Holographic D3 Probes

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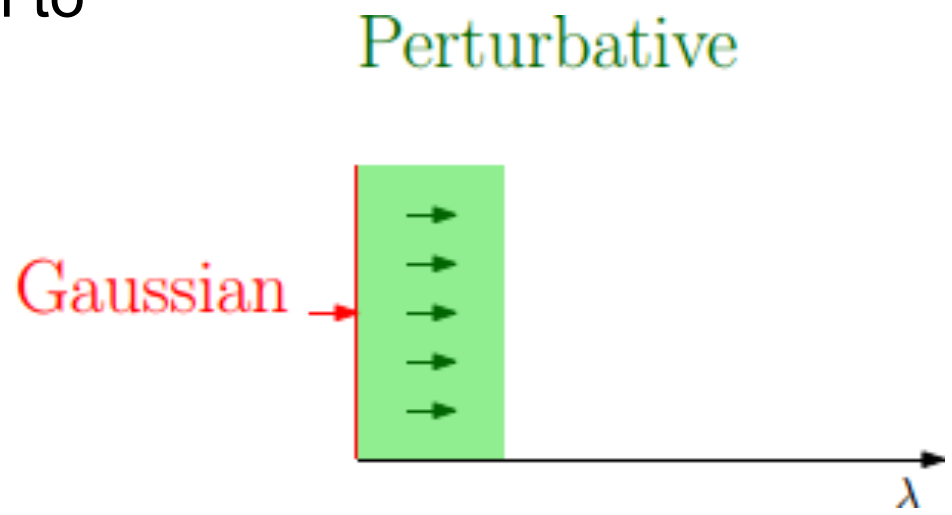
Includes results of previous col·laborations with **Blai Garolera** (UCR)

Durham, May 25th 2015

Our standard approach to QFTs:



Needs

- Quasiparticle description
- All couplings small

Not applicable to

- QCD physics
- High T_c superconductors
- ...

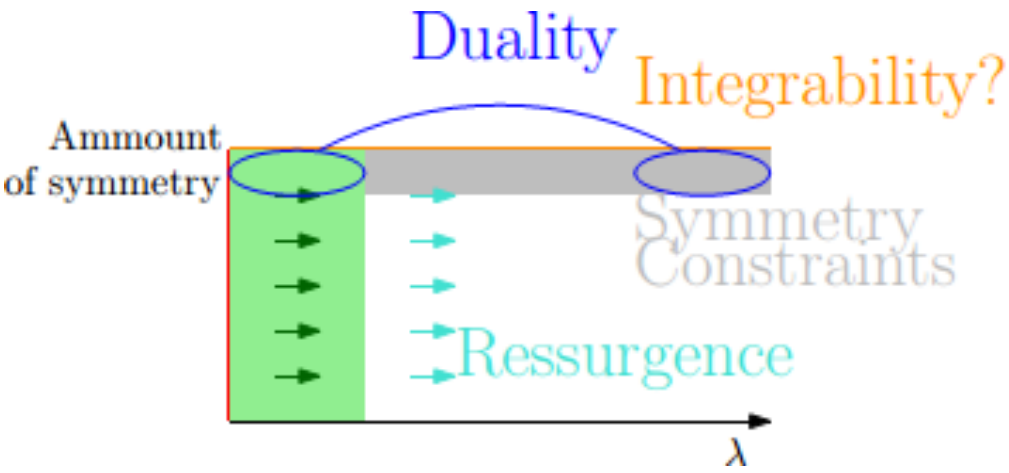
Non-perturbative QFT problems are hard.

General approaches (ressurgence) still far from controlled

Obtain information in simplified problems:

Lattice simulations	Static configurations
Localization	SUSY, particular observables
Bootstrap	Conformality
Integrability	Particular theories and regimes
Holography	
...	...

Symmetries are usually a great help



Focus in N=4 SYM in 4D

- ♦ Conformal
- ♦ Maximal SUSY

Admits

- ♦ Bootstrapping
- ♦ Localization
- ♦ Holographic (useful at large N and strong coupling)
- ♦ Integrability at large N?

$$\mathcal{L}_{\mathcal{N}=4} = \mathrm{Tr}\left(-\frac{F_{\mu\nu}F^{\mu\nu}}{2g_{YM}^2} + \frac{\theta F_{\mu\nu}\tilde{F^{\mu\nu}}}{8\pi^2}\right) +$$
$$+\mathrm{Tr}\left(-\sum_I D_\mu \Phi^I D^\mu \Phi^I + \frac{g_{YM}^2}{2}\sum\left[\Phi^I, \Phi^J\right]\right) -$$
$$-\mathrm{Tr}\sum_a i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda_a +$$
$$+\mathrm{Tr}\left(\sum_{a,b,I} g_{YM} C_I^{ab} \lambda_a \left[\Phi^I, \lambda_b\right]\right) +$$
$$+\mathrm{Tr}\left(\sum_{a,b,I} g_{YM} \bar{C}_{Iab} \bar{\lambda}^a \left[\Phi^I, \bar{\lambda}^b\right]\right)$$

N=Gauge group rank

Far from QCD

- ♦ Not confining (at zero T)
- ♦ No mass gap (conformal)
- ♦ Only adjoint matter

Higgs it to get generic massive charges:

$$SU\left(N+1\right) \to SU\left(N\right) \times U\left(1\right)$$

$$\Phi_{i\,N+1,N+1} = M\theta_i + \phi_{i\,N+1,N+1}$$

Obtain W bosons with { Mass (take it large); Fundamental charge }

$$\int dy \left\langle \omega(x)^\dagger \omega(x) \omega(y) \omega(y)^\dagger \right\rangle \approx \int \mathcal{D}A_\mu \mathcal{D}\Phi_i e^{-S_{SU(N)}} \int \mathcal{D}x(s) e^{\int ds \left(iA_\mu \dot{x}^\mu + |\dot{x}| \Phi_i \theta^i - |\dot{x}| M \right)}$$

Wilson-loop-like object coupled to the scalars.

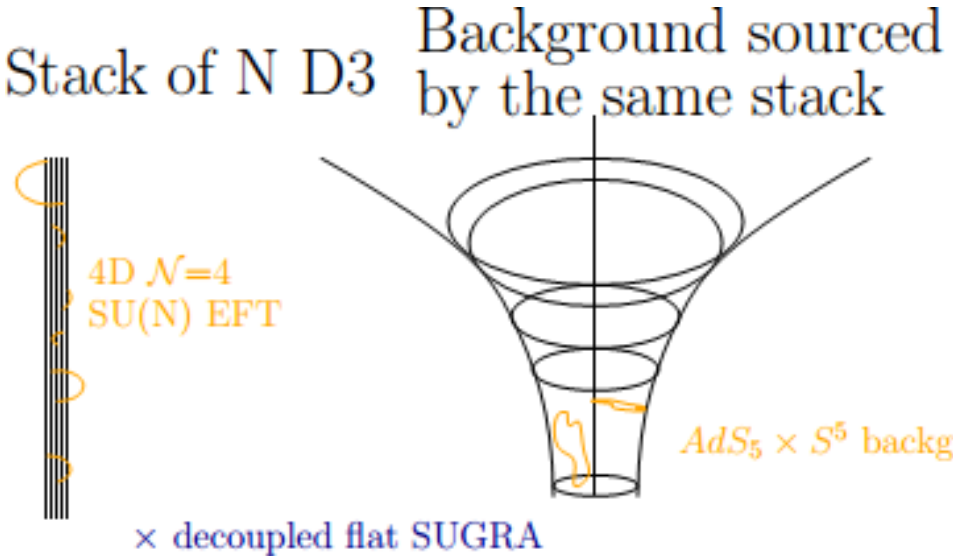
Out of them, loops in generic representations.

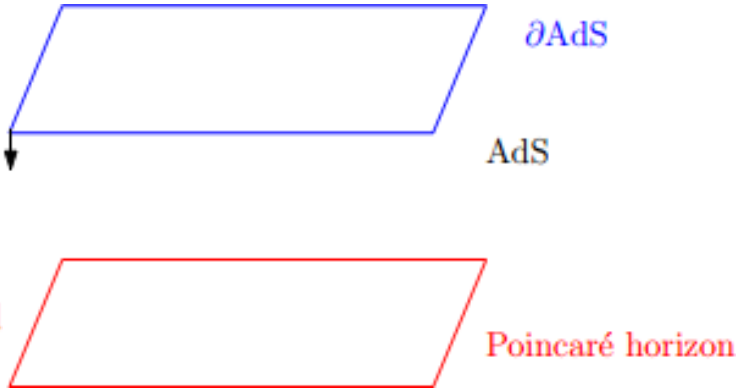
$$W_R\left(C\right) = \frac{1}{\dim R} \mathrm{Tr}_R \mathcal{P} e^{i \oint_C \left(A_\mu \dot{x}^\mu + |\dot{x}| \Phi_I \theta^I \right) ds}$$

[Drukker, Gross, Ooguri 9904191]

Holography:

Low-E stack of D3 branes admit two dual descriptions





$\mathcal{N} = 4$ 4D SYM

$SU(N)$

$\lambda = g_{YM}^2 N$

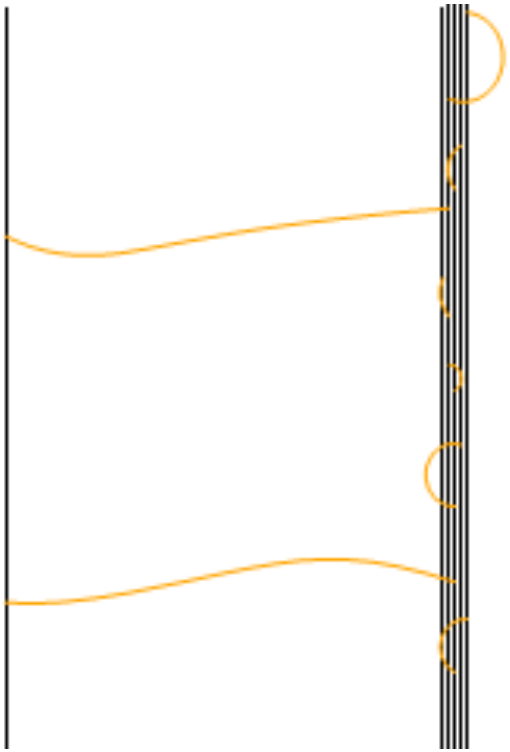
Type IIB $AdS_5 \times S^5$

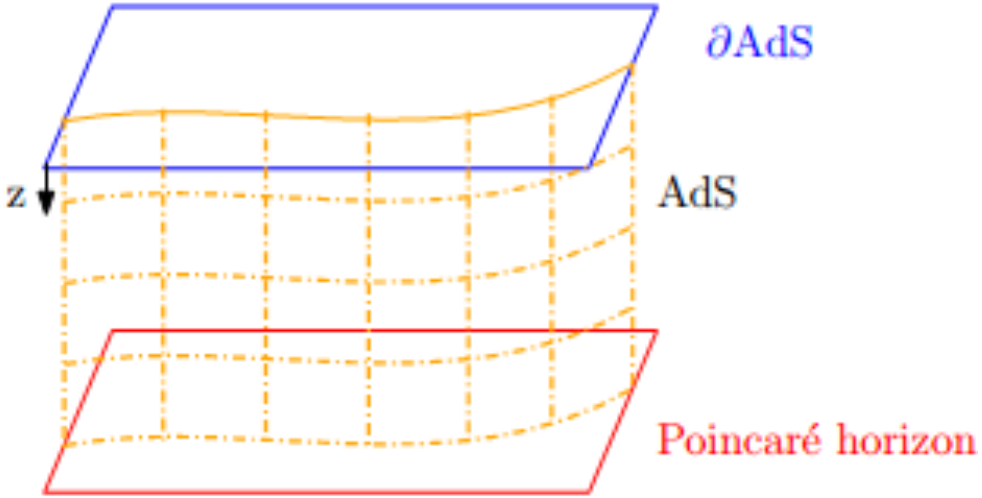
$R^2 = \alpha' \sqrt{\lambda}$

$\lambda = 4\pi g_s N$

[Maldacena 9711200]

How is the symmetry breaking realized?



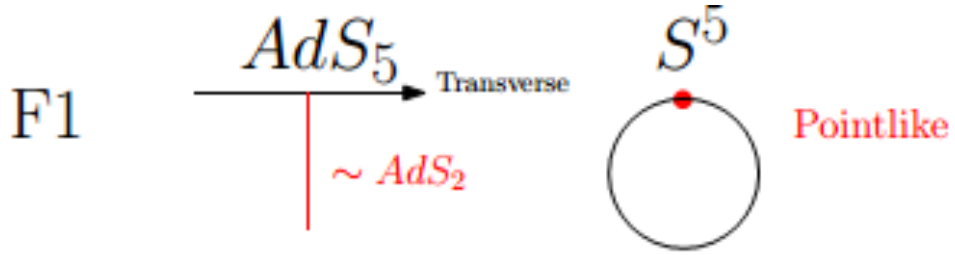


Restrict ourselves to the infinitely massive case.

[Rey,Yee 9803001]
[Maldacena 9803002]

Quark represented by a string:

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int d^2\sigma e^{\phi/2} \sqrt{\det g_{\mu\nu} \partial_i X^\mu \partial_j X^\nu}$$



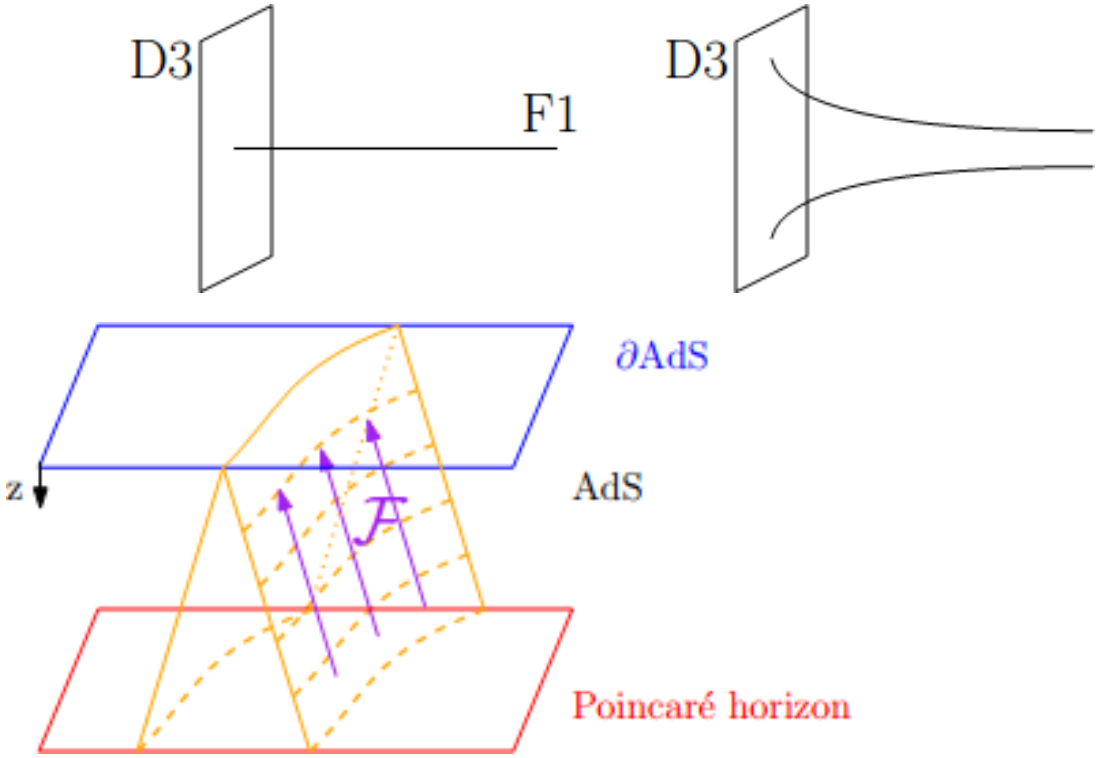
We will work in the semiclassical probe approximation

- First-quantized 1+1D deffect
- In the semi-classical approximation
- Neglecting backreaction

What about the higher representations?

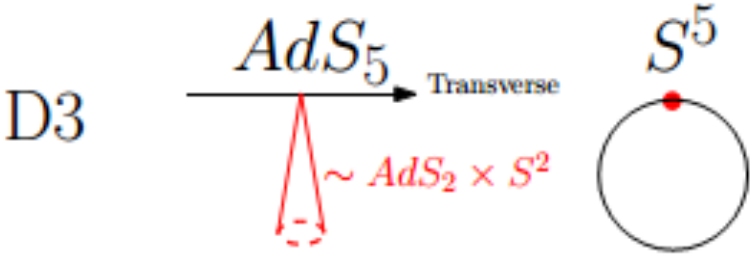
Stack of open strings ⟷ ? ⟷ D-brane they end on

Use the idea of Brane instantons:

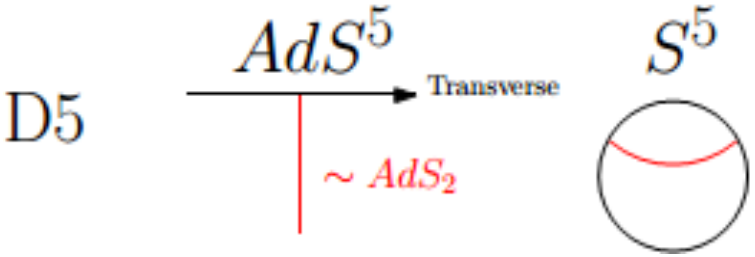


[Callan, Maldacena 9708147]

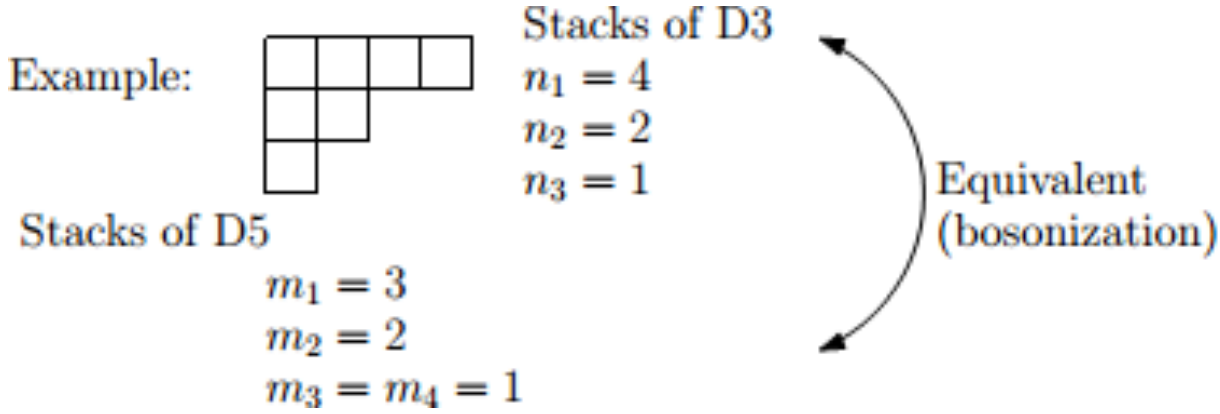
Use of D3 branes for symmetric BPS probes



Use of D5 branes for antisymmetric BPS probes



General picture: Stacks of either type



Expected to be valid for non-BPS states as well.

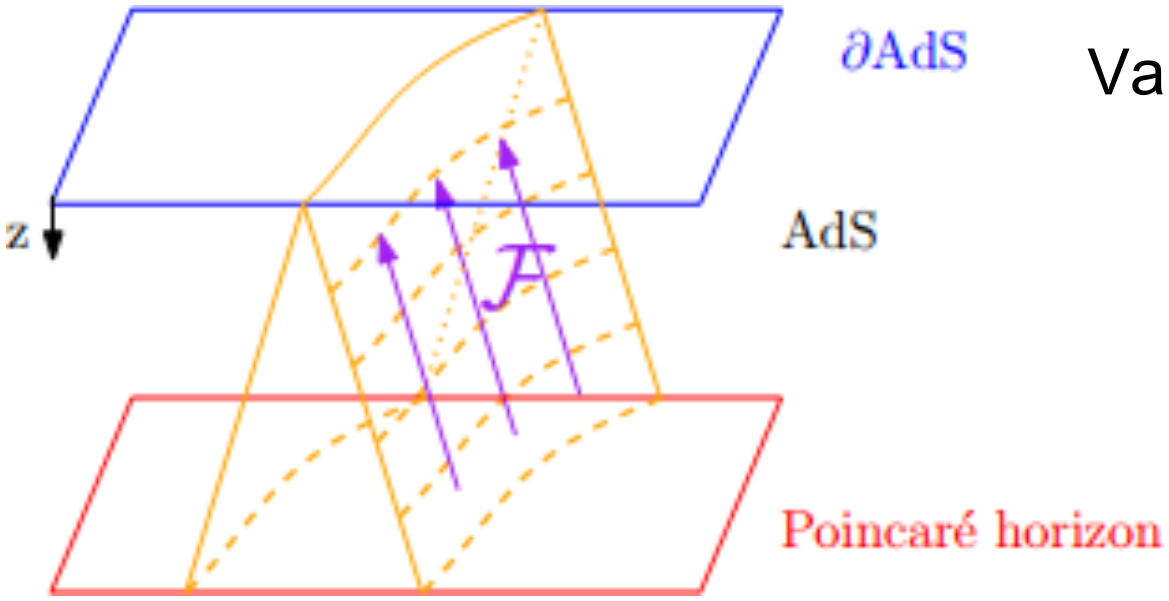
[Drukker, Fiol 0501109]
[Yamaguchi 0603208]
[Gomis, Passerini 0604007]

Pure k-(a)sym probe represented by a D3 (D5) brane

$$S_{DBI} = -T_{D3} \int d^4\xi e^{\phi/2} \sqrt{\det g_{\mu\nu} \partial_i X^\mu \partial_j X^\nu + \mathcal{F}}$$

$$S_{WZ} = T_{D3} \int e^{\mathcal{F}} \wedge C$$

Flux related to the charge it's representing.



Validity of DBI restricted to low curvature

How to use probe holography

$$\mathcal{Z}_{\mathcal{N}=4,J} = e^{-\mathcal{W}_{\mathcal{N}=4,J}}$$

$$\mathcal{Z}_{grav,\phi(z_0)} \sim e^{-S_{sol,\phi(z_0)}}$$

$$\langle \mathcal{O} \rangle_W = \frac{\langle \mathcal{O} W \rangle_0}{\langle W \rangle_0}$$

Sources

Boundary conditions

- Wilson loop's vev
- Correlation, probe-local operator
- Energy and momentum
- Defect operators

[Witten 9802150]

[Gubser, Klebanov, Polyakov 9802109]

How to use probe holography

$$\langle \mathcal{O} \rangle_W = \frac{\langle \mathcal{O} W \rangle_0}{\langle W \rangle_0} \qquad \mathcal{Z}_{Grav} \sim e^{-S_{bulk} - S_{probe}}$$

- Wilson loop's vev

  Direct comparison with pure N=4

  $$\langle W \rangle_0 = \frac{\mathcal{Z}_W}{\mathcal{Z}_0}$$

  $$\langle W \rangle = e^{-S_{probe}}$$

- Correlation, probe-local operator
- Energy and momentum
- Defect operators

[Rey,Yee 9803001]
[Maldacena 9803002]

How to use probe holography

$$\langle \mathcal{O} \rangle_W = \frac{\langle \mathcal{O} W \rangle_0}{\langle W \rangle_0} \qquad \mathcal{Z}_{Grav} \sim e^{-S_{bulk} - S_{probe}}$$

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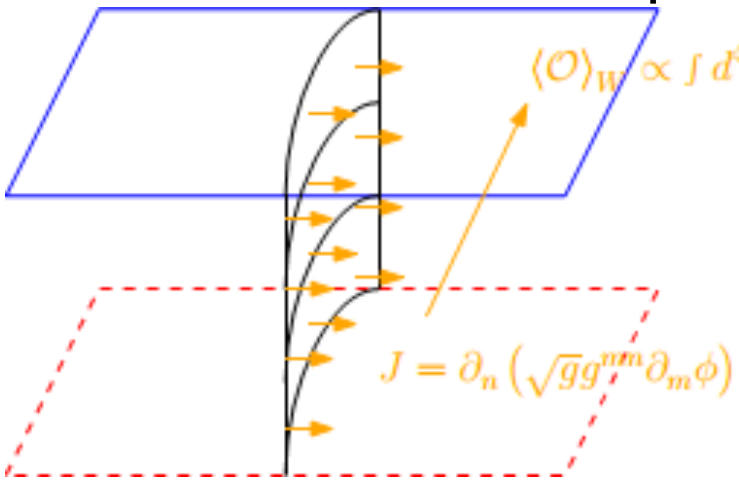

Standard recipe:

$$\langle \mathcal{O}(x) W \rangle = - \frac{\delta (S_{bulk} + S_{probe})}{\delta \phi(x)} \Big|_{\phi}$$

Source ⟶ Field ⟶ Falloff

- Energy and momentum
- Defect operators

[Callan, Güijosa 9906153]

How to use probe holography

$$\langle \mathcal{O} \rangle_W = \frac{\langle \mathcal{O} W \rangle_0}{\langle W \rangle_0} \qquad \mathcal{Z}_{Grav} \sim e^{-S_{bulk} - S_{probe}}$$

- Wilson loop's vev $\langle W \rangle = e^{-S_{probe}}$
- Correlation, probe-local operator $\langle \mathcal{O}(x) W \rangle = - \frac{\delta (S_{bulk} + S_{probe})}{\delta \phi(x)} \Big|_{\phi}$
- Energy and momentum

  Full results for the stress-energy tensor

  E-p of the probe already contain information.

  Small velocities: $\Delta E_{rad} = 2\pi B(\lambda, N) \int dt \dot{v}^2$

- Defect operators

[Mikhailov 0305196]
[Correa, Henn, Maldacena, Sever 1202.4455]

How to use probe holography

$$\langle \mathcal{O} \rangle_W = \frac{\langle \mathcal{O} W \rangle_0}{\langle W \rangle_0} \qquad \mathcal{Z}_{Grav} \sim e^{-S_{bulk} - S_{probe}}$$

- Wilson loop's vev $\langle W \rangle = e^{-S_{probe}}$
- Correlation, probe-local operator $\langle \mathcal{O}(x) W \rangle = - \frac{\delta (S_{bulk} + S_{probe})}{\delta \phi(x)} \Big|_{\phi}$
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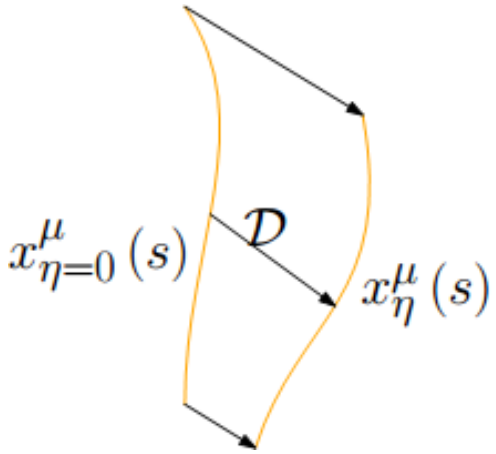
- Defect operators



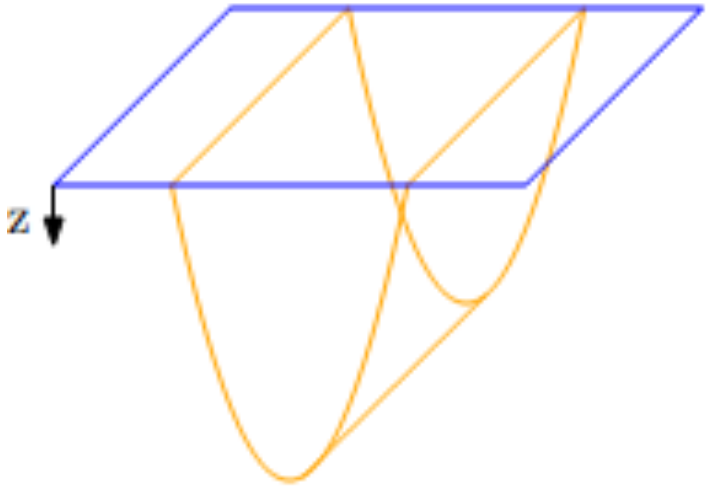
Linearized fluctuations of embedding scalars

$$\kappa_{ij} = \lim_{\omega \to 0} \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \langle\langle \mathbb{D}_i(\tau) \mathbb{D}_j(0) \rangle\rangle$$

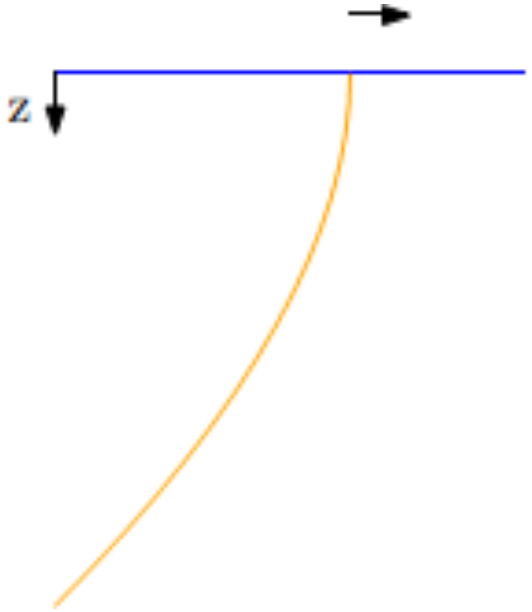
[Correa, Henn, Maldacena, Sever 1202.4455]

[Fiol, Garolera, GT 1302.6991]

Useful string configurations:



[Rey,Yee 9803001]
[Maldacena 9803002]

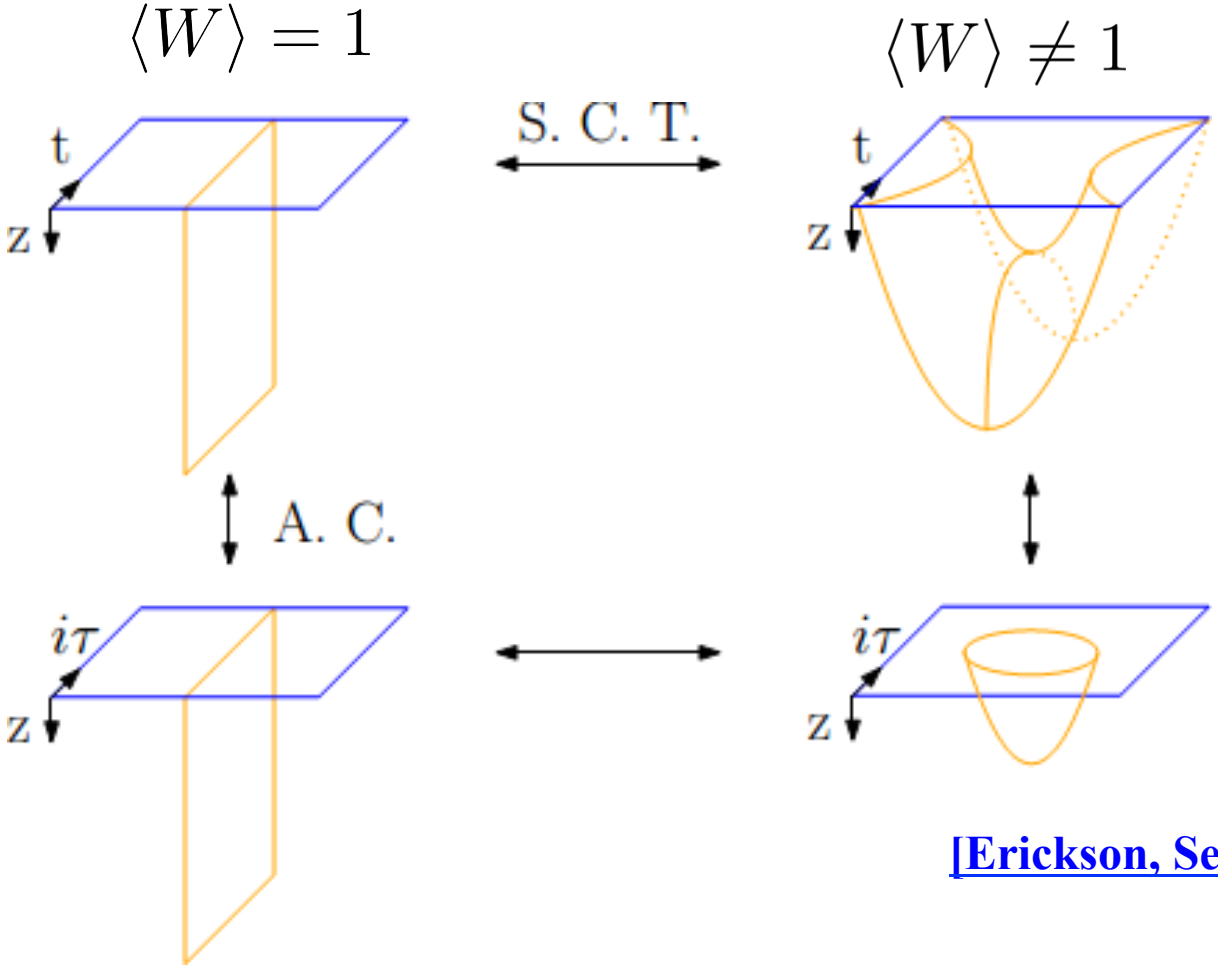


[HKKKY 0605158]
[Gubser 0605182]

Usually solvable when reduced to 1D

Applicability to nontrivial backgrounds

At T=0 we can use conformal symmetry and SUSY



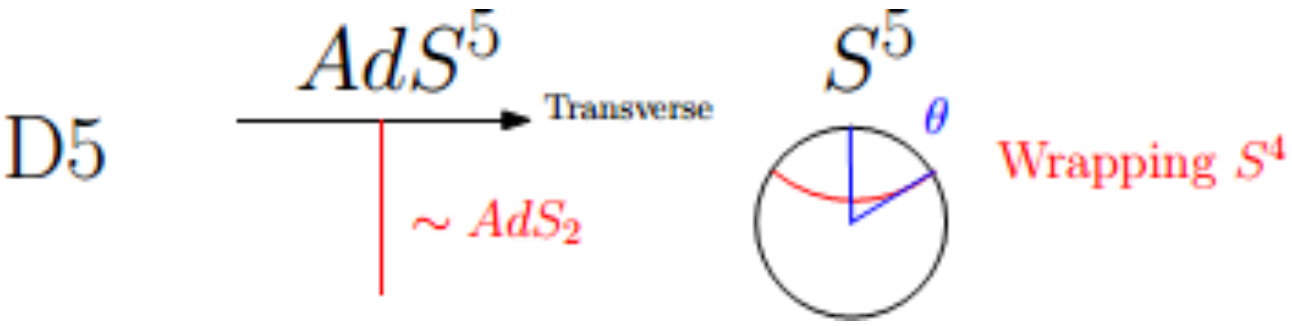
Conformal anomaly

Computable by localization

Nontrivial holography check

[Erickson, Semenoff, Zarembo 0003055]
[Drukker, Gross 0010274]
[Pestun 0712.2824]

Results for D5 branes: k-antisymmetric representation



$$\theta - \sin\theta\cos\theta = \pi\frac{N-k}{N}$$

$$S_{D5}|_{renor.} = \frac{2N}{3\pi}\sin^3\theta\ S_{F1}|_{renor.}$$

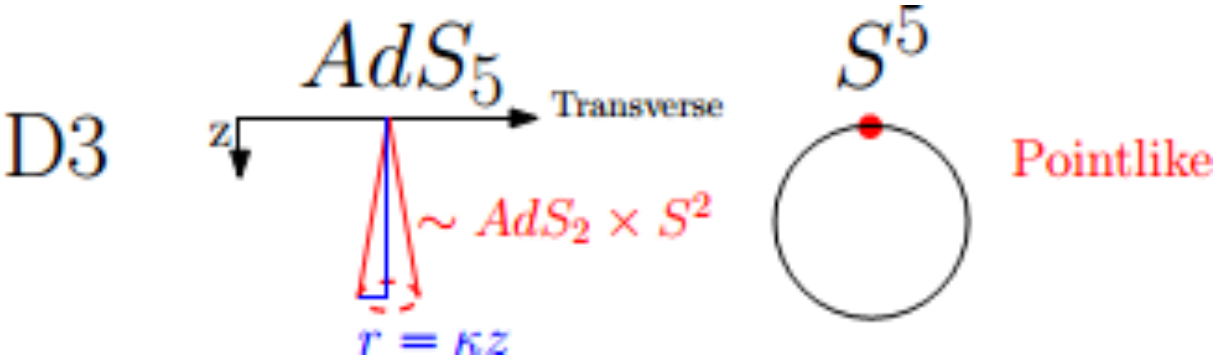
Straightforward from string results

Should not be trusted at very low and very large k

[Hartnoll 0606178]

Results for D3 branes: k-symmetric representations

Solution known for the static case (+ hyperbola, circle)



$$\kappa = k\frac{\sqrt{\lambda}}{4N}$$

Validity:

- Probe approximation $\left(\frac{N}{\lambda}\right)^2 \gg k$
- Dirac-Born-Infield validity $\kappa \gg 1/\lambda \Rightarrow k \gg \frac{N}{\lambda^{3/4}}$

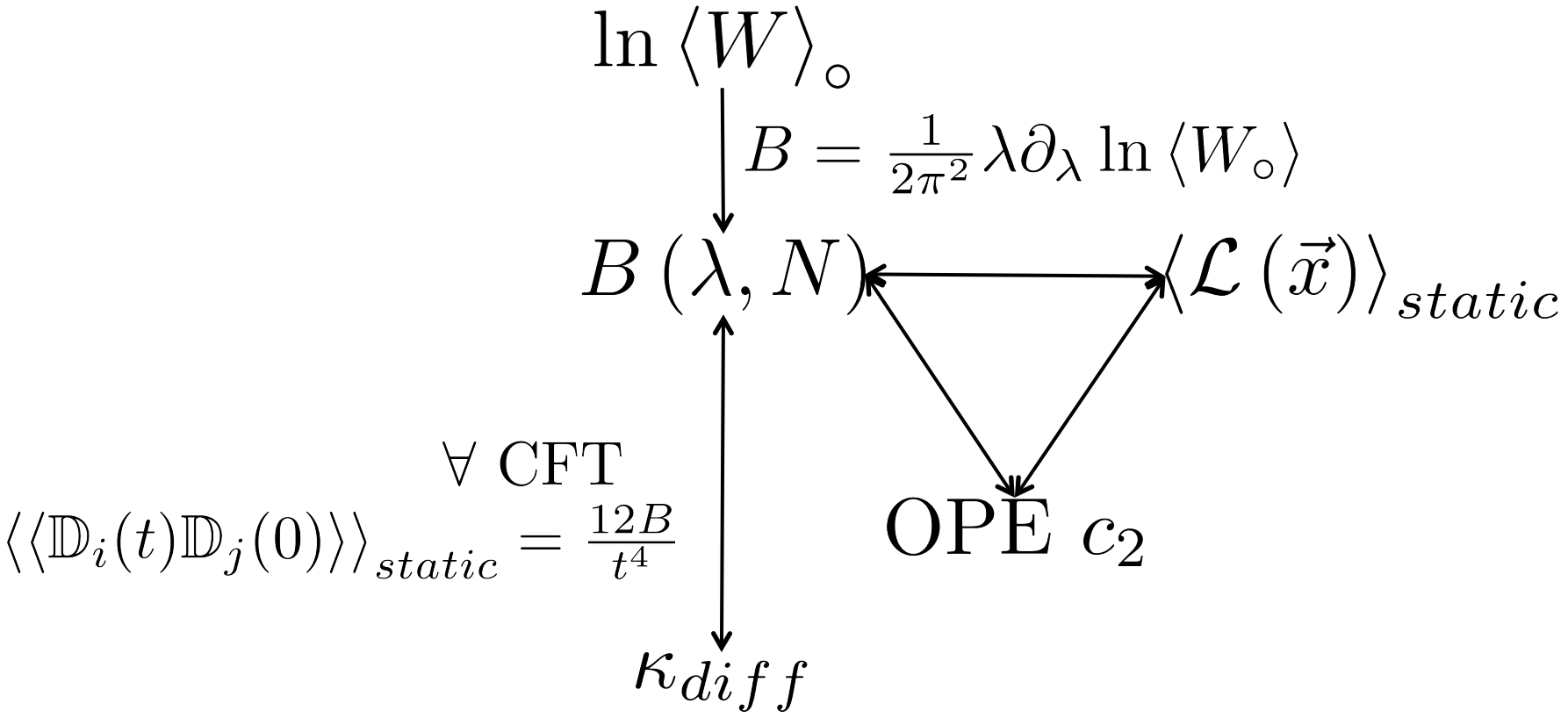
k=1 excluded in principle

[Drukker, Fiol 0501109]

Summary of simple results:

	String	D3 brane $\kappa = k\frac{\sqrt{\lambda}}{4N}$
$\ln \langle W \rangle_\circ$	$\sqrt{\lambda}$	$\sqrt{\lambda}\sqrt{1+\kappa^2}\frac{k}{2} + 2N \sinh^{-1}\kappa$
	[BCFM 9809188]	[Drukker, Fiol 0501109]
$\langle \mathcal{L}(\vec{x}) \rangle_{static}$	$\frac{\sqrt{\lambda}}{16\pi^2 x^4}$	$\frac{\sqrt{\lambda}}{16\pi^2 x^4}\sqrt{1+\kappa^2}k$
	[Callan, Güijosa 9906153]	[Fiol, Garlolera, Lewkowykz 1202.5292]
$B(\lambda, N)$	$\frac{\sqrt{\lambda}}{4\pi^2}$	$\frac{\sqrt{\lambda}}{4\pi^2}\sqrt{1+\kappa^2}k$
	[Mikhailov 0305196]	[Fiol, Garolera 1106.5418]
OPE c_2	$\frac{\sqrt{2\lambda}}{N}$	$\frac{\sqrt{2\lambda}}{N}\sqrt{1+\kappa^2}k$
	[BCFM 9809188]	[Giombi, Ricci, Trancanelli 0608077]
κ_{diff}	$4\pi\sqrt{\lambda}T^3$	$4\pi\sqrt{\lambda}T^3\sqrt{1+\kappa^2}k$
	[Xiao 0804.1343]	[Fiol, Garolera, GT 1302.6991]

Not a coincidence, related observables:



[Correa, Henn, Maldacena, Sever 1202.4455]

[Fiol, Garolera, Lewkowykz 1202.5292]

Still, nontrivial check of holography:

$$\langle W\rangle_{\circ,\square} = \frac{1}{N} L^1_{N-1}\left(\frac{-\lambda}{4N}\right) e^{\frac{\lambda}{8N}}$$

$$\downarrow \frac{\sqrt{\lambda}}{4N} = ctt,\ N \gg 1$$

$$\ln\langle W\rangle_{\circ,\square} \sim \sqrt{\lambda}\sqrt{1+\frac{\lambda}{(4N)^2}}\frac{k}{2} + 2N\sinh^{-1}\frac{\sqrt{\lambda}}{4N}$$

Compare to: $\sqrt{\lambda}\sqrt{1+\kappa^2}\frac{k}{2} + 2N\sinh^{-1}\kappa$

$$\kappa = k\frac{\sqrt{\lambda}}{4N}$$

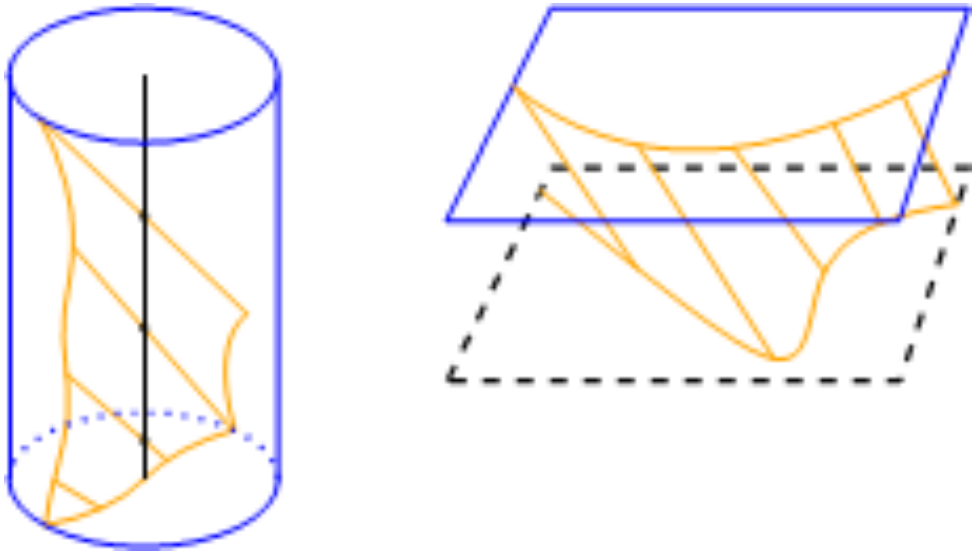
Perfect match for k=1! Resummation of N^{-1} corrections!

What part of this could be kept in less symmetric cases?

[Drukker, Fiol 0501109]

Known string solutions for all timelike Wilson line:

$$S_{NG} \sim -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{\det g_{ind}}$$



$$X^\mu(\tau, z) = x^\mu(\tau) + z v^\mu(\tau)$$
$$v^\mu v_\mu = 1$$

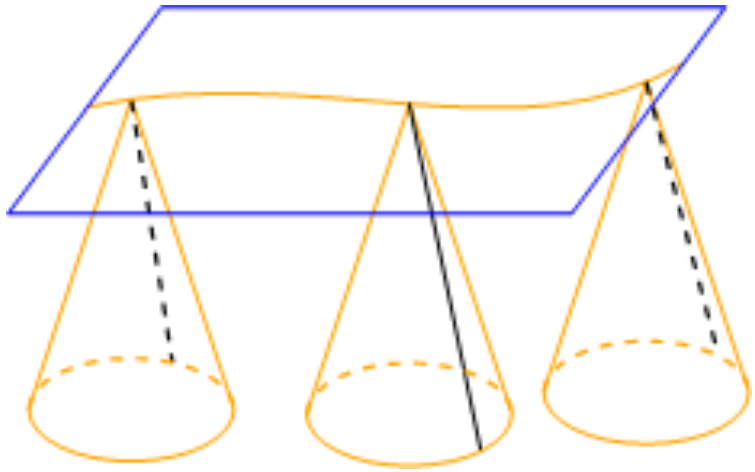
[Mikhailov 0305196]

Let's do the same for D3 branes:

Now every point sources a spherical light shell.

Motion and acceleration $\longrightarrow$ Anisotropy?

Allow for a local choice of angle origin



Unitary vector for a direction:

$$n^\mu(\tau,\theta,\phi)$$
$$n^\mu n_\mu = -1$$
$$n^\mu v_\mu = 0$$

$$X^\mu(\tau,z,\theta,\phi) = x^\mu(\tau) + z\Delta^\mu(\tau,\theta,\phi)$$
$$\Delta^\mu = \sqrt{1+\kappa^2}v^\mu + \kappa n^\mu$$

[Fiol, Güijosa, Pedraza 1410.0692]

But what electromagnetic fluxes F solve the e.o.m.s?

$$S_{DBI+WZ} \sim -T_{D3} \int d^4\xi \left(\sqrt{\det g_{ind} + \mathcal{F}} - C_4 \right)$$

Analysis of known solutions & their boosts suggests

$$\mathcal{F}_{ij} = -\frac{g_{ind\,ij}}{\sqrt{1+\kappa^2}};\ j > i$$
$$\mathcal{F}_{ij} = \frac{g_{ind\,ij}}{\sqrt{1+\kappa^2}};\ j < i$$
$$\mathcal{F}_{ii} = 0$$

This ansatz satisfies all equations of motion.

Also, it greatly simplifies the calculations:

$$\sqrt{-\det\left(g_{ind} + \mathcal{F}\right)} = \frac{\kappa}{\sqrt{1+\kappa^2}} \sqrt{-\det g_{ind}}$$

[Fiol, Güijosa, Pedraza 1410.0692]

Given n, we are able to build a full solution.

$$\Delta^\mu = \sqrt{1+\kappa^2} v^\mu + \kappa n^\mu$$

$$X^\mu(\tau, z, \theta, \phi) = x^\mu(\tau) + z\Delta^\mu(\tau, \theta, \phi)$$

$$g_{ind} = \begin{pmatrix} \frac{1+2\kappa z v_\mu \partial_\tau n^\mu + (\partial_\tau \Delta^\mu)^2}{z^2} & \frac{\sqrt{1+\kappa^2}}{z^2} & \kappa \partial_\tau \Delta^\mu \partial_\theta n_\mu & \kappa \partial_\tau \Delta^\mu \partial_\phi n_\mu \\ \frac{\sqrt{\kappa^2+1}}{z^2} & 0 & 0 & 0 \\ \kappa \partial_\tau \Delta^\mu \partial_\theta n_\mu & 0 & -\kappa^2 & 0 \\ \kappa \partial_\tau \Delta^\mu \partial_\phi n_\mu & 0 & 0 & -\kappa^2 \sin^2(\theta) \end{pmatrix}$$

$$\mathcal{F}_{ij} = -\frac{g_{ind\,ij}}{\sqrt{1+\kappa^2}};\ j > i$$

$$\mathcal{F}_{ij} = \frac{g_{ind\,ij}}{\sqrt{1+\kappa^2}};\ j < i$$

$$\mathcal{F}_{ii} = 0$$

[Fiol, Güijosa, Pedraza 1410.0692]

Wasn't n a gauge freedom?

$$\mathcal{F}_{ij} = -\frac{g_{ind\,ij}}{\sqrt{1+\kappa^2}};\ j > i$$
$$\mathcal{F}_{ij} = \frac{g_{ind\,ij}}{\sqrt{1+\kappa^2}};\ j < i$$
$$\mathcal{F}_{ii} = 0$$

Non-covariant Ansatz

The solution generated by any n is still a solution

We can change coordinates in any solution

Several solutions...

... of different flux ...

... satisfying the ansatz in one frame

Can we check that this choice is physical?

- Energy on the worldvolume:

$$\epsilon = T_{D3}\left(-\frac{\det\left(g_{\mathrm{ind}\,\hat{\mu}\hat{\nu}}+2\pi\alpha' F_{\hat{\mu}\hat{\nu}}\right)_{\hat{\mu},\hat{\nu}>0}}{z^2\sqrt{-\det\left(g_{\mathrm{ind}\,\hat{k}\hat{l}}+2\pi\alpha' F_{\hat{k}\hat{l}}\right)}} - \frac{1}{z^4}\partial_{\hat{1}}X^{\mu}\partial_{\hat{2}}X^{\nu}\partial_{\hat{3}}X^{\rho}\epsilon_{0\mu\nu\rho}\right)$$

For a static D3, the dependence on n becomes explicit

$$\epsilon = T_{D3}\left(\frac{\kappa\sin\theta}{z^2}\left(1 - z^2\kappa^2\left(\partial_\tau n^{\nu}\right)^2\right)\right)$$

Spacelike, ergo positive contribution to energy

Can we check that this choice is physical?

- Preserved SUSY:

Let us focus on a static D3. The flux choice freedom is:

$$\mathcal{F}_{\tau\theta} = -R^2\kappa^2 f(\tau)$$
$$\mathcal{F}_{\tau\phi} = -R^2\kappa^2 \sin^2\theta g(\tau)$$

Build projector Γ, and look for global eigenvector killings

$$(\gamma_t K - \gamma_z)\,\epsilon_0^- = 0$$
$$(\gamma_z + \gamma_t K)\,\epsilon_0^+ = 0$$
$$f(\tau)\left(\gamma_{zr} + \kappa + \sqrt{\kappa^2+1}\gamma_{rt}\right) M\left(\epsilon_0^-\right)^\star = 0$$
$$g(\tau)\left(\gamma_{zr} + \kappa + \sqrt{\kappa^2+1}\gamma_{rt}\right) M\left(\epsilon_0^-\right)^\star = 0$$
$$f(\tau)\left(\gamma_{zr} + \kappa - \sqrt{\kappa^2+1}\gamma_{rt}\right)\gamma_i\gamma_t M\left(\epsilon_0^+\right)^\star = 0;\ i \in \{r,t,z\}$$
$$g(\tau)\left(\gamma_{zr} + \kappa - \sqrt{\kappa^2+1}\gamma_{rt}\right)\gamma_i\gamma_t M\left(\epsilon_0^+\right)^\star = 0;\ i \in \{r,t,z\}$$

Unless f=g=0, only a 0 spinor field can satisfy them.

Only the Drukker-Fiol brane is BPS

Summary and concluding remarks:

♦ D3 brane holography aims at improving non-perturbative results for charges in strongly-coupled backgrounds

♦ So far has only been used for most supersymmetric configurations

♦ Fiol-Güijosa-Pedraza's solution constructs a D3 brane for any path, and might allow computations for generic paths

♦ There are still issues to be understood on this solution and its applicability