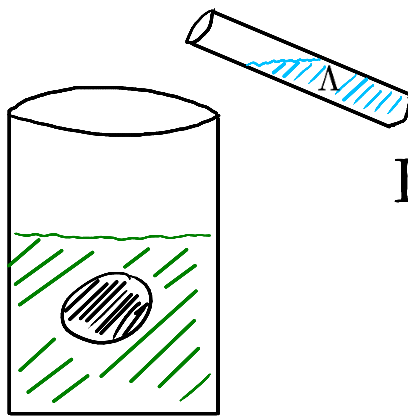
Black Hole Chemistry



Durham Student Seminar
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January 19, 2014

Work with Robert Mann and Miok Park
University of Waterloo

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Smarr Relations and Thermodynamics

First Law of Thermodynamics:

Electric potential

$$dM = T\,dS + \Phi\, dQ$$

Energy !

Smarr Relation:

$$M = 2(TS) + \Phi Q$$

Smarr 1973
PRL 30, 2

Let's look at the Reissner-Nordstrom black hole.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_1^2$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

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$$\Phi = \frac{Q}{r_h}$$

$$M = 2(TS) + \Phi Q = \frac{r_h}{2} - \frac{Q^2}{2r_h} + \frac{Q^2}{r_h}$$

Does the first law hold?

$$T = \frac{f'(r_h)}{4\pi} = \frac{1}{4\pi}\left(\frac{1}{r_h} - \frac{Q^2}{r_h^3}\right) \qquad S = \frac{A}{4} = \pi r_h^2$$

$$\Phi = \frac{Q}{r_h}$$

$$M = 2(TS) + \Phi Q = \frac{r_h}{2} - \frac{Q^2}{2r_h} + \frac{Q^2}{r_h}$$

$$\left(\frac{\partial M}{\partial r_h}\right)_Q = \frac{1}{2} - \frac{Q^2}{r_h^2} = T\frac{\partial S}{\partial r_h} \qquad \left(\frac{\partial M}{\partial Q}\right)_{r_h} = \frac{Q}{r_h} = \Phi$$

Suggested refs: Poisson GR Text, 1402.1443

How do we know we had the mass correct?

ADM Mass

$$\mathcal{M}_{ADM} \equiv -\frac{1}{8\pi}\int_{\infty} d^2\theta\sqrt{\sigma}(k-k_0) \qquad k \equiv \sigma^{AB}k_{AB}$$

Brown-York Quasilocal Mass

$$\mathcal{M}_{BY} \equiv \int ds \cdot u^{\alpha}\Psi^{\beta}T^{(BY)}_{\alpha\beta} \qquad T^{(BY)}_{\alpha\beta} = T_{\alpha\beta} + \frac{2}{\sqrt{-\gamma}}\frac{\delta S_{ct}}{\delta\gamma^{\alpha\beta}}$$

Komar Mass

$$\mathcal{M}_K \equiv -\frac{1}{8\pi}\int_{\infty} dS_{\alpha\beta}\nabla^{\alpha}\xi^{\beta}_t \qquad dS_{\alpha\beta} = -2n_{[\alpha}r_{\beta]}\sqrt{\sigma}d^2\theta$$

Smarr Relations and Thermodynamics

What happens when we add a cosmological constant?

Smarr Relations and Thermodynamics

What happens when we add a cosmological constant?

1. The metric requires another lengthscale (l).

2. The previous Smarr relation may fail to hold.

Smarr Relations and Thermodynamics

First Law of Thermodynamics:

$$dM = T\,dS + V\,dP + \Phi\,dQ$$

Enthalpy!

Smarr Relation:

$$M = 2(TS - PV) + \Phi Q$$

$$(D-3)M = (D-2)TS - 2PV + (D-3)\Phi Q$$

Kastor, Ray, Traschen
arXiv:0904.2765

Let's look at the Reissner-Nordstrom AdS black hole.

$$ds^2 = -\frac{r^2}{l^2} f(r) dt^2 + \frac{l^2 dr^2}{r^2 f(r)} + r^2 d\Omega_k^2$$

$$f(r) = 1 + k\frac{l^2}{r^2} - \frac{2Ml^2}{r^3} + \frac{Q^2 l^2}{r^4}$$

Let's look at the Reissner-Nordstrom AdS black hole.

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$$T = \frac{r_h^2}{l^2}\frac{f'(r_h)}{4\pi} = \frac{3r_h}{4\pi l^2} + \frac{k}{4\pi r_h} - \frac{Q^2}{4\pi r_h^3} \qquad S = \frac{A}{4} = \frac{\omega_{k,2} r_h^2}{4}$$

$$\Phi = \frac{Q}{r_h}$$

$$M_{k=1} \stackrel{?}{=} 2(TS) + \Phi Q = \frac{3r_h^2}{2l^2} + \frac{r_h}{2} - \frac{Q^2}{4r_h} + \frac{Q^2}{r_h} \neq \frac{r_h^3}{2l^2} + \frac{r_h}{2} + \frac{Q^2}{2r_h}$$

Maybe try adding the pressure term...

$$P = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi l^2} \qquad V = \frac{4\pi r_h^3}{3}$$

$$T = \frac{r_h^2}{l^2}\frac{f'(r_h)}{4\pi} = \frac{3r_h}{4\pi l^2} + \frac{k}{4\pi r_h} - \frac{Q^2}{4\pi r_h^3} \qquad S = \frac{A}{4} = \frac{\omega_{k,2} r_h^2}{4}$$

$$\Phi = \frac{Q}{r_h}$$

$$M_{k=1} \stackrel{?}{=} 2(TS) - 2PV + \Phi Q = \frac{3r_h^2}{2l^2} + \frac{r_h}{2} - \frac{r_h^3}{l^2} - \frac{Q^2}{4r_h} + \frac{Q^2}{r_h}$$

Why are we doing this, anyway?

Smarr Relations and Thermodynamics

We want to compute thermodynamics and (eventually) universality classes.

Smarr gives the definition of pressure and TD volume:

$$P = \frac{-1}{8\pi} \Lambda \qquad V_{TD} = \frac{4}{3} \pi r_h^3$$

(for Reissner-Nordström AdS Black Holes in 3+1 dimensions)

We can then find the equation of state:

$$P_{(V,T)} = \frac{T}{2r_h} - \frac{1}{8\pi r_h^2} + \frac{Q^2}{8\pi r_h^4}$$

and identify Gibbs free energy and Helmholtz free energy.

Critical Exponents

$$\Delta\rho(T) = \rho_{liq} - \rho_{gas} \Big|_{P=P_c}$$

α : $C_V \sim |T-T_c|^{-\alpha}$

γ : $K_T \equiv \frac{-1}{v} \frac{\partial v}{\partial P}\Big|_T \sim |T-T_c|^{-\gamma}$

δ : $|P-P_c| \sim |v-v_c|^{\delta}$

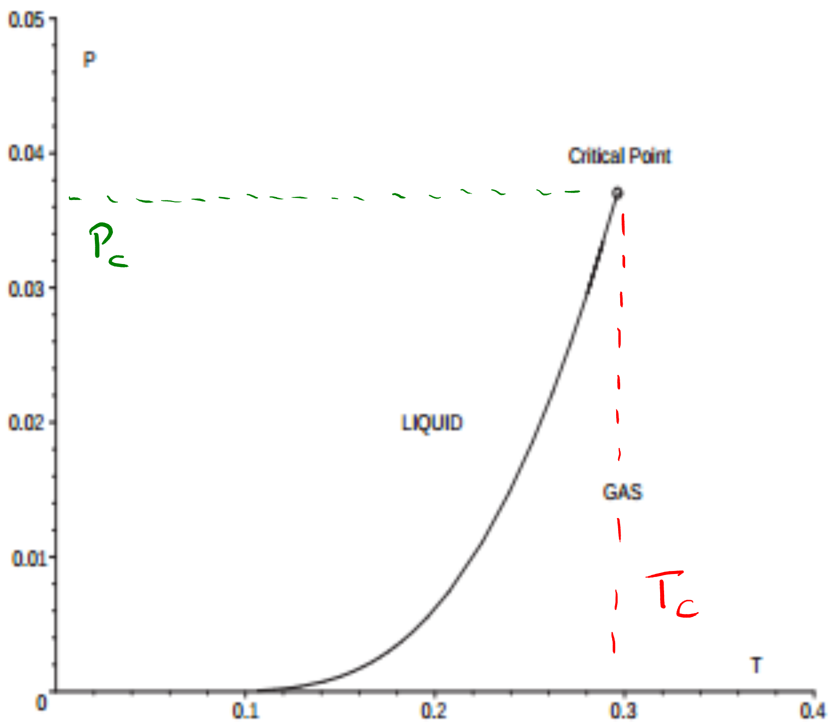
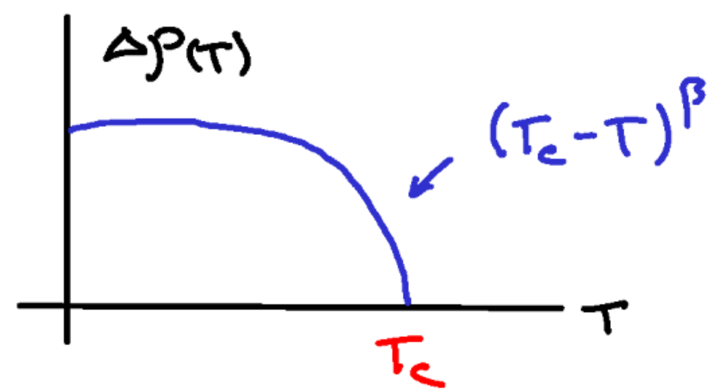


FIG. 5. **Coexistence line of liquid–gas phase.** Fig. displays the coexistence line of liquid and gas phases of the Van der Waals fluid in (P, T)-plane. The critical point is highlighted by a small circle at the end of the coexistence curve.

order parameter



Smarr Relations and Thermodynamics

The 3+1 Dimensional Reissner-Nordström AdS Black Hole:

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4} \qquad v \equiv 2 l_P^2 r_+$$

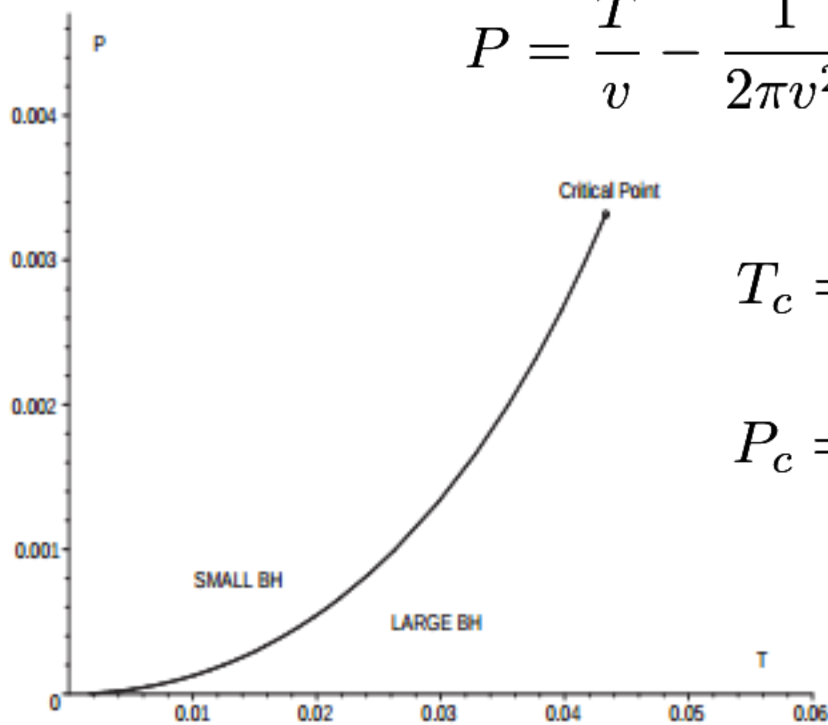


FIG. 9. **Coexistence line of charged AdS black hole.** Fig. displays the coexistence line of small–large black hole phase transition of the charged AdS black hole system in (P, T)-plane. The critical point is highlighted by a small circle at the end of the coexistence line.

$$T_c = \frac{\sqrt{6}}{18\pi Q}$$

$$P_c = \frac{1}{96\pi Q^2}$$

α	0
β	1/2
γ	1
δ	3

Kubiznak, Mann
arXiv:1205.0559

Smarr Relations and Thermodynamics

The Van der Waals Fluid: $\left(P + \frac{a}{v^2}\right)(v - b^2) = kT$

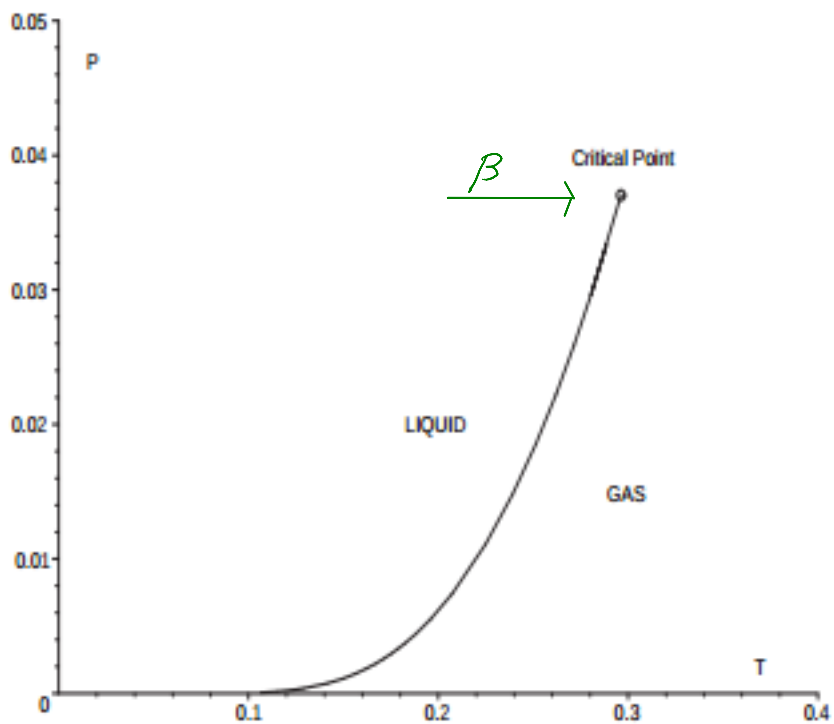


FIG. 5. **Coexistence line of liquid–gas phase.** Fig. displays the coexistence line of liquid and gas phases of the Van der Waals fluid in (P, T)-plane. The critical point is highlighted by a small circle at the end of the coexistence curve.

α	0
β	$\frac{1}{2}$
γ	1
δ	3

Lifshitz-Symmetric Spacetimes

Anti de Sitter Spacetime

$$ds^2 = -\left(\frac{r^2}{L^2}\right)dt^2 + L^2\frac{dr^2}{r^2} + r^2 d\Omega^2$$

Lifshitz ~ Introduce an Anisotropy in Time

$$t \to \lambda^z t \quad \text{while} \quad x \to \lambda x$$

$$ds^2 = -\left(\frac{r^{2z}}{L^{2z}}\right)dt^2 + L^2\frac{dr^2}{r^2} + r^2 d\Omega^2$$

~ Conjectured dual to certain condensed matter systems

Lifshitz-Symmetric Spacetimes

For example: $z = 3, D = 3$

$$ds^2 = -\left(\frac{r}{l}\right)^{2z} f(r)dt^2 + \frac{l^2 dr^2}{f(r)r^2} + r^2 d\Omega_k^2 \qquad f(r) = 1 - \frac{ml^2}{r^2}$$

$$\mathcal{S} = \int d^D x\sqrt{-g}\left(\frac{1}{\kappa}\left[R - 2\Lambda\right] + aR^2 + bR^{\mu\nu}R_{\mu\nu} + c\left[R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right]\right)$$

$$a = -\frac{3l^2}{4\kappa}, \quad b = \frac{2l^2}{\kappa}, \quad c = 0, \quad \Lambda = -\frac{13}{2l^2}$$

Lifshitz-Symmetric Spacetimes

For example: $z = 3, D = 3$

$$ds^2 = -\left(\frac{r}{l}\right)^{2z} f(r)dt^2 + \frac{l^2 dr^2}{f(r)r^2} + r^2 d\Omega_k^2 \qquad f(r) = 1 - \frac{ml^2}{r^2}$$

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$$a = -\frac{3l^2}{4\kappa}, \quad b = \frac{2l^2}{\kappa}, \quad c = 0, \quad \Lambda = -\frac{13}{2l^2}$$

$$T = \frac{r_h^4}{l^4}\frac{f'(r_h)}{4\pi} = \frac{r_h^3}{2\pi l^4} \qquad S = 2\pi r_h$$

$$(D-3)M = (D-2)TS - 2PV$$

We don't know M or V. This is a problem!

Smarr Relations and Thermodynamics

Smarr obeys Eulerian Scaling:

$$(D-3)M = (D-2)TS - 2PV$$

Eulerian Scaling:

$$f(\alpha^p x, \alpha^q y) = \alpha^r f(x,y)$$

implies

$$r f(x,y) = p\frac{\partial f}{\partial x}x + q\frac{\partial f}{\partial y}y$$

(take $\frac{\partial}{\partial \alpha}$: $r\alpha^{r-1} f(x,y) = p\alpha^{p-1} x \frac{\partial f}{\partial x} + q\alpha^{q-1} y \frac{\partial f}{\partial y}$)

then let $\alpha = 1$

Smarr Relations and Thermodynamics

Smarr obeys Eulerian Scaling:

$$(D-3)M = (D-2)TS - 2PV$$

through the first law:

$$dM = TdS + VdP \rightsquigarrow \frac{\partial M}{\partial S} = T \; , \quad \frac{\partial M}{\partial P} = V$$

We have $M(S, P) \sim L^{D-3}$ * conjecture

$S \sim L^{D-2}$ (Wald / Iyer ~> BH entropy ~ area)

$P \sim L^{-2}$ (Kastor, Traschen)

Smarr Relations and Thermodynamics

We obtain a nonlinear set of equations:

Equations:

① Smarr $(D-3)M = (D-2)TS - 2PV$

② 1^{st} Law $dM = TdS + VdP$

$\hookrightarrow \frac{\partial M}{\partial r_+} = T\frac{\partial S}{\partial r_+}$; $\frac{\partial M}{\partial \ell} = V\frac{\partial P}{\partial \ell}$

③ Eulerian $M \sim L^{D-3} = \ell^{\alpha} r_+^{\beta}$;

$\alpha + \beta = D - 3$

Smarr Relations and Thermodynamics

We obtain a nonlinear set of equations:

Equations:

1. Smarr $$(D-3)M = (D-2)TS - 2PV$$

2. 1st Law $$dM = TdS + VdP$$

   $$\hookrightarrow \quad \frac{\partial M}{\partial r_+} = T\frac{\partial S}{\partial r_+} \; ; \quad \frac{\partial M}{\partial \ell} = V\frac{\partial P}{\partial \ell} + T\frac{\partial S}{\partial \ell}$$

3. Eulerian $$M \sim L^{D-3} = \ell^{\alpha} r_+^{\beta} \; ;$$

   $$\alpha + \beta = D - 3$$

Smarr Relations and Thermodynamics

Is there an exact solution?

Smarr Relations and Thermodynamics

Is there an exact solution?

Yes!

$$M = \sum_{ij} M_0^{(ij)} r_h^{\beta_{ij}} l^{\alpha_{ij}}$$

$$TS = \sum_{ij} T_0^{(i)} S_0^{(j)} r_h^{\beta_{ij}} l^{\alpha_{ij}} = \sum_i T^{(i)} \sum_j S^{(j)}$$

$$V = \sum_{ij} V_0^{(ij)} r_h^{\hat{\beta}_{ij}} l^{\hat{\alpha}_{ij}}$$

$$M_0^{(ij)} = \frac{T^{(i)}}{\beta_{ij} l^{\alpha_{ij}} \cdot r_h^{\beta_{ij}-1}} \frac{\partial S^{(j)}}{\partial r_h}$$

$$V^{(ij)} = \frac{T^{(i)}}{2P\beta_{ij}} \left[(D-2)\beta_{ij} S^{(j)} - (D-3) r_h \frac{\partial S^{(j)}}{\partial r_h} \right]$$

A Lifshitz Smarr Relation

Can we define a charge?

A Lifshitz Smarr Relation

Can we define a charge?

Yes. It's a little more complicated, but proceeds in the same manner.

$$M = M_0 + \sum_{i=1}^{N} M_i q_i^{\mathfrak{a}_i} \qquad (D-3)M_i = (D-3)T_i S - 2V_i P + (D-3)\frac{\Phi_i Q_i}{q_i}$$

$$M_i = \frac{(D-2)}{(D-3)(1-a_i)-\alpha_i} T_i S$$

A Lifshitz Smarr Relation

Can we define a mass via other methods?

A Lifshitz Smarr Relation

Can we define a mass via other methods?

In some cases, yes!

e.g. Gim, Kim, Li; arXiv:1403.4704

z=3, D=3

$$ds^2 = -\left(\frac{r^2}{l^2}\right)^z \left(1 - \frac{ml^2}{r^2}\right) dt^2 + \frac{1}{\frac{r^2}{l^2}\left(1 - \frac{ml^2}{r^2}\right)} dr^2 + r^2 d\phi^2$$

$$\mathcal{S} = \int d^3x \sqrt{-g} \left[\frac{1}{\kappa} (R - 2\Lambda) + \mathcal{F}(R, R_{\mu\nu}, R_{\mu\nu\alpha\beta}) \right]$$

Quasilocal background subtraction $\rightsquigarrow M = \frac{r_+^4}{4G\ell^4}$

(Brown-York mass with thermodynamically guessed-at counterterms)

A Lifshitz Smarr Relation

Is the thermodynamic mass consistent?

A Lifshitz Smarr Relation

Is the thermodynamic mass consistent?

For the unambiguous cases, yes.

Quasilocal background subtraction $\rightsquigarrow$ $M = \dfrac{r_+^4}{4G\ell^4}$

$$S = \frac{2\pi r_+}{G}, \quad T = \frac{r_+^3}{2\pi \ell^4}, \quad P = \frac{\Lambda}{8\pi G}$$

↳ then

$$M = \frac{r_+^4}{4Gl^4} \qquad V = \frac{8\pi r_+^4}{13l^2}$$

A Lifshitz Smarr Relation

Are our Smarr relation assumptions in agreement?

$$M = \frac{r_+^4}{4Gl^4} \qquad V = \frac{8\pi r_+^4}{13l^2}$$

$$0 = TS - 2VP$$

* prefactors chosen to obey Eulerian scaling; $M \sim L^0$, $P \sim L^{-2}$, $S \sim L^1$

A Lifshitz Smarr Relation

Other spacetimes for which our method agrees:

Reissner-Nordström AdS ($z=1$, $D=4$) ADM ✓

Non-rotating BTZ ($z=1$, $D=3$) ADM ✓

AdS-Taub-NUT ($z=1$, $D=4$) ADM ✓

5D Lifshitz with Higher Curvature terms ($z=2$, $D=5$) BY + TD counterterms ✓

Lifshitz with Einstein-Dilaton-Maxwell ($z=D$) Wald formula ✓
Komar integral ✓

A Lifshitz Smarr Relation

Dependent lenthscales yield ambiguous results.

4D Lifshitz with Maxwell Charge (z=4, D=4); Pang, arXiv:0901.2777

$$f(r) = 1 + \frac{kl^2}{10r^2} - \frac{3k^2l^4}{400r^4} - \frac{q^2l^2}{2r^4}$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}H^2 - \frac{m^2}{2}B^2 - \frac{1}{4}F^2\right]$$

$$T = -\frac{kr_h^2}{20\pi l^3} + \frac{3k^2}{400\pi l} + \frac{q^2}{2\pi l^3}$$

$$S = \frac{A}{4} = \frac{\omega_{k,2}r_h^2}{4}$$

$$Q = \frac{1}{4\pi}\int_{\mathcal{H}} \star F = \frac{q\omega_{k,2}}{2\pi}$$

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The problem here: horizon radius depends on the cosmological lengthscale when q = 0. We have a number of ways to resolve this.

$$f(r) = 1 + \frac{kl^2}{10r^2} - \frac{3k^2l^4}{400r^4} - \frac{q^2l^2}{2r^4}$$

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$$Q = \frac{1}{4\pi}\int_{\mathcal{H}} \star F = \frac{q\omega_{k,2}}{2\pi}$$

Create a fictitious mass in the metric function:

$$f(r) = 1 + \frac{kl^2}{10r^2} - \frac{3k^2l^4}{400r^4} + \tilde{m}\frac{l^p}{r^p}$$

$$f(r) = 1 + \frac{kl^2}{10r^2} - \frac{3k^2l^4}{400r^4} + \tilde{m}\frac{l^p}{r^p}$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}H^2 - \frac{m^2}{2}B^2 - \frac{1}{4}F^2\right]$$

$$T = -\frac{kr_h^2}{20\pi l^3} + \frac{3k^2}{400\pi l} + \frac{q^2}{2\pi l^3}$$

$$S = \frac{A}{4} = \frac{\omega_{k,2}r_h^2}{4}$$

$$Q = \frac{1}{4\pi}\int_{\mathcal{H}} \star F = \frac{q\omega_{k,2}}{2\pi}$$

For p = (D+z-2), we obtain results that agree with a Wald formula:

$$M = 0$$

$$V = \frac{\pi l^3}{6000}$$

A Lifshitz Smarr Relation

Why does this happen?

$$dM = TdS + VdP$$

$$\left(\frac{\partial M}{\partial S}\right)_P = T \qquad \left(\frac{\partial M}{\partial P}\right)_S = V$$

If we have $r_h = r_h(l)$ then these quantities are not independent.

We have a couple of options.

A Lifshitz Smarr Relation

We have a couple of options.

1. Manually separate the quantities in the metric.

2. Re-think how the action contributes new lengthscales.

A Lifshitz Smarr Relation

We have a couple of options.

1. Manually separate the quantities in the metric.

$$f(r) \to f(r) + \tilde{m}\frac{l^p}{r^p}$$

$p = (D + z - 2)$ works in all cases we can check!

A Lifshitz Smarr Relation

We have a couple of options.

2. Re-think how the action contributes new lengthscales.

$$\mathcal{S} = \int d^D x \sqrt{-g}\,[R - 2\Lambda + \alpha f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) + \beta F_{\mu\nu}F^{\mu\nu}]$$

lengthscale r_h lengthscale ℓ lengthscale λ lengthscale Q

$\rightsquigarrow$ can generate arbitrary Smarr

$$(D-3)M = (D-2)TS - 2PV + (D-3)\Phi_a Q + (\cdots)\Phi_\lambda \lambda$$

A Lifshitz Smarr Relation

We have a couple of options.

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$$\mathcal{S} = \int d^D x \sqrt{-g}\,[R - 2\Lambda + \alpha f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) + \beta F_{\mu\nu}F^{\mu\nu}]$$

lengthscale r_h

lengthscale ℓ

lengthscale λ

lengthscale Q

$\rightsquigarrow$ if $r_h = r_h(\ell)$, what varies? Maybe r_h is fundamental :

$$dM = TdS + V\cancelto{0}{dP} \qquad \text{and} \qquad M = \left(\int \frac{\partial S}{\partial r_h}\, dr_h\right) T$$

A Lifshitz Smarr Relation

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2. Re-think how the action contributes new lengthscales.

$$\mathcal{S} = \int d^D x \sqrt{-g}\,[R - 2\Lambda + \alpha f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) + \beta F_{\mu\nu}F^{\mu\nu}]$$

lengthscale r_h

lengthscale ℓ

lengthscale λ

lengthscale Q

$\rightsquigarrow$ if $r_h = r_h(\ell)$, what varies? Maybe ℓ is fundamental:

$\cancel{dM} = TdS + VdP$ and $M = 0$

0

A Lifshitz Smarr Relation

We have a couple of options.

2. Re-think how the action contributes new lengthscales.

$$\mathcal{S} = \int d^D x \sqrt{-g}\left[R - 2\Lambda + \alpha f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) + \beta F_{\mu\nu}F^{\mu\nu}\right]$$

lengthscale r_h

lengthscale ℓ

lengthscale λ

lengthscale Q

$\rightsquigarrow$ if $r_h = r_h(\ell)$, what varies? Maybe we choose pieces of each!

$\rightsquigarrow$ equivalent to fictitious mass approach.

A Lifshitz Smarr Relation

How come people use other Smarr relations?

A simplification occurs for planar black holes. In some cases:

$$[T] = \frac{r_h^z}{l^{z+1}}$$ while the two lengthscales are independent.

Via our method, the Smarr relation will simplify to

$$(D + z - 2)M = (D - 2)TS$$

A Lifshitz Smarr Relation

Via our method, the Smarr relation will simplify to

$$(D + z - 2)M = (D - 2)TS$$

This is analogous to pretending pressure does not contribute (which is why this appeared for k = 0 black holes before 2009). In that case there is only one lengthscale, the horizon radius.

Extension to Numerical Solutions

This method works particularly well for numerical solutions.

$$TS = \sum_{ij} T_0^{(i)} S_0^{(j)} r_h^{\beta_{ij}} l^{\alpha_{ij}} = \sum_i T^{(i)} \sum_j S^{(j)}$$

If both series independently converge, do thermodynamic mass and volume converge? Typically entropy is finite and known, so we require:

$$T = \sum_i T^{(i)} \quad \text{absolutely convergent}$$

Conclusion

Anisotropy in Time ~ mass becomes a tricky concept

Lifshitz Smarr Relation ~ gives enough
information to obtain a TD
mass and volume

Next...

Is a Maxwell field necessary for a finite-T critical point?

What is the universality class?

Attempt with numerical solutions!

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Questions?