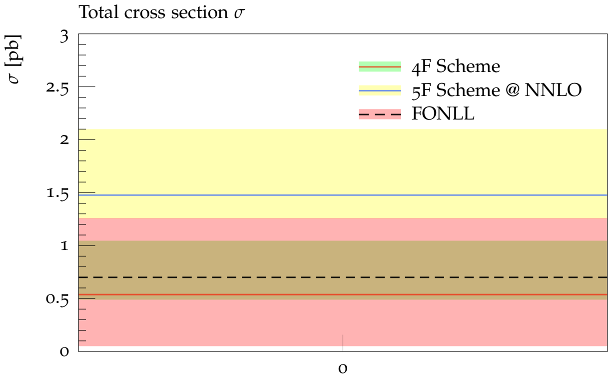




With correct mass dependence!

Good bye plot

Now they seem to agree better!



Outline

1. b quarks at LHC
   - 4F vs 5F scheme
   - The α_S running
   - Problems!
2. Including mass effects
   - Towards a 5F-Improved scheme
3. FONLL
4. $b\bar{b} \to h$ @ $\mathcal{O}(\alpha_S^2)$
5. Conclusions

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- b initiated processes still are to be handled with care
- FONLL can be extended to any of them
- Need people to do the calculation, though!
- Hopefully in the next few years this will become automated...

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