Dark Matter Monopoles

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Magnetic Monopoles

Maxwell's Equations:

$$\nabla \cdot E = 4\pi \rho_e$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

$$\nabla \times B = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c} J_e$$

$\nabla \cdot B = 0 \implies$ No Magnetic Monopoles

No magnetic monopoles have been discovered

Flux of monopoles less than $10^{-18}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$IceCube Collaboration 2014

Magnetic Monopoles

Maxwell's Equations with monopoles:

$$\nabla \cdot E = 4\pi\rho_e$$

$$\nabla \cdot B = 4\pi\rho_m$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} - \frac{4\pi}{c}J_m$$

$$\nabla \times B = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c}J_e$$

More beautiful?

Polchinski 03: The existence of magnetic monopoles seems like one of the safest bets that one can make about physics not yet seen.

Quantum Magnetic Monopoles

Dirac 1931 investigated monopoles in a quantum theory.

If we assume that monopoles exist and we consider a **stationary monopole**, with magnetic charge g_m, and a particle, with electric charge e, moving around it.

Angular Momentum of the system is proportional to $g_m e$, and is quantized:

$$g_m e = 2\pi n$$

Dirac's Interpretation: The existences of **one** monopole would explain the quantization of electric charge

Dirac Monopole not a particle since it has a divergence which would lead to infinite mass.

't Hooft Polyakov monopole

't Hooft 1974 and Polyakov 1974 independently discovered how to remove the singularities of the Dirac monopole and constructed a quantum theory with monopoles.

They found that monopoles arise in SU(2) gauge theories, with an adjoint scalar, when the theory is spontaneously broken to U(1)

At large distances the SU(2) monopole is similar to the Dirac monopole and would lead to charge quantization.

At short distances we would resolve the SU(2) structure and there would be no divergences

't Hooft Polyakov monopole

We will consider an SU(2) theory with an adjoint scalar, Φ (or equivalently and SO(3) theory with a real vector)

$$\mathcal{L}_D = -\frac{1}{2}\mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}(D_\mu\Phi(D^\mu\Phi)^\dagger) - \lambda_\phi \mathrm{Tr}(\Phi\Phi^\dagger)^2 + m^2\,\mathrm{Tr}(\Phi\Phi^\dagger)$$

$$\Phi = \phi_a \frac{\sigma_a}{2}, \quad A_\mu = A_\mu^a \frac{\sigma_a}{2}, \quad D_\mu\Phi = \partial_\mu\Phi + ie[A_\mu, \Phi]$$

The negative mass squared term leads to spontaneous symmetry breaking such that:

$$\langle\Phi\rangle = \frac{\langle\phi_3\rangle}{2}\sigma_3 = \frac{v}{2}\sigma_3 = \frac{1}{2}\begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$

The theory has two massive gauge bosons $W^\pm$, one massless gauge boson γ and one massive higgs boson.

't Hooft Polyakov monopole

We now want to construct a monopole as an extend field configuration

For the monopole to have finite energy we need
$V(\Phi) = 0 \implies |\Phi|^2 = m^2/\sqrt{\lambda_\phi}$ at the two-sphere of infinity.

The Higgs Field is therefore a map from $S^2 \to S^2$

This means that there is a **topological winding** number that is conserved (counts the number of times S^2 is wrapped around S^2).

One can show that this winding number gives the magnetic charge, and that the monopole will have charge of:

$$g_m = \frac{4\pi}{e} N$$

't Hooft Polyakov monopole

A magnetic monopole is a **semi-classical particle**, localized in space.

The mass of the monopole is given by:

$$M_m \geq \frac{4\pi}{e} v = \frac{M_W}{\alpha_e}$$

The equality is reached in the BPS limit where $\lambda_\phi \to 0$.

For non-BPS monopoles the mass can reach 1.8 times the BPS mass Preskill 84.

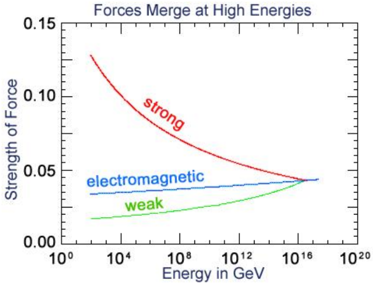
We will approximate monopole mass with:

$$M_m = \frac{4\pi}{e} v$$

Monopoles and Grand Unification

At high energy SU(3)×SU(2)×U(1) could unify to e.g. SU(5)

This would happen at an energy scale of around $10^{15} - 10^{16}$GeV.



The breaking of SU(5) to SU(3)×SU(2)×U(1) would give rise to magnetic monopoles with mass of the GUT scale.

The same is true for other GUT theories like SO(10).

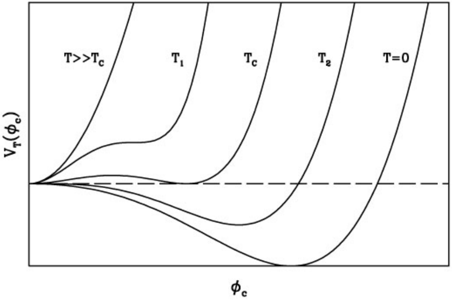
This led to the **cosmological monopole** problem

Cosmological Production of Monopoles

Monopoles can be produced in cosmological **phase transitions** Kibble 79.

The production depends on the type of phase transition.

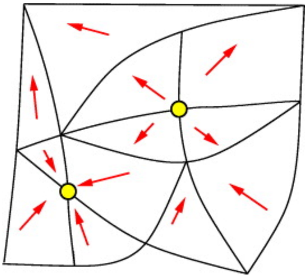
In a **first order phase transition**, there is an energy barrier and the phase transition is discontinuous. The phase transition will happen via nucleation of bubbles of the true vacuum spreading at the speed of light.
In a **second order phase transition**, there is no barrier and the phase transition is smooth. Correlation lengths diverge.



Kibble Mechanism

Kibble 79 Considered a second order phase transition in the early universe

Even if correlation length is diverging, the causal horizon sets maximum size of correlations $d_H \leq H^{-1}$.



In each region $|\Phi|^2 = v^2$, but Φ will point in a random direction.

This will give rise to **non-trivial topologies**.

We would then expect about **one** monopole per d_H^3

$$\frac{n_{\rm m}}{T^3} \geq \left(\frac{T_c}{\sqrt{\frac{45}{4\pi^3 g_\star}} M_{\rm Pl}} \right)^3 \qquad (1)$$

Kibble-Zurek Mechanism

Zurek 1985 improved on Kibble's argument and found that the frozen correlation length is **smaller** than the Kibble estimate.

The relaxation time and the correlation length during the phase transition:

$$\tau = \frac{\tau_0}{\sqrt{|\epsilon(T)|}}, \quad \text{and} \quad \zeta = \zeta_0 \, |\epsilon(T)|^{-\nu}, \quad \epsilon(T) = \frac{T - T_c}{T_c}$$

Even if the correlation length is **diverging**, the relaxation time is also increasing. At a time $t_\star$ the relaxation time is **too long for the field to keep up** with the increasing correlation length and the correlation length is frozen out.

$$d_h = \zeta(t_\star)$$

ν is a critical exponent. $\nu = 1/2$ classically, but can be changed by quantum corrections.

Has been experimentally verified in condensed matter systems.

Kibble-Zurek Mechanism

This gives a density of monopoles today of:

$$\Omega_{\rm m}\, h^2 \;=\; 1.5\times 10^9 \left(\frac{M_{\rm m}}{1\ \text{TeV}}\right)\left(\frac{30\ T_c}{M_{\rm Pl}}\right)^{\frac{3\nu}{1+\nu}}$$

Murayama et al 2010

Monopole Problem:
For a GUT monopole $M_m \sim 10^{16}$GeV and $T_c \sim 10^{14}$GeV. This gives

$$\Omega_{\rm m}\, h^2 \;\sim 10^{19}$$

Monopoles would dramatically over-close the Universe
A first order PT would not help. We get approximately one monopole per bubble, which still leads to

$$\Omega_{\rm m}\, h^2 \;\sim 10^{11}$$

This was the main motivation for Inflation

Dark Matter

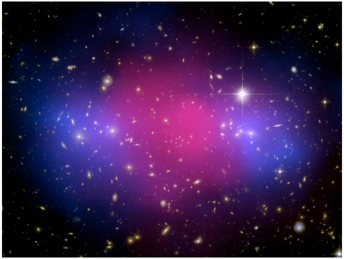
We know from many sources that the universe has to contain **Dark Matter**.

Planck Satellite have measured

$$\Omega_{dm} h^2 = 0.1187 \pm 0.0017$$

In the standard scenario Dark Matter is **cold** and **collisionless**.

Weak scale cross sections and masses give the right abundance(WIMP miracle)



Bullet Cluster
Source: NASA

Challenges to Cold Collisionless Dark Matter

Cold Collissionless Dark Matter is very successful at scales $\geq$ 1Mpc

On smaller scales there are discrepancies between observations and simulations

- **Core-vs-Cusp Problem**:
  CCDM simulations predict cusps at the centre of galaxies while cores have been seen observationally.
- **Too-big-to-fail problem**:
  Simulations predict $O(10)$ subhalos with $v >$30km/s for the Milky Way while there are none with $v >$25km/s

Warm Dark Matter and Self-Interacting Dark Matter have been proposed as solutions

Might be explained by **Baryonic Effects**

Self-Interacting Dark Matter

Self-Interacting Dark Matter can solve the problems with CCDM due to **energy transfer** between DM particles Spregel et al 2000

This flattens out the cores and decreases the number of sub-halos

Energy Transfer captured by **transfer cross section**:

$$\sigma_{\mathrm{T}} = \int d\Omega \, (1 - \cos\theta) \, \frac{d\sigma}{d\Omega}$$

Bullet Cluster give limit $\sigma_T / M_{DM} < 1.25 cm^2/g$ for $v \sim 1000$km/s
Randall et al 2008

Galaxies give limits of $\sigma_T / M_{DM} < 0.1 - 1 cm^2/g$ for $v \sim 200$km/s
Tulin et al 2013

Need $\sigma_T / M_{DM} = 0.1 - 10 cm^2/g$ at $v \sim 30$ km/s to solve problems
Zavala et al 2013,Buckley et al 2014

We will consider **velocity dependent self-interacting** Dark Matter

Dark Radiation

In the universe there can also be non SM radiation, called **Dark Radiation**

$$\rho_{\mathrm{rel}} = g_\star(T) \times \frac{\pi^2}{30} T^4$$

$g_\star$ counts the number of relativistic degrees of freedom.
In the SM this consists of photons and neutrinos, and is often written:

$$g_\star = g_\gamma + \frac{7}{8} g_\nu N_{\mathrm{eff}} \times \left(\frac{T_\nu}{T}\right)^4 = 2 + \frac{7}{8} 2 N_{\mathrm{eff}} \left(\frac{4}{11}\right)^{\frac{4}{3}}$$

In the SM we expect $N_{eff} = 3.046$

The amount of Dark Radiation impacts BBN and CMB so we get limits:

From CMB: $N_{\mathrm{eff}} = 3.30 \pm 0.27$ Planck Collaboration 2013

From BBN: $N_{\mathrm{eff}} = 3.24 \pm 1.2$ Cyburt et al 2004

Model

The aim is to see if **monopoles** could make up part of Dark Matter

We will consider a model where we extend the SM with an SU(2) group with an adjoint scalar

$$\mathcal{L}_D = -\frac{1}{2}\mathrm{Tr}F'_{\mu\nu}F'^{\mu\nu} + \mathrm{Tr}(D_\mu\Phi(D^\mu\Phi)^\dagger) - \lambda_\phi \mathrm{Tr}(\Phi\Phi^\dagger)^2 + m^2\,\mathrm{Tr}(\Phi\Phi^\dagger)$$

$$\mathcal{L}_{SM-D} = \lambda_p Tr(\Phi\Phi^\dagger)HH^\dagger$$

We get spontaneous symmetry breaking in the hidden sector $|\Phi|^2 = w^2$

To avoid **hierarchy problem** we need $\lambda_p w^2 \lesssim m_h^2$

Cosmological Implications

This model introduces magnetic monopoles, heavy vector bosons, a massless vector boson and a neutral scalar that mixes with the Higgs.

The phase transition in the hidden sector will produced **magnetic monopoles** which are stable and could therefore be part of Dark Matter

The heavy $W'_{\pm}$ particles are **stable**, due to hidden electric charge, and will therefore also contribute to Dark Matter

The massless γ' can not decay into anything and will contribute to Dark Radiation

The neutral scalar is **not stable** as it mixes with the Higgs. Could change Higgs phenomenology at detectors.

Extra Dark Radiation

The massless γ' contributes to **dark radiation**

The amount is determined by how much entropy in the hidden sector is transfered to radiation.

$$\frac{g^h_{\star s}(T^h_M)\, T^{h\,3}_M}{g^h_{\star s}(T_D)\, T^3_D} = \frac{g^{sm}_{\star s}(T_M)\, T^3_M}{g^{sm}_{\star s}(T_D)\, T^3_D}$$

$$\Delta N_{\rm eff}(T_M) = 4.4 \times \left(\frac{g^h_{\star s}(T_D)}{g^h_{\star s}(T^h_M)} \frac{g^{sm}_{\star s}(T_M)}{g^{sm}_{\star s}(T_D)} \right)^{4/3} ,$$

If we have n charged degrees of freedom in the hidden sector we get:

$$\Delta N_{\rm eff}(T_{CMB}) = 0.022 \times (2+n)^{4/3}$$

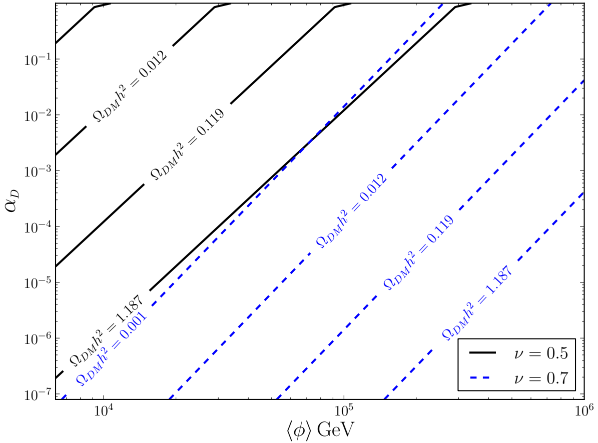
$$\Delta N_{\rm eff}(T_{BBN}) = 0.08 \times (2+n)^{4/3}$$

The Planck limit gives $n < 14$ and the BBN limit gives $n < 7$, $n = 6$ in simplest model.

Monopoles as Dark Matter

If the phase transition is second order monopoles are produced via the **Kibble-Zurek mechanism**:

$$\Omega_{\mathrm{m}}\, h^2 \;=\; 1.5 \times 10^9 \left(\frac{M_{\mathrm{m}}}{1\ \mathrm{TeV}}\right) \left(\frac{30\ T_c}{M_{\mathrm{Pl}}}\right)^{\frac{3\nu}{1+\nu}}$$

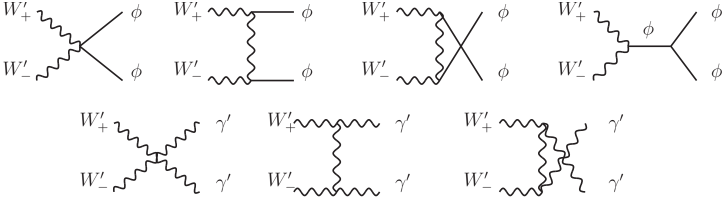


There could be monopole anti-monopole anhiliations (Preskill 1984), but in our case they do no change the abundance significantly due to the lack of a charged plasma.

Vector Dark Matter

$W'_\pm$ are stable due to being **charged** under remaining U(1)

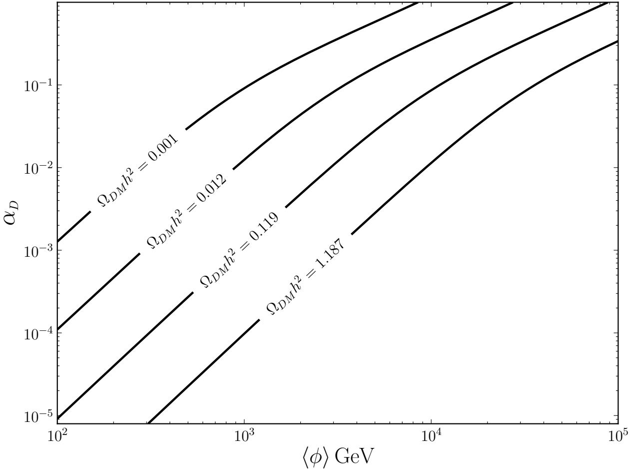
Relic density due to standard thermal freeze out



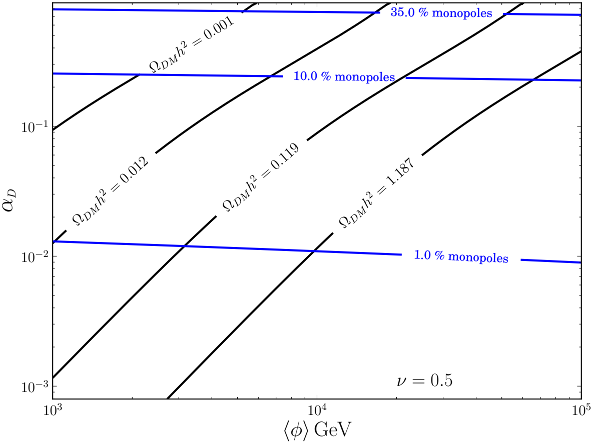
$$\langle \sigma v \rangle_{\mathrm{pert}} = \frac{1579\, g_D^4}{2304\pi M_{W'}^2} - \frac{5\, g_D^2\, \lambda_\phi}{192\pi M_{W'}^2} + \frac{3\, \lambda_\phi^2}{64\pi M_{W'}^2}$$

The cross section is enhanced due to **Sommerfeld enhancement**:

$$\langle \sigma v \rangle \;=\; S \,\langle \sigma v \rangle_{\rm pert}\,, \quad S = \frac{\alpha_D \pi}{v} \frac{1}{1 - \exp\left[-\frac{\alpha_D \pi}{v}\right]}$$

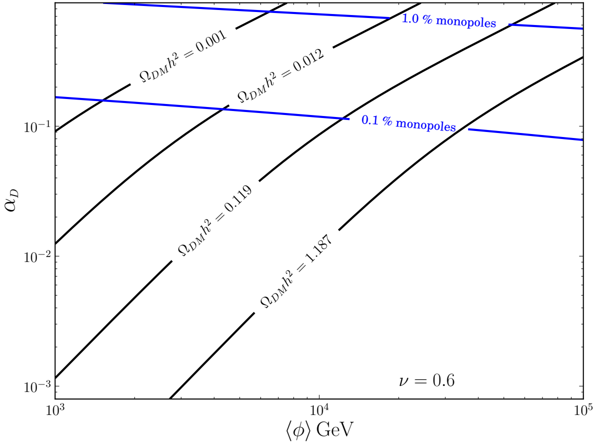


Combined Monopole and Vector Dark Matter



Combined relic density with $\nu = 0.5$.

Combined Monopole and Vector Dark Matter



Combined relic density with $\nu = 0.6$.

Self Interactions

Both monopoles and Vector Dark Matter is charged under a **long range force** due to non-broken U(1)

The interactions are via a Yukawa potential

$$V(r) = \frac{\alpha_e}{r} e^{-m_{\gamma'} r}$$

where $m_{\gamma'}$ is an effective mass due to interactions with the plasma

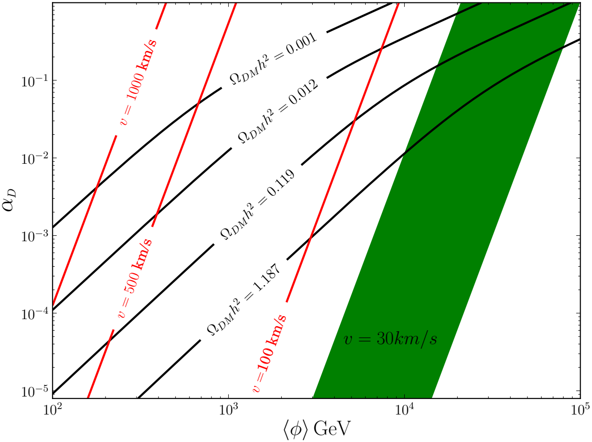
$$m_{\gamma'} \; = \; \frac{1}{l_{\mathrm{D}}} \; = \; \frac{(4\pi\alpha_D\rho)^{1/2}}{M_{\mathrm{DM}}\, v}$$

We can use the classical limit where the transfer cross section becomes:

$$\sigma_{\mathrm{T}} = \frac{16\pi\alpha_D^2}{M_{\mathrm{DM}}^2\, v^4} \, \log\left(1 + \frac{M_{\mathrm{DM}}^2\, v^2}{2\alpha_D m_{\gamma'}^2}\right)$$

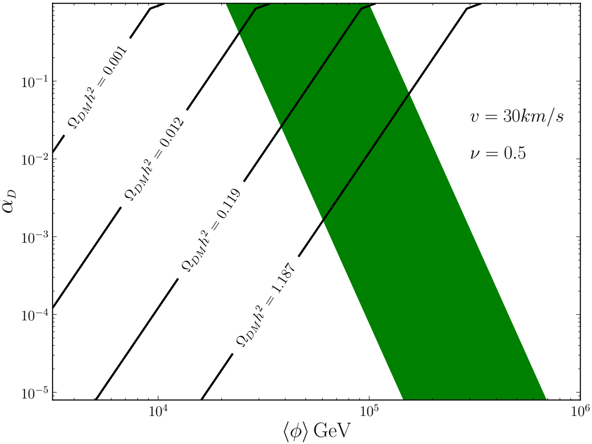
For monopoles $\alpha_D \to 1/\alpha_D$

Self Interactions of Vector Dark Matter



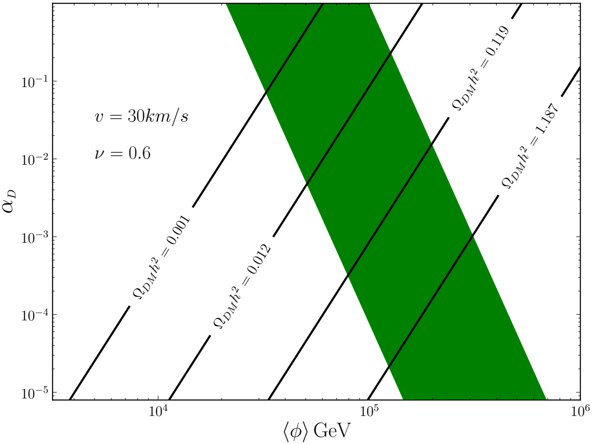
In the green region $\sigma_T/m_{DM} = 0.1 - 10cm^2/g$ for v=30km/s.
The red lines show $\sigma_T/m_{DM} = 1$

Self Interactions of Monopole Dark Matter



In the green region $\sigma_T/m_{DM} = 0.1 - 10cm^2/g$ for v=30km/s.
Critical exponent $\nu = 0.5$

Self Interactions of Monopole Dark Matter



In the green region $\sigma_T/m_{DM} = 0.1 - 10cm^2/g$ for v=30km/s.
Critical exponent $\nu = 0.6$

Comments about self-interactions

The long range Coulomb force can help **solve** the too-big-too-fail and the core-vs-cusp problem with CCDM

In this regime, due to the strong **velocity dependence** this would not cause any problems on larger scales

We have not taken into account interactions between monopoles and vectors if we have two component DM.

Open questions about how DM would behave if there are two components with **different** self-interactions

Cosmological simulations **necessary**.

Conclusions

We investigated an extension of the SM with an SU(2) hidden sector

- These models would have an extra contribution to Dark Radiation which is within current limits
- The model can have both Vector and Monopole Dark matter
- The fraction of monopole density can be up to about 30%
- Due to long range forces between dark matter this model could help solve the too-big-too-fail and the core-vs-cusp problem.