Flavour Changing IR limits

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Motivation

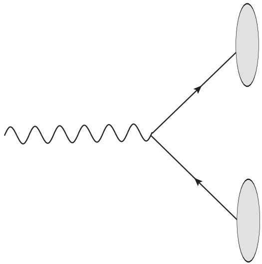
Leading order (LO) calculations typically come with very large scale uncertainties. We need tools to reduce this scale uncertainty to gain any physical insight. Two options

- Go to higher orders in $\alpha_s(\mu^2)$ - Next-to-Leading (NLO) corrections,
- Exponentiate logarithms in regions of the phase space where our perturbative series breaks down - resummation,

Both improve scale uncertainties in different regimes.
For certain processes even NLO corrections are insufficient, then we've got to consider the Next-to-Next-to-Leading (NNLO) corrections.

Example

Consider the simplest case, $e^+e^- \to q\bar{q}$



Blobs denote the jet algorithm, it looks at the final state (2 partons) and maps how many jets we'd expect given the parton momentum configuration (in this case 2 jets).

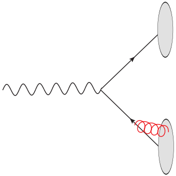
Problems

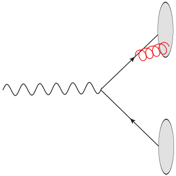
- Not very physical! Jets consist of many hadrons, we're saying that one parton has produced many hadrons?
- Unphysical scale only appears in $\alpha_s(\mu^2)$ at LO. This means that variations in the scale result in massive changes in the result for the total cross section.

NLO Corrections

One extra power of α_s. This implies we are left with two possibilities

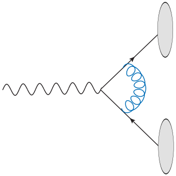
- Real corrections





Jet algorithm now maps 3 partons $\rightarrow$ 2 jets

- Virtual corrections



Jet algorithm maps 2 partons $\rightarrow$ 2 jets

Problems - Real corrections

For massless partons,

$$s_{ij} = E_i E_j (1 - \cos(\theta_{ij})) \tag{1}$$

Matrix elements are generated from products of invariants. Divergences arise as $s_{ij} \to 0$.

- $E_{i/j} \to 0$ - soft singularity
- $\cos(\theta_{ij}) \to 1$ - collinear singularity

Problems - Virtual corrections

Virtual matrix element is explicitly divergent, i.e. a power series in ϵ.

$$|M_2^1(1_q, 2_{\bar{q}})|^2 = \overbrace{\left(-\frac{1}{\epsilon^2} - \frac{3}{2}\frac{1}{\epsilon}\right)}^{\text{Catani pole structure}} |M_2^0(1_q, 2_{\bar{q}})|^2 + \text{finite} \tag{2}$$

Need a way to regulate the implicit singularities of the real corrections and the explicit singularities of the virtual corrections to obtain a finite result.

Subtraction

$$\mathrm{d}\hat{\sigma}_{ij}^{NLO} = \int_{\mathrm{d}\sigma_{n+1}} \left(\mathrm{d}\hat{\sigma}_{ij}^{R,NLO} - \mathrm{d}\hat{\sigma}_{ij}^{S,NLO}\right) + \int_{\mathrm{d}\sigma_{n}} \left(\mathrm{d}\hat{\sigma}_{ij}^{V,NLO} - \mathrm{d}\hat{\sigma}_{ij}^{T,NLO}\right), \tag{3}$$

$\mathrm{d}\hat{\sigma}_{ij}^{S,NLO}$ mimics the implicit singularity structure of $\mathrm{d}\hat{\sigma}_{ij}^{R,NLO}$.
$\mathrm{d}\hat{\sigma}_{ij}^{T,NLO}$ has the same explicit singularity structure as $\mathrm{d}\hat{\sigma}_{ij}^{V,NLO}$.

$$\mathrm{d}\hat{\sigma}_{ij}^{T,NLO} = -\int_{\mathrm{d}\sigma_{1}} \mathrm{d}\hat{\sigma}_{ij}^{S,NLO} \tag{4}$$

The integration of the real subtraction term can either be analytical (e.g. Catani-Seymour dipoles) or numerical (e.g. FKS).

When should we consider higher order corrections?

Higher orders in $\alpha_s(\mu^2)$ imply more pain

- We need more ingredients (matrix elements, subtraction etc) - brain power pain
- More complicated functions to evaluate - numerical pain

We need to pick processes for higher order corrections selectively, prioritise processes we know will have large higher order corrections (large K factor).
How can we know when higher order corrections are large?

New channels

$$\mathrm{d}\sigma = \sum_{ij} \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} \overbrace{f_i(\xi_1, \mu_F^2) f_j(\xi_2, \mu_F^2)}^{\text{parton distribution functions}} \underbrace{\mathrm{d}\hat{\sigma}_{ij}(\alpha_s(\mu^2, \mu, \mu_F))}_{\text{Partonic cross section}} \qquad (5)$$

Consider Drell-Yan, $pp \to e^+e^-$
LO, one production channel

- $q\bar{q} \to e^+e^-$

NLO, three channels

- $q\bar{q} \to ge^+e^-$
- $qg \to qe^+e^-$ - dominant channel
- $\bar{q}g \to \bar{q}e^+e^-$

We know this process will have a very large K factor because the Parton Distribution Functions (PDFs) are gluon dominated at the LHC.

Drell-Yan - real corrections

$q(1)g(2) \to q(3)e^+e^-$
To be classified as a real correction to Drell-Yan, jet algorithm does not need to resolve any jets. $q(3)$ can go unresolved. How can it go unresolved?

- q(3) soft - doesn't make sense

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- ~~q(3) soft - doesn't make sense~~
- q(1) q(3) collinear - would factorise onto

$$P_{q\bar{q}\to g}(z)M_2^0(\hat{1}_g, \hat{2}_g) \tag{6}$$

No pure gluon matrix element exists for Drell-Yan, therefore limit does not exist.

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  No pure gluon matrix element exists for Drell-Yan, therefore limit does not exist.

- g(2) q(3) collinear - would factorise onto

$$P_{qg\to q}(z)M_2^0(\hat{1}_q, \hat{2}_{\bar{q}}) \tag{7}$$

  $q\bar{q}$ matrix element does exist, therefore we have one divergence

Problem?

$$\mathrm{d}\hat{\sigma}_{ij}^{NLO} = \int_{\mathrm{d}\sigma_{n+1}} \left(\mathrm{d}\hat{\sigma}_{ij}^{R,NLO} - \mathrm{d}\hat{\sigma}_{ij}^{S,NLO} \right) + \int_{\mathrm{d}\sigma_n} \left(\mathrm{d}\hat{\sigma}_{ij}^{V,NLO} - \mathrm{d}\hat{\sigma}_{ij}^{T,NLO} \right), \quad (8)$$

- *qg* real correction has a divergence, we need to perform subtraction to remove it.
- there is no '*qg*' virtual matrix element to cancel the pole structure of the real subtraction term?

Mass Factorisation

$$\mathrm{d}\sigma = \sum_{ij} \int \frac{d\xi_1}{\xi_1}\frac{d\xi_2}{\xi_2} \overbrace{f_i(\xi_1, \mu_F^2) f_j(\xi_2, \mu_F^2)}^{\text{parton distribution functions}} \underbrace{\mathrm{d}\hat{\sigma}_{ij}(\alpha_s(\mu^2, \mu, \mu_F))}_{\text{Partonic cross section}} \tag{9}$$

- With processes with initial state QCD we have to be careful with our definition of what resides in the hard scattering process.
- PDFs contain all interactions not deemed to be part of the hard process.
- The 'trick' is to introduce a new factorisation scale μ_F and with it a divergent counter term piece explicit in ϵ.
- Our collinear singularity is now cancelled against this counter term, as opposed to a virtual matrix element, rendering the cross section finite.

$$-\underbrace{\Gamma^1_{qg;qq}(z_1, z_2)}_{\substack{\text{mass factorisation}\\ \text{splitting kernel}}} \overbrace{M_2^0(1_q, 2_{\bar{q}})}^{\substack{\text{leading order}\\ \text{matrix element}}} \tag{10}$$

Explicitly

$$\mathrm{d}\hat{\sigma}_{ij}^{NLO} = \int_{\mathrm{d}\sigma_{n+1}} \left(\mathrm{d}\hat{\sigma}_{ij}^{R,NLO} - \mathrm{d}\hat{\sigma}_{ij}^{S,NLO}\right) + \int_{\mathrm{d}\sigma_{n}} \left(\mathrm{d}\hat{\sigma}_{ij}^{V,NLO} - \mathrm{d}\hat{\sigma}_{ij}^{T,NLO}\right), \tag{11}$$

where now,

$$\mathrm{d}\hat{\sigma}_{ij}^{T,NLO} = -\int_{\mathrm{d}\sigma_1} \mathrm{d}\hat{\sigma}_{ij}^{S,NLO} - \mathrm{d}\hat{\sigma}_{ij,NLO}^{MF}. \tag{12}$$

Rendering the whole cross section finite.

Antenna Subtraction

Exploit the universal factorisation of QCD in Infrared (IR) limits.
We pick simple processes and recycle their pole structures for use in more complicated processes.

$$M_3^0(1_q, i_g, 2_{\bar{q}}) \overset{i_g}{\underset{}{\overset{\text{unresolved}}{\longrightarrow}}} \overbrace{A_3^0(1_q, i_g, 2_{\bar{q}})}^{\text{Antenna function}} \underbrace{M_2^0((\widetilde{1i})_q, (\widetilde{i2})_{\bar{q}})}_{\text{reduced matrix element}} . \tag{13}$$

The antenna function is only dependent on the flavour of the partons neighbouring it in colour ordering. There is no dependence on other particles in the process (this information all resides in the reduced matrix element).
The antenna function contains all the singularities between the hard radiators and the unresolved emission (1/i collinear limit, 2/i collinear limit and i soft).

Consider the $e^+e^- \to q\bar{q}g$ 3 jet process at LO, one of the NLO corrections comes from

$$e^+e^- \to q(a)\bar{q}(b)g(i)g(j) \tag{14}$$

at Leading Colour the subtraction term for this process looks like

$$\sum_{ij}\Big[+d_3^0(a_q, i_g, j_g)M_3^0(\bar{a}_q, \widetilde{(ij)}_g, b_{\bar{q}}) \tag{15}$$

$$+d_3^0(b_{\bar{q}}, j_g, i_g)M_3^0(a_q, \widetilde{(ij)}_g, \bar{b}_{\bar{q}})\Big] \tag{16}$$

in line 15: a/i collinear limit, i soft limit, part of i/j limit
in line 16: b/j collinear limit, j soft limit, part of i/j limit
We split the i/j limit between the two antenna in a smart way to avoid double counting the full limit.

Cross this channel into the initial state

What happens when we cross this process into the initial state? For example consider

$$q(1)g(2) \to q(i)g(j)e^+e^- \qquad (17)$$

Try the same as above? We now have:

$$\sum_{ij}\Big[+d_3^0(1_q, j_g, 2_g)M_3^0(\bar{1}_q, \bar{2}_g, i_{\bar{q}}) \qquad (18)$$

$$+d_3^0(i_q, 2_g, j_g)M_3^0(1_q, \bar{2}_g, \widetilde{(ij)}_{\bar{q}})\Big] \qquad (19)$$

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$$+d_3^0(i_q, 2_g, j_g) M_3^0(1_q, \bar{2}_g, \widetilde{(ij)}_{\bar{q}}) \Big] \tag{19}$$

Problem in 2/i collinear limit. We've factorised onto a matrix element where parton 2 is a gluon but it has flavour changed to a quark.

Solution

'Pick out' the 2/i limit and only the 2/i limit using partial fractioning. Multiply this part of the antenna by a matrix element where $q\bar{q}$ are the in initial state.

$$\sum_{ij}\Big[+D_3^0(1_q, j_g, 2_g)M_3^0(\bar{1}_q, \bar{2}_g, i_{\bar{q}}) \tag{20}$$

$$+d^0_{3,g\to q}(i_q, 2_g, j_g)M_3^0(1_q, \widetilde{(ij)}_g, \bar{2}_{\bar{q}})\Big] \tag{21}$$

in line 20: 1/j collinear limit, j soft limit, full 2/j limit
in line 21: i/2 collinear limit only

Punchline

- Your subtraction term is dependent on the initial state configuration of your process, otherwise you get a mismatch of your reduced matrix elements.
- Use partial fractioning technique to isolate the singularities you want.

Beyond NLO

Consider now the Real-Real corrections to our $qg \to qe^+e^-$ process. One such contribution comes from

$$q(1)g(2) \to q(i)g(j)g(k)e^+e^- \tag{22}$$

This subtraction term is very long and complicated! However lets focus on one line from it...

$$D_4^0(i_q, 2_g, j_g, k_g) M_3^0(1_q, \bar{2}_g, \widetilde{(ijk)}_{\bar{q}}) \tag{23}$$

- deals with lots of singularities (e.g. $2_g/j_g/k_g$ triple collinear splitting function),

$$D_4^0(i_q, 2_g, j_g, k_g)M_3^0(1_q, \bar{2}_g, \widetilde{(ijk)}_{\bar{q}}) \tag{23}$$

- deals with lots of singularities (e.g. $2_g/j_g/k_g$ triple collinear splitting function),
- but like at NLO, we have a flavour changing configurations (e.g. $2_g/i_q/j_g$ triple collinear limit) factorising onto a mismatched matrix element.

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- but like at NLO, we have a flavour changing configurations (e.g. $2_g/i_q/j_g$ triple collinear limit) factorising onto a mismatched matrix element.
- Like at NLO, this is a serious problem...

Partial Fractioning

Can we apply what we did at NLO to NNLO?

Comparison

$$D_3^0(1_q, 3_g, 4_g) = \frac{1}{s_{134}^2}\left(\frac{2s_{134}^2 s_{14}}{s_{13}s_{34}} + \frac{2s_{134}^2 s_{13}}{s_{14}s_{34}} + \frac{s_{14}s_{34} + s_{34}^2}{s_{13}} \right.$$
$$\left. + \frac{s_{13}s_{34} + s_{34}^2}{s_{14}} + \frac{2s_{13}s_{14}}{s_{34}} + 5s_{134} + s_{34}\right) \quad (24)$$

Comparison

$$D_4^0(1_q, 3_g, 4_g, 5_g) \quad = \quad d_4^0(1,3,4,5) + d_4^0(1,5,4,3) \;, \tag{25}$$

$$d_4^0(1,3,4,5) =$$

$$\begin{aligned}
&\frac{1}{s_{1345}^2}\Bigg\{ \frac{s_{14}}{2 s_{13} s_{15} s_{34} s_{45}} \left[3 s_{14} s_{35}^2 + 3 s_{14}^2 s_{35} + 2 s_{14}^3 + s_{35}^3\right] \\
&+ \frac{s_{14}}{s_{13} s_{15} s_{34}} \left[6 s_{14} s_{35} + 3 s_{14} s_{45} + 4 s_{14}^2 + 3 s_{35} s_{45} + 3 s_{35}^2 + s_{45}^2\right] \\
&+ \frac{s_{14}}{s_{13} s_{15}} \left[3 s_{14} + 3 s_{35} + 3 s_{45}\right] + \frac{s_{14}}{s_{13} s_{34} s_{135} s_{345}} \Big[3 s_{14} s_{45}^2 + 3 s_{14}^2 s_{45} \\
&+ 2 s_{14}^3 + s_{45}^3\Big] + \frac{s_{14}}{s_{13} s_{135} s_{345}} \left[3 s_{14} s_{45} + 3 s_{14}^2 - 2 s_{35} s_{45} - s_{35}^2 + s_{45}^2\right] \\
&+ \frac{s_{14}^3}{s_{13} s_{34} s_{135}} + \frac{s_{35}^3 s_{45}}{2 s_{13} s_{15} s_{134} s_{145}} - \frac{s_{35}^3}{2 s_{13} s_{15} s_{134}} \\
&+ \frac{s_{15}}{s_{13} s_{45} s_{134} s_{145}} \left[3 s_{15} s_{35}^2 + 3 s_{15}^2 s_{35} + s_{15}^3 + s_{35}^3\right]
\end{aligned} \tag{26}$$

Highly Trivial

$$+\frac{1}{s_{13}s_{34}s_{45}}\Big[9s_{14}s_{15}s_{35}+4s_{14}s_{15}^2+6s_{14}s_{35}^2+6s_{14}^2s_{15}+9s_{14}^2s_{35}+4s_{14}^3$$
$$+3s_{15}s_{35}^2+4s_{15}^2s_{35}+2s_{15}^3+s_{35}^3\Big]+\frac{1}{s_{13}s_{34}s_{345}}\Big[s_{14}s_{15}s_{45}+4s_{14}s_{15}^2+s_{14}s_{45}^2$$
$$+6s_{14}^2s_{15}+3s_{14}^2s_{45}+4s_{14}^3+s_{15}s_{45}^2-2s_{15}^2s_{45}+2s_{15}^3\Big]$$
$$+\frac{1}{s_{13}s_{34}}\Big[17s_{14}s_{15}+16s_{14}s_{35}+11s_{14}s_{45}+15s_{14}^2+12s_{15}s_{35}+5s_{15}s_{45}$$
$$+10s_{15}^2+5s_{35}s_{45}+5s_{35}^2+2s_{45}^2\Big]+\frac{s_{34}}{s_{13}s_{134}^2}\Big[2s_{15}s_{35}+2s_{15}s_{45}+s_{15}^2$$
$$+2s_{35}s_{45}+s_{35}^2+s_{45}^2\Big]+\frac{s_{35}}{s_{13}s_{45}s_{145}s_{135}}\left[3s_{34}s_{35}^2+3s_{34}^2s_{35}+s_{34}^3+s_{35}^3\right]$$
$$+\frac{s_{35}}{s_{13}s_{145}s_{135}}\left[-6s_{34}s_{35}+3s_{34}s_{45}-3s_{34}^2+3s_{35}s_{45}-3s_{35}^2-s_{45}^2\right]$$
$$+\frac{s_{35}}{s_{13}s_{135}^2}\left[2s_{14}s_{34}+2s_{14}s_{45}+s_{14}^2+2s_{34}s_{45}+s_{34}^2+s_{45}^2\right]$$
$$+\frac{1}{s_{13}s_{45}s_{134}s_{345}}\left[5s_{15}s_{35}^3+9s_{15}^2s_{35}^2+7s_{15}^3s_{35}+2s_{15}^4+s_{35}^4\right]$$

Trivial

$$
\begin{aligned}
&+\frac{1}{s_{13}s_{45}s_{134}}\left[2s_{15}s_{34}s_{35} - 4s_{15}s_{35}^2 + s_{15}^2 s_{34} - 5s_{15}^2 s_{35} - 2s_{15}^3 + s_{34}s_{35}^2 - s_{35}^3\right] \\
&+\frac{1}{s_{13}s_{45}s_{145}}\Big[3s_{15}s_{34}s_{35} + 2s_{15}s_{34}^2 + 2s_{15}s_{35}^2 - s_{15}^2 s_{34} + s_{15}^2 s_{35} + s_{15}^3 \\
&-3s_{34}s_{35}^2 - 3s_{34}^2 s_{35} - s_{34}^3 - s_{35}^3\Big] + \frac{1}{s_{13}s_{45}s_{135}s_{345}}\Big[-5s_{14}s_{35}^3 + 9s_{14}^2 s_{35}^2 \\
&-7s_{14}^3 s_{35} + 2s_{14}^4 + s_{35}^4\Big] + \frac{1}{s_{13}s_{45}s_{135}}\Big[4s_{14}s_{34}^2 + 6s_{14}s_{35}^2 + 7s_{14}^2 s_{34} - 7s_{14}^2 s_{35} \\
&+6s_{14}^3 + 4s_{34}s_{35}^2 + 3s_{34}^2 s_{35} + s_{34}^3\Big] + \frac{1}{s_{13}s_{45}s_{345}}\Big[10s_{14}s_{15}^2 + 6s_{14}s_{35}^2 \\
&+10s_{14}^2 s_{15} - 7s_{14}^2 s_{35} + 6s_{14}^3 + 6s_{15}s_{35}^2 + 7s_{15}^2 s_{35} + 6s_{15}^3\Big] \\
&+\frac{1}{s_{13}s_{45}}\Big[17s_{14}s_{15} + 8s_{14}s_{34} + 6s_{14}s_{35} + 18s_{14}^2 + 5s_{15}s_{34} + 4s_{15}s_{35} + 5s_{15}^2 \\
&-s_{34}s_{35} - s_{34}^2 + 2s_{35}^2\Big] + \frac{1}{s_{13}s_{134}s_{145}}\Big[3s_{15}s_{35}s_{45} + 6s_{15}s_{35}^2 + 6s_{15}^2 s_{35} \\
&+s_{15}^2 s_{45} + 2s_{15}^3 + 3s_{35}^2 s_{45} + \frac{3}{\ } s_{35}^3\Big]
\end{aligned}
\qquad (28)
$$

Ok not so trivial

$$
\begin{aligned}
&+\frac{1}{s_{13}s_{134}s_{345}}\Big[6s_{15}s_{35}s_{45}+9s_{15}s_{35}^2\\
&+s_{15}s_{45}^2+9s_{15}^2s_{35}+3s_{15}^2s_{45}+3s_{15}^3+s_{35}s_{45}^2+3s_{35}^2s_{45}+3s_{35}^3\Big]\\
&+\frac{1}{s_{13}s_{134}}\Big[4s_{15}s_{34}-13s_{15}s_{35}-6s_{15}s_{45}-7s_{15}^2+4s_{34}s_{35}+3s_{34}s_{45}\\
&-5s_{35}s_{45}-7s_{35}^2-2s_{45}^2\Big]+\frac{1}{s_{13}s_{145}}\Big[-2s_{15}s_{34}-s_{15}s_{35}\\
&+s_{15}^2+6s_{34}s_{35}-s_{34}s_{45}+2s_{34}^2-3s_{35}s_{45}+4s_{35}^2\Big]\\
&+\frac{1}{s_{13}s_{135}}\left[2s_{14}s_{34}-s_{14}s_{35}+6s_{14}^2-s_{34}s_{35}-2s_{34}s_{45}+3s_{35}s_{45}-s_{35}^2-s_{45}^2\right]\\
&+\frac{1}{s_{13}s_{345}}\Big[2s_{14}s_{15}+2s_{14}s_{35}+2s_{14}s_{45}+s_{14}^2+4s_{15}s_{35}+2s_{15}s_{45}+s_{15}^2\\
&+s_{35}s_{45}+2s_{35}^2\Big]+\frac{1}{s_{13}}\left[14s_{14}+2s_{15}+2s_{34}+3s_{35}-2s_{45}\right]
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&-\frac{4s_{13}s_{15}^2}{s_{45}s_{145}s_{345}}+\frac{s_{14}s_{15}}{s_{34}s_{134}^2}\left[-4s_{35}-4s_{45}\right]-\frac{4s_{14}s_{15}^2s_{45}}{s_{34}^2s_{134}s_{345}}\\
&+\frac{s_{14}}{s_{34}^2s_{134}}\left[2s_{14}s_{15}+2s_{14}s_{35}+2s_{14}s_{45}+4s_{15}s_{35}-4s_{15}s_{45}-2s_{35}s_{45}-2s_{45}^2\right]\\
&+\frac{s_{14}^2}{s_{34}^2s_{134}^2}\left[2s_{15}s_{35}+2s_{15}s_{45}+2s_{15}^2+2s_{35}s_{45}+s_{35}^2+s_{45}^2\right]\\
&+\frac{s_{15}}{2s_{34}s_{45}s_{134}s_{145}}\left[3s_{15}s_{35}^2+3s_{15}^2s_{35}+2s_{15}^3+s_{35}^3\right]+\frac{s_{15}}{s_{34}s_{45}s_{145}}\Bigg[2s_{13}s_{15}\\
&+2s_{15}s_{35}+\frac{3}{2}s_{35}^2\Bigg]+\frac{s_{15}}{s_{34}s_{134}s_{345}}\left[-4s_{14}s_{15}-2s_{14}s_{45}-4s_{14}^2-8s_{15}^2-2s_{45}^2\right]\\
&+\frac{s_{15}s_{35}}{s_{45}^2s_{345}}\left[-4s_{14}-4s_{15}+4s_{35}\right]+\frac{s_{15}s_{35}}{s_{45}s_{135}s_{345}}\left[2s_{14}+s_{15}+\frac{1}{2}s_{35}\right]\\
&+\frac{s_{15}s_{35}^2}{s_{45}^2s_{345}^2}\left[4s_{14}+2s_{15}\right]+\frac{s_{15}}{s_{45}^2s_{145}}\Big[2s_{13}s_{15}+2s_{15}s_{34}+2s_{15}s_{35}\\
&-2s_{34}s_{35}-2s_{35}^2\Big]+\frac{s_{15}}{s_{45}^2}\left[2s_{15}+2s_{34}-6s_{35}\right]
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&+\frac{s_{15}}{s_{45}s_{134}s_{345}}\left[-6s_{14}s_{15}-2s_{14}s_{35}-2s_{14}^2-6s_{15}s_{35}-4s_{15}^2-8s_{35}^2\right]\\
&+\frac{s_{15}}{s_{45}s_{134}}\left[-s_{14}+\frac{5}{2}s_{15}+\frac{1}{2}s_{34}+\frac{15}{2}s_{35}\right]+\frac{s_{15}}{s_{45}s_{145}^2}\left[4s_{34}s_{35}+2s_{34}^2+2s_{35}^2\right]\\
&+\frac{s_{15}}{s_{45}s_{145}s_{135}}\left[\frac{3}{2}s_{15}s_{34}-\frac{1}{2}s_{15}s_{35}-s_{15}^2-\frac{3}{2}s_{34}s_{35}-\frac{3}{2}s_{34}^2-s_{35}^2\right]\\
&+\frac{s_{15}}{s_{45}s_{145}}\left[-2s_{15}+\frac{11}{2}s_{34}+4s_{35}\right]+\frac{s_{15}}{s_{45}s_{135}}\left[-2s_{14}-2s_{34}\right]\\
&+\frac{s_{15}}{s_{45}s_{345}}\left[-6s_{14}-6s_{15}-6s_{35}\right]+\frac{15s_{15}}{2s_{45}}+\frac{s_{15}^2}{s_{45}^2s_{145}^2}\Big[2s_{13}s_{34}+2s_{13}s_{35}\\
&+2s_{34}s_{35}+s_{34}^2+s_{35}^2\Big]+\frac{s_{45}}{s_{34}^2s_{345}}\left[-8s_{14}s_{15}+4s_{14}s_{45}-4s_{14}^2+4s_{15}s_{45}\right]\\
&+\frac{s_{45}^2}{s_{34}^2s_{345}^2}\left[4s_{13}s_{15}+4s_{14}s_{15}+2s_{14}^2+2s_{15}^2\right]+\frac{1}{s_{34}^2}\Big[-2s_{13}s_{15}+2s_{14}s_{15}\\
&+2s_{14}s_{35}-6s_{14}s_{45}+2s_{14}^2-2s_{15}s_{35}-2s_{15}s_{45}+s_{35}^2+s_{45}^2\Big]\\
&+\frac{1}{s_{34}s_{45}s_{134}}\Big[\frac{1}{2}s_{14}s_{15}s_{35}+3s_{14}s_{15}^2+s_{14}^2s_{15}+\frac{1}{2}s_{14}^2s_{35}+s_{14}^3+\frac{3}{2}s_{15}s_{35}^2
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{s_{34}s_{45}}\Big[4s_{13}s_{15}+16s_{14}s_{15}+11s_{14}s_{35}+7s_{14}^2\\
&+16s_{15}s_{35}+8s_{15}^2+\frac{9}{2}s_{35}^2\Big]+\frac{s_{45}}{s_{34}s_{135}s_{345}}\Big[2s_{14}s_{15}-6s_{14}s_{45}-6s_{14}^2\\
&+\frac{3}{2}s_{15}s_{45}-s_{15}^2-\frac{5}{2}s_{45}^2\Big]+\frac{s_{45}}{s_{34}s_{345}^2}\left[8s_{13}s_{15}+8s_{14}s_{15}+4s_{14}^2+4s_{15}^2\right]\\
&+\frac{1}{s_{34}s_{134}s_{145}}\Big[9s_{15}s_{35}s_{45}+\frac{9}{2}s_{15}s_{35}^2+5s_{15}s_{45}^2+6s_{15}^2s_{35}+\frac{11}{2}s_{15}^2s_{45}+\frac{7}{2}s_{15}^3\\
&+6s_{35}s_{45}^2+\frac{9}{2}s_{35}^2s_{45}+s_{35}^3+\frac{5}{2}s_{45}^3\Big]+\frac{1}{s_{34}s_{134}s_{135}}\Big[\frac{9}{2}s_{15}s_{35}s_{45}+3s_{15}s_{35}^2\\
&+\frac{3}{2}s_{15}s_{45}^2+\frac{5}{2}s_{15}^2s_{35}+\frac{3}{2}s_{15}^2s_{45}+s_{15}^3+\frac{9}{2}s_{35}s_{45}^2+6s_{35}^2s_{45}+\frac{5}{2}s_{35}^3+s_{45}^3\Big]\\
&+\frac{1}{2s_{34}s_{134}}\Big[-5s_{14}s_{15}+5s_{14}s_{35}-3s_{14}s_{45}-7s_{14}^2-22s_{15}s_{35}-19s_{15}s_{45}\\
&-35s_{15}^2-27s_{35}s_{45}-17s_{35}^2-13s_{45}^2\Big]\\
&+\frac{1}{s_{34}s_{145}s_{345}}\left[-6s_{13}s_{15}s_{45}-2s_{13}s_{15}^2+6s_{15}s_{45}^2+5s_{15}^2s_{45}+2s_{15}^3+4s_{45}^3\right]
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
&+\frac{1}{s_{34}s_{145}}\left[\frac{13}{2}s_{13}s_{15}+\frac{13}{2}s_{15}s_{35}+\frac{1}{2}s_{15}^2+7s_{35}s_{45}+3s_{35}^2-s_{45}^2\right]\\
&+\frac{1}{s_{34}s_{135}}\Big[+s_{14}s_{35}+7s_{14}s_{45}+4s_{14}^2+\frac{3}{2}s_{15}s_{35}+\frac{1}{2}s_{15}s_{45}+s_{15}^2+\frac{7}{2}s_{35}s_{45}\\
&+\frac{5}{2}s_{35}^2+\frac{11}{2}s_{45}^2\Big]+\frac{1}{s_{34}s_{345}}\Big[-8s_{13}s_{15}-14s_{14}s_{15}+7s_{14}s_{45}-8s_{14}^2\\
&+7s_{15}s_{45}-16s_{15}^2-\frac{9}{2}s_{45}^2\Big]+\frac{1}{2s_{34}}\left[23s_{14}-13s_{15}+5s_{35}+4s_{45}\right]\\
&+\frac{1}{2s_{45}s_{134}s_{145}}\Big[-3s_{15}s_{34}s_{35}-s_{15}s_{34}^2-3s_{15}s_{35}^2-3s_{15}^2s_{35}-3s_{15}^3-3s_{34}s_{35}^2\\
&-3s_{34}^2s_{35}-s_{34}^3-s_{35}^3\Big]+\frac{1}{s_{134}^2}\left[s_{15}^2+4s_{35}s_{45}+2s_{35}^2+2s_{45}^2\right]\\
&+\frac{1}{s_{134}s_{145}}\Big[\frac{3}{2}s_{15}s_{34}-3s_{15}s_{35}-6s_{15}s_{45}-\frac{9}{2}s_{15}^2+\frac{3}{2}s_{34}s_{45}+\frac{1}{2}s_{34}^2-3s_{35}s_{45}\\
&-\frac{3}{2}s_{35}^2-4s_{45}^2\Big]+\frac{1}{s_{134}s_{135}}\left[+\frac{3}{2}s_{15}s_{34}-\frac{9}{2}s_{15}s_{35}-\frac{3}{2}s_{15}^2-9s_{35}s_{45}-6s_{35}^2\right]\\
&+\frac{1}{2s_{134}s_{345}}\Big[-5s_{14}s_{35}-3s_{14}s_{45}+3s_{14}^2-22s_{15}s_{35}-12s_{15}s_{45}+12s_{15}^2
\end{aligned}
$$

Beautiful

$$
\begin{aligned}
&+\frac{1}{2s_{134}}\left[-3s_{14}+29s_{15}+18s_{35}+23s_{45}\right]\\
&+\frac{1}{2s_{145}s_{135}}\left[3s_{15}s_{35}-3s_{15}s_{45}-3s_{15}^2+9s_{35}s_{45}-2s_{45}^2\right]\\
&+\frac{1}{2s_{145}s_{345}}\left[-2s_{15}s_{35}+3s_{15}s_{45}+s_{15}^2-3s_{35}s_{45}+s_{35}^2+5s_{45}^2\right]\\
&+\frac{1}{s_{145}}\left[-4s_{15}-4s_{45}\right]+\frac{1}{s_{135}^2}\left[4s_{14}s_{45}+s_{14}^2+2s_{34}s_{45}+2s_{45}^2\right]\\
&+\frac{1}{s_{135}s_{345}}\left[2s_{14}s_{15}+7s_{14}s_{35}-6s_{14}s_{45}-4s_{14}^2-\frac{3}{2}s_{15}s_{35}+\frac{3}{2}s_{15}s_{45}-s_{15}^2\right.\\
&\left.+\frac{5}{2}s_{35}s_{45}-\frac{5}{2}s_{35}^2-\frac{5}{2}s_{45}^2\right]+\frac{1}{s_{135}}\left[2s_{14}-2s_{35}+7s_{45}\right]\\
&+\frac{1}{s_{345}^2}\left[6s_{13}s_{15}+8s_{14}s_{15}+3s_{14}^2+4s_{15}^2\right]\\
&\left.+\frac{1}{s_{345}}\left[\frac{7}{2}s_{14}+\frac{15}{2}s_{15}+3s_{35}-\frac{5}{2}s_{45}\right]+8\right\}. \qquad (34)
\end{aligned}
$$

Hmm...

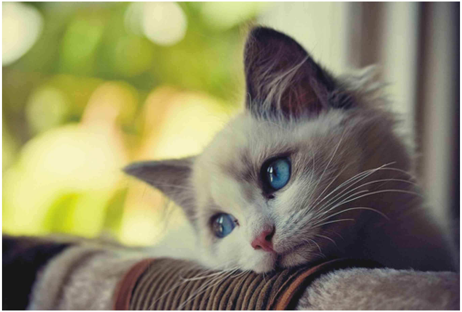
$$D_4^0(i_q, 2_g, j_g, k_g) \tag{35}$$

Will Mathematica save us?

By the way we've constructed the antenna partial fractioning is impossible.

- the gluon ordering in the antenna exhibits full line reversal and cyclic symmetries amongst the gluon triplet
- there is a subleading colour 'unphysical' limit (in this case between $(i_q, 2_g, k_g)$).

Singularly structure of D_4^0 is a total mess and cannot be decomposed analytically in a symmetric way that is analytically integrable.



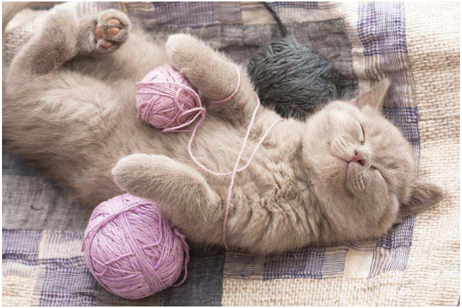
A new Approach?

- It turns out there exists a carefully constructed block of $X_3^0 \times X_3^0$ antenna which fully cancel the bad singularities we've introduced by using this antenna.
- We then use another set of X_0^4 to add back in the correct limit we wanted in this flavour changing configuration.

After much pain and suffering, the numerical integration is being performed and we should (hopefully) have real results in due course.

Conclusions

- Your subtraction term is highly dependent on the initial state configuration of your process.
- For processes with quarks in the final state we end up factorisating onto reduced matrix elements that don't share the same initial state configuration as the process we're considering ('flavour changing limits')
- For certain processes this proved to be a huge stumbling block for the antenna formalism but now everything seems to be under control.



Thanks for listening!