An Introduction to Mathieu Moonshine

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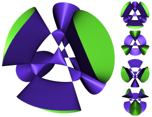
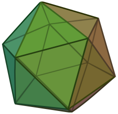
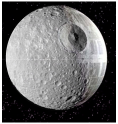


ATRAP

Moonshine, n.

- An illusive shadow

Dictionary of Archaic Words, J. O. Halliwell, London

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- An illusive shadow
- A dish composed partly of eggs

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Outline

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- Cancellation of the Weyl anomaly for the Superstring gives a critical dimension $D = 10$.
- Phenomenologically interesting models are formed by compactifying on a Calabi-Yau manifold.
- The only compact Calabi-Yau two-folds are $K3$ and the torus T^4.

We will try to write a partition function for the internal $c = 4(1 + \frac{1}{2}) = 6$ theory.

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Gepner Models

- Compactifying the superstring on the tensor product of $\mathcal{N} = 2$ theories leads to a consistent compactified string theory[1]

[1] Doron Gepner. "Space-time supersymmetry in compactified string theory and superconformal models". In: *Nuclear Physics B* 296.4 (1988), pp. 757–778.

[2] Doron Gepner. "Exactly solvable string compactifications on manifolds of SU (N) holonomy". In: *Physics Letters B* 199.3 (1987), pp. 380–388.

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- Compactifying on a product of such minimal models whose central charges add to $c_{tot} = 6$ is equivalent to compactifying on a Calabi-Yau 2-fold[2]
- $\mathcal{N} = 2$ minimal models are exactly solvable QFTs.

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The $\mathcal{N} = 2$ (2D) SCA

The $\mathcal{N} = 2$ superconformal algebra contains the energy-momentum operator $T(z)$ of conformal dimension-2, two supercurrents $G^{+}(z), G^{-}(z)$ of dimension $\frac{3}{2}$, as well as an operator of dimension 1, $J(z)$. The modes of the fields satisfy

Definition

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}\delta_{m+n,0}m(m^2-1) \tag{1}$$

where d_ϕ is the conformal dimension of ϕ, i.e $d_G = \frac{3}{2}$ and $d_J = 1, d_Q$ and $m, n \in \mathbb{Z}$, $r, s \in \mathbb{Z}$ in the Ramond sector and $r, s \in \mathbb{Z} + \frac{1}{2}$ in the Neveu-Schwarz sector.

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These may be parameterised in the NS sector as

$$h^{NS,k}_{l,m} = \frac{l(l+2) - m^2}{4(k+2)} \tag{5}$$

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$$0 \leq l \leq k, \quad |m| \leq l, \quad l \equiv m \ (2) \tag{7}$$

Characters

Definition

The character of a representation is

$$ch(\tau, z) := Tr\left(q^{L_0 - \frac{c}{24}} y^{J_0}\right) \tag{8}$$

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The character in the NS sector at level k can be given by

$$\mathrm{ch}_{l,m}^{NS(k)} = q^{h_{l,m}^{NS(k)} - \frac{c_k}{24}} y^{Q_{l,m}^{NS(k)}} \prod_{n=1}^{\infty} \frac{(1 + yq^{n-\frac{1}{2}})(1 + y^{-1}q^{n-\frac{1}{2}})}{(1 - q^n)^2}$$

$$\times \prod_{n=1}^{\infty} \frac{(1 - q^{(k+2)(n-1)+l+1})(1 - q^{(k+2)n-(l+1)})(1 - q^{(k+2)n})^2}{(1 + yq^{(k+2)n-\frac{1}{2}(l+m+1)})(1 + y^{-1}q^{(k+2)(n-1)+\frac{1}{2}(l+m+1)})(1 + y^{-1}q^{(k+2)n-\frac{1}{2}(l-m+1)})(1 + yq^{(k+2)(n-1)+\frac{1}{2}(l-m+1)})}$$

Modular Properties

The characters can be written in terms of functions with known modular transformations allowing the modular properties of the characters to be calculated

$$\mathrm{ch}_{l,m}^{NS(k)}(\tau+1,z) = \exp\left\{2\pi i(h_{l,m}^{NS(k)} - \frac{c_k}{24} - \frac{m}{2(k+2)})\right\}\mathrm{ch}_{l,m}^{NS(k)}\tau, z + \frac{1}{2} \quad (10)$$

$$\mathrm{ch}_{l,m}^{NS(k)}(\frac{-1}{\tau},\frac{z}{\tau}) = \frac{1}{k+2}\exp\left\{\frac{\pi i k z^2}{\tau(k+2)}\right\} \times \sum_{l'=0}^{k}\sum_{m'=-(k+1)}^{k+2}\sin\frac{\pi(l+1)(l'+1)}{k+2}\exp\left\{\frac{\pi i m m'}{k+2}\right\}\mathrm{ch}_{l',m'}^{NS(k)}(\tau,z) \quad (11)$$

Spectral Flow

The $\mathcal{N} = 2$ SCA has a continuous automorphism given by[3]

$$\begin{aligned} L_n &\to L_n + \eta J_n + \frac{c}{6}\eta^2\delta_{n,0} \\ J_n &\to J_n + \frac{c}{3}\eta\delta_{n,0} \\ G_r^{\pm} &\to G_{r\mp\eta}^{\pm} \end{aligned} \tag{12}$$

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These flows correspond to the shifts $z \to z \mp \frac{\tau}{2}$ and $z \to z \mp \tau$ in our characters respectively, under which we find the following transformations

$$\mathrm{ch}_{l,m}^{NS(k)}(\tau, z + \frac{\tau}{2}) = q^{-\frac{c_k}{24}} y^{-\frac{c_k}{12}} \mathrm{ch}_{l,m}^{R(k)}(\tau, z) \tag{13}$$

$$\mathrm{ch}_{l,m}^{(NS,R)(k)}(\tau, z + \tau) = q^{-\frac{c_k}{6}} y^{-\frac{c_k}{6}} \mathrm{ch}_{l,m-2}^{(R,NS)(k)}(\tau, z) \tag{14}$$

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Outline

The 1^6 Model

In a Gepner model, we consider a product $\prod_i k_i^{m_i}$ such that

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Definition

An orbit is a combination of characters which has integral U(1) charge and is invariant under two-fold spectral flow.

The 1^6 model orbits

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These characters transform into each other under spectral flow

$$A \to q^{-\frac{1}{6}} y^{-\frac{1}{6}} B \tag{16}$$

$$B \to q^{-\frac{1}{6}} y^{-\frac{1}{6}} C \tag{17}$$

$$C \to q^{-\frac{1}{6}} y^{-\frac{1}{6}} A \tag{18}$$

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We use this to construct our first orbit NS_1

$$NS_1 := A^6 + B^6 + C^6 \to q^{-1} y^{-1} NS_1 \tag{19}$$

Orbits under modular transformations

Using our previous results about the modular transformations of the characters we can calculate the S transform of A

$$A \to \mathrm{ch}^{NS}_{0,0}(-\frac{1}{\tau}, \frac{z}{\tau}) = \frac{1}{\sqrt{3}} e^{\pi i \frac{z^2}{3\tau}} (A + B + C) \tag{20}$$

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Similarly we can calculate

$$A^6 \to \frac{1}{27} e^{2\pi i \frac{z^2}{\tau}} (A + B + C)^6 \quad (21)$$

$$B^6 \to \frac{1}{27} e^{2\pi i \frac{z^2}{\tau}} (A - e^{\frac{1}{3}\pi i} B + e^{\frac{2}{3}\pi i} C)^6 \quad (22)$$

$$C^6 \to \frac{1}{27} e^{2\pi i \frac{z^2}{\tau}} (A + e^{\frac{2}{3}\pi i} B - e^{\frac{1}{3}\pi i} C)^6 \quad (23)$$

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Using this

$$\begin{aligned} NS_1 \xrightarrow{S} \frac{1}{27} e^{2\pi i \frac{z^2}{\tau}} \{ & 3(A^6 + B^6 + C^6) + 90(A^4BC + AB^4C + ABC^4) \\ & + 60(A^3B^3 + A^3C^3 + B^3C^3) + 270A^2B^2C^2 \} \end{aligned} \tag{24}$$

Orbits Continued

We have now found a total of four orbits

$$NS_1 := A^6 + B^6 + C^6 \quad NS_2 := A^3B^3A^3C^3 + B^3C^3 \tag{25}$$

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We can calculate the matrix $S_{i,j}$ of the S transforms of the orbits

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For the 1^6 model we find

$$S = \frac{1}{27} \begin{pmatrix} 3 & 60 & 270 & 90 \\ 3 & -21 & 27 & 9 \\ 1 & 2 & 9 & -6 \\ 3 & 6 & -54 & 9 \end{pmatrix} \tag{28}$$

We note that this matrix satisfies $S^2 = 1$.

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without summation. We can now write an S-invariant combination as

$$\begin{aligned}
\sum_i D_i \overline{NS_i} NS_i &\xrightarrow{S} \sum_{i,j,k} D_i S_{i,j} S_{i,k} \overline{NS_j} NS_k \\
&= \sum_{i,j,k} D_j S_{j,i} S_{i,k} \overline{NS_j} NS_k \\
&= \sum_{j,k} D_J \delta_{j,k} \overline{NS_j} NS_k \\
&= \sum_j \overline{NS_j} NS_j
\end{aligned} \tag{31}$$

Symmetry Enhancement and $\mathcal{N} = 4$

Alvarez-Gaumé and Freedman[4] showed that a sigma model on a hyperkähler manifold has $\mathcal{N} = 4$ symmetry.

[4] Luis Alvarez-Gaume and Daniel Z Freedman. "Geometrical structure and ultraviolet finiteness in the supersymmetric σ-model". In: *Communications in Mathematical Physics* 80.3 (1981), pp. 443–451.

[5] Tohru Eguchi et al. "Superconformal algebras and string compactification on manifolds with $SU(n)$ holonomy". In: *Nuclear Physics B* 315.1 (1989), pp. 193–221.

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- Unitarity bounds $h \geq \frac{k}{4}\ (R), h \geq l\ (NS), c = 6k, 0 \leq l \leq k$

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- $L_n, G^a_r, \bar{G}^a_r, J^i_m$
- HWS $|h, l\rangle$
- Unitarity bounds $h \geq \frac{k}{4}$ $(R), h \geq l$ $(NS), c = 6k, 0 \leq l \leq k$
- Massless representation when the bound is saturated.

[4] Alvarez-Gaume and Freedman, "Geometrical structure and ultraviolet finiteness in the supersymmetric σ-model".

[5] Eguchi et al., "Superconformal algebras and string compactification on manifolds with $SU(n)$ holonomy".

The Partition Function and the Elliptic Genus

We can find orbits in the other sectors by using the spectral flow and can write down a modular invariant combination of orbits

$$Z = \sum_i D_i(NS_i\overline{NS_i} + \widetilde{NS}_i\widetilde{\overline{NS_i}} + R_i\overline{R_i} + \widetilde{R}_i\widetilde{\overline{R_i}}) \tag{32}$$

where the $\widetilde{NS}, \widetilde{R}$ sectors are the NS, R sectors with $(-1)^F$ insertions.

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Definition

The Elliptic Genus of an $\mathcal{N} = (4,4)$ conformal field theory corresponding to a sigma model on a target space $\mathcal{M}$ is defined as

$$\varepsilon_{\mathcal{M}}(\tau, z) := Tr_{\mathcal{H}^R}\left((-1)^F q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}} y^{2J_0^3}\right) \tag{33}$$

More Elliptic Genus

The elliptic genus is simply the partition function in the $\widetilde{R}$ sector

$$Z_{\widetilde{R}}(\tau, z; \bar{\tau}, \bar{z}) := \mathrm{Tr}_{\mathcal{H}^R}\left((-1)^F q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L_0} - \frac{\bar{c}}{24}} y^{2J_0^3} \bar{y}^{2\bar{J}_0^3}\right) \tag{34}$$

with the right-movers projected out

$$\varepsilon_{\mathcal{M}}(\tau, z) = Z_{\widetilde{R}}(\tau, z; \bar{\tau}, \bar{z} = 0) \tag{35}$$

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$$\varepsilon_{\mathcal{M}}(\tau, z) = Z_{\widetilde{R}}(\tau, z; \bar{\tau}, \bar{z} = 0) \tag{35}$$

It is also independent of $\bar{\tau}$ since, decomposing $\mathcal{H}^R$ into left and right movers $\mathcal{H}^R = \bigoplus_{(j,j^*)\in\mathcal{J}} \mathcal{H}_j^R \otimes \mathcal{H}_{j^*}^R$

$$\begin{aligned}\varepsilon_{\mathcal{M}}(\tau, z) &= \sum_{(j,j^*)\in\mathcal{J}} \mathrm{Tr}_{\mathcal{H}_j^R}\left((-1)^{F_L} q^{L_0 - \frac{c}{24}} y^{2J_0^3}\right) \times \mathrm{Tr}_{\mathcal{H}_{j^*}^R}\left((-1)^{F_L} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right) \\ &= \sum_{(j,j^*)\in\mathcal{J}} ch_j^R(\tau, z + \frac{1}{2}) \times I_{j^*}\end{aligned} \tag{36}$$

where I_{j^*} is just the Witten Index of the representation j^*.

The Elliptic Genus of 1[6]

$$\varepsilon(\tau, z) = Z_{\widetilde{R}}(\tau, z; \bar{\tau}, \bar{z} = 0)$$

[6] Eguchi et al., "Superconformal algebras and string compactification on manifolds with $SU(n)$ holonomy".

The Elliptic Genus of 1[6]

$$\begin{aligned} \varepsilon(\tau, z) &= Z_{\widetilde{R}}(\tau, z; \bar{\tau}, \bar{z} = 0) \\ &= \sum_{i=1}^{d+d'} D_i \widetilde{R}_i(\tau, z) \bar{\widetilde{R}}_i(\bar{\tau}, \bar{z} = 0) \end{aligned} \tag{37}$$

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Note the second factor is the Witten Index. Using details of the representation theory of $\mathcal{N} = 4$[6]

$$\begin{aligned} \varepsilon(\tau, z) =& - 2\widetilde{R}_1(\tau, z) + \sum_{i=2}^{d} D_i \widetilde{R}_i(\tau, z) \\ =& - 2\mathrm{ch}_0^{\widetilde{R}}(l = \frac{1}{2}; \tau, z) + \sum_{i=2}^{d} D_i a_i \mathrm{ch}_0^{\widetilde{R}}(l = 0; \tau, z) \\ &+ \left(-2F_1(\tau) + \sum_{i=2}^{d} D_i a_i F_i(\tau)\right) \hat{\mathrm{ch}}^{\widetilde{R}}(h = \frac{1}{4}; \tau, z) \end{aligned} \tag{38}$$

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Note the second factor is the Witten Index. Using details of the representation theory of $\mathcal{N} = 4$[6]

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\begin{aligned}
\varepsilon(\tau, z) =& 20 \mathrm{ch}_0^{\widetilde{R}}(l = 0; \tau, z) - 2\mathrm{ch}_0^{\widetilde{R}}(l = \frac{1}{2}; \tau, z) \\
& + 2\left(\mathbf{45} q + \mathbf{231} q^2 + \mathbf{770} q^3 + \mathbf{2277} q^4 + \dots\right) \hat{\mathrm{ch}}^{\widetilde{R}}(h = \frac{1}{4}; \tau, z)
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And Now For Something Completely Different

Outline

A sporadic group

Theorem

The Classification of Finite Simple Groups.

This theorem states that all finite simple groups fall into one of the following families:

1. Cyclic groups of order n for n prime.
2. Alternating groups of degree at least 5.
3. Simple Lie type groups.
4. The 26 sporadic simple groups.

A sporadic group

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1. Cyclic groups of order n for n prime.
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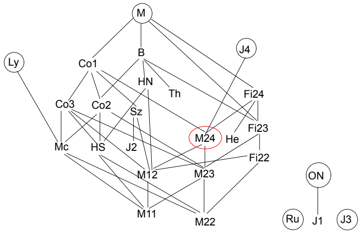
4. The 26 sporadic simple groups.

M_{24} is one of the sporadic finite simple groups. It is a subgroup of the well known Monster group M, as shown below.



The Golay Code

Linear codes are linear subspaces of vector spaces over finite fields.

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There is a unique $[24, 12, 8]$ code up to equivalency, $\mathcal{G}_{24}$. This code is known as the Extended Binary Golay Code.

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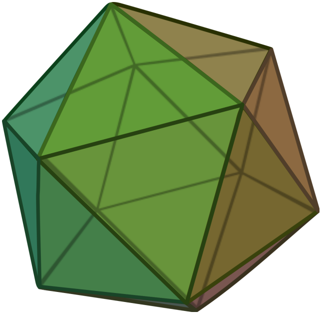

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Lexicographic Code

$$\begin{aligned}
c_0 &= (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)\\
c_1 &= (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1)\\
c_2 &= (0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,1,1,1,1)\\
c_3 &= (0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1)\\
&\vdots
\end{aligned}$$

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The Mathematical Game of Mogul

H H H H H H H H H H H H H H H H H H H H H H H H

$\downarrow$

H H T H H H T H H T H H H H H T H H H H H T H T

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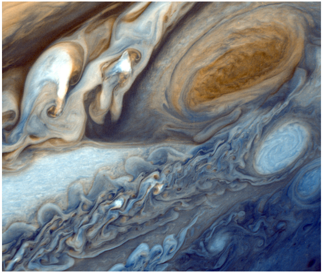
The Golay code was used to transmit photos back from the Voyager spacecraft.



M_{24}

We can define M_{24} in many different ways, however one that suits us is the following.

Definition

$$M_{24} := Aut(\mathcal{G}_{24}) \tag{39}$$

That is, $M_{24} = \{\tau \in S_{24} |\ \tau(c) \in \mathcal{G}_{24} \quad \forall c \in \mathcal{G}_{24}\}$

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M_{24} has order $2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23 = 244823040$

M_{24} Representation Theory

K_g	1A	2A	2B	3A	3B	4A	4B	4C	5A	6A	6B	7A	7B	8A	10A	11A	12A	12B	14A	14B	15A	15B	21A	21B	23A	23B
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ρ_1	23	7	-1	5	-1	-1	3	-1	3	1	-1	2	2	1	-1	1	-1	-1	0	0	0	0	-1	-1	0	0
ρ_2	45	-3	5	0	3	-3	1	1	0	0	-1	α_+^-	α_-^-	-1	0	1	0	1	α_-^+	α_+^+	0	0	α_+^-	α_-^-	-1	-1
$\overline{\rho_2}$	45	-3	5	0	3	-3	1	1	0	0	-1	α_-^-	α_+^-	-1	0	1	0	1	α_+^+	α_-^+	0	0	α_-^-	α_+^-	-1	-1
ρ_3	231	7	-9	-3	0	-1	-1	3	1	1	0	0	0	-1	1	0	-1	0	0	0	β_+^-	β_-^-	0	0	1	1
$\overline{\rho_3}$	231	7	-9	-3	0	-1	-1	3	1	1	0	0	0	-1	1	0	-1	0	0	0	β_-^-	β_+^-	0	0	1	1
ρ_4	252	28	12	9	0	4	4	0	2	1	0	0	0	0	2	-1	1	0	0	0	-1	-1	0	0	-1	-1
ρ_5	253	13	-11	10	1	-3	1	1	3	-2	1	1	1	-1	-1	0	0	1	-1	-1	0	0	1	1	0	0
ρ_6	483	35	3	6	0	3	3	3	-2	2	0	0	0	-1	-2	-1	0	0	0	0	1	1	0	0	0	0
ρ_7	770	-14	10	5	-7	2	-2	-2	0	1	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	γ_+^-	γ_-^-
$\overline{\rho_7}$	770	-14	10	5	-7	2	-2	-2	0	1	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	γ_-^-	γ_+^-
ρ_8	990	-18	-10	0	3	6	2	-2	0	0	-1	α_+^-	α_-^-	0	0	0	0	1	α_+^-	α_-^-	0	0	α_+^-	α_-^-	1	1
$\overline{\rho_8}$	990	-18	-10	0	3	6	2	-2	0	0	-1	α_-^-	α_+^-	0	0	0	0	1	α_-^-	α_+^-	0	0	α_-^-	α_+^-	1	1
ρ_9	1035	27	35	0	6	3	-1	3	0	0	2	-1	-1	1	0	1	0	0	-1	-1	0	0	-1	-1	0	0
ρ_{10}	1035	-21	-5	0	-3	3	3	-1	0	0	1	$2\alpha_+^-$	$2\alpha_-^-$	-1	0	1	0	-1	0	0	0	0	α_-^+	α_+^+	0	0
$\overline{\rho_{10}}$	1035	-21	-5	0	-3	3	3	-1	0	0	1	$2\alpha_-^-$	$2\alpha_+^-$	-1	0	1	0	-1	0	0	0	0	α_+^+	α_-^+	0	0
ρ_{11}	1265	49	-15	5	8	-7	1	-3	0	1	0	-2	-2	1	0	0	-1	0	0	0	0	0	1	1	0	0
ρ_{12}	1771	-21	11	16	7	3	-5	-1	1	0	-1	0	0	-1	1	0	0	-1	0	0	1	1	0	0	0	0
ρ_{13}	2024	8	24	-1	8	8	0	0	-1	-1	0	1	1	0	-1	0	-1	0	1	1	-1	-1	1	1	0	0
ρ_{14}	2277	21	-19	0	6	-3	1	-3	-3	0	2	2	2	-1	1	0	0	0	0	0	0	0	-1	-1	0	0
ρ_{15}	3312	48	16	0	-6	0	0	0	-3	0	-2	1	1	0	1	1	0	0	-1	-1	0	0	1	1	0	0
ρ_{16}	3520	64	0	10	-8	0	0	0	0	-2	0	-1	-1	0	0	0	0	0	1	1	0	0	-1	-1	1	1
ρ_{17}	5313	49	9	-15	0	1	-3	-3	3	1	0	0	0	-1	-1	0	1	0	0	0	0	0	0	0	0	0
ρ_{18}	5544	-56	24	9	0	-8	0	0	-1	1	0	0	0	0	-1	0	1	0	0	0	-1	-1	0	0	1	1
ρ_{19}	5796	-28	36	-9	0	-4	4	0	1	-1	0	0	0	0	1	-1	-1	0	0	0	1	1	0	0	0	0
ρ_{20}	10395	-21	-45	0	0	3	-1	3	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	-1	-1

Outline

Mathieu Moonshine

When considering the 1^6 model we calculated the elliptic genus as

$$\begin{aligned}\varepsilon(\tau,z) =& 20\mathrm{ch}_0^{\widetilde{R}}(l=0;\tau,z) - 2\mathrm{ch}_0^{\widetilde{R}}(l=\frac{1}{2};\tau,z) \\ &+ 2\left(\mathbf{45}q + \mathbf{231}q^2 + \mathbf{770}q^3 + \mathbf{2277}q^4 + \dots\right)\hat{\mathrm{ch}}^{\widetilde{R}}(h=\frac{1}{4};\tau,z)\end{aligned}$$

[7] Terry Gannon. "Much ado about Mathieu". In: *arXiv preprint arXiv:1211.5531* (2012).

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We can now see that these coefficients are all sums of dimensions of irreducible representations of M_{24}.

Gannon[7] introduced the *twining elliptic genera* for $g \in M_24$

$$\phi_g(\tau, z) = \mathrm{ch}_{H_{00}}(g)\mathrm{ch}^R_{\frac{1}{4},0} + \sum_{n=0}^{\infty} \mathrm{ch}_{H_n}(g)\mathrm{ch}^R_{n+\frac{1}{4},\frac{1}{2}}(\tau, z) \qquad (40)$$

and proved that all H_n are indeed representations of M_{24}.

[7] Gannon, "Much ado about Mathieu".

Final Comments on the Elliptic Genus

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When a CFT has a sigma-model construction the two notions of elliptic genus agree.


UNCLE COOTER'S
GENUINE
MOONSHINE
XXX
Hooch, Fire-water, Booze, Snake Oil,
Adult Beverage, Medicine, Elixir,
Sauce, Home Brew, Hair-of-the-dog.
NOT FOR BABIES

Questions?